Form & Shape Analysis I

Multivariate Morphometrics, Thin-Plate Spines & Geometric Morphometrics

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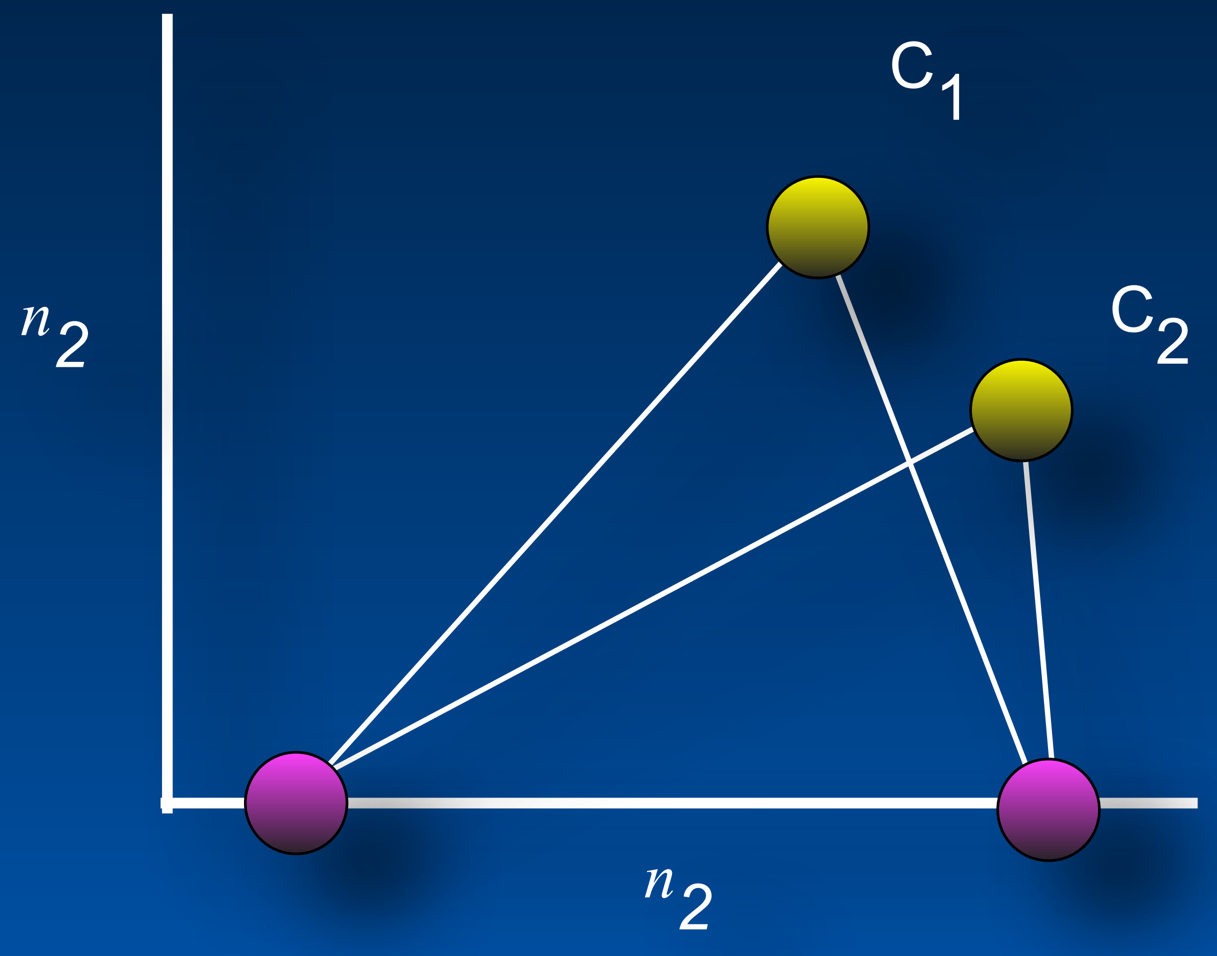
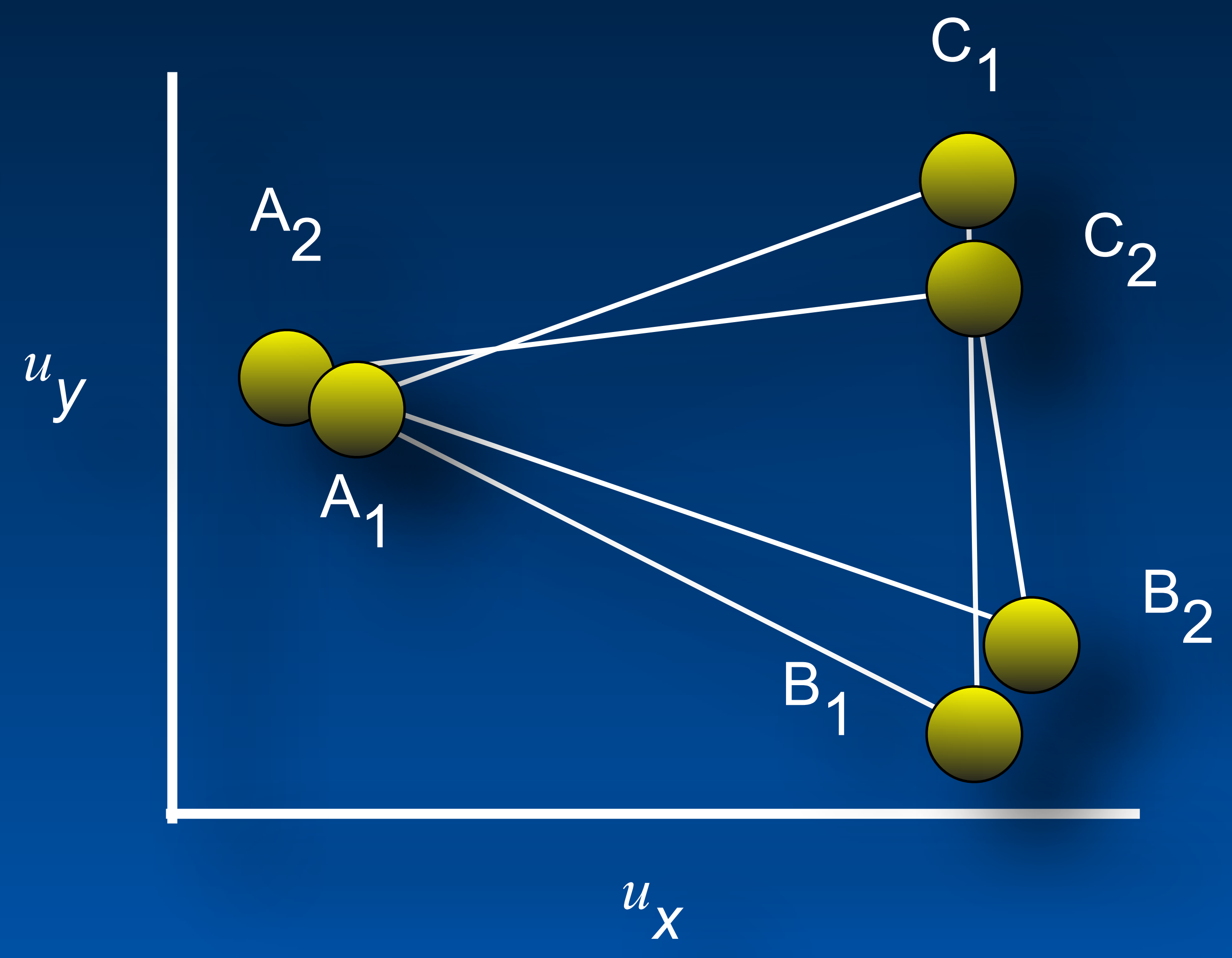
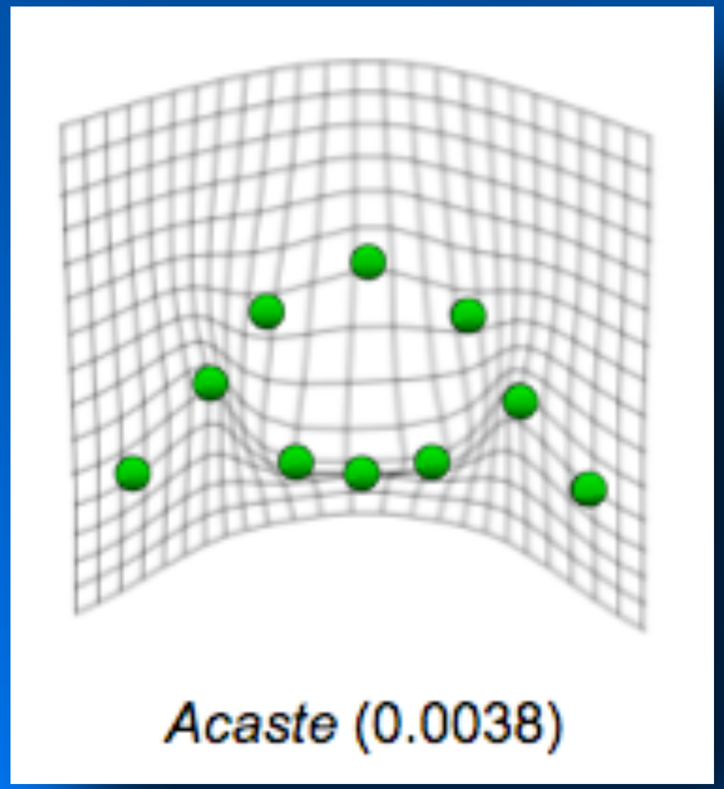
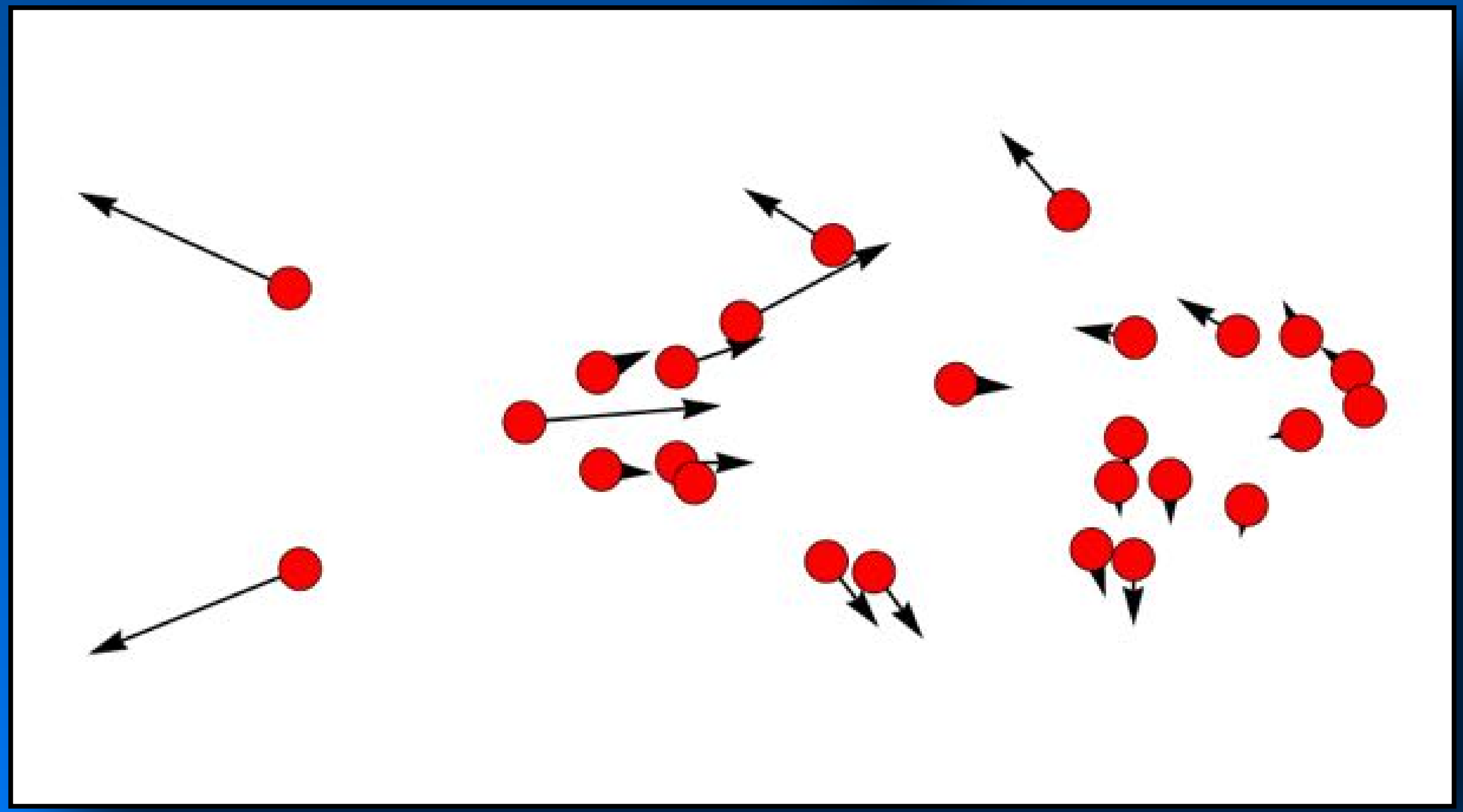
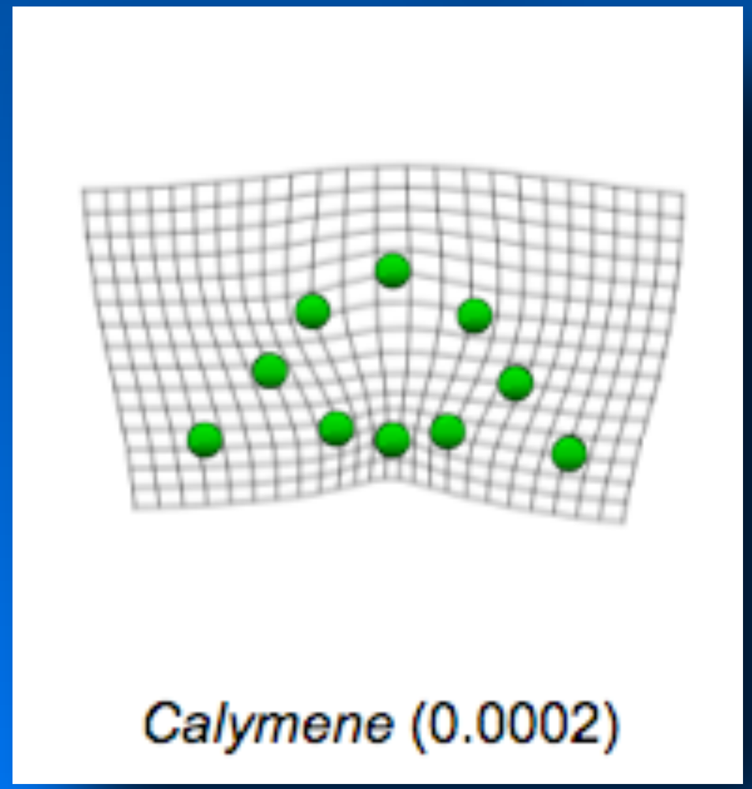








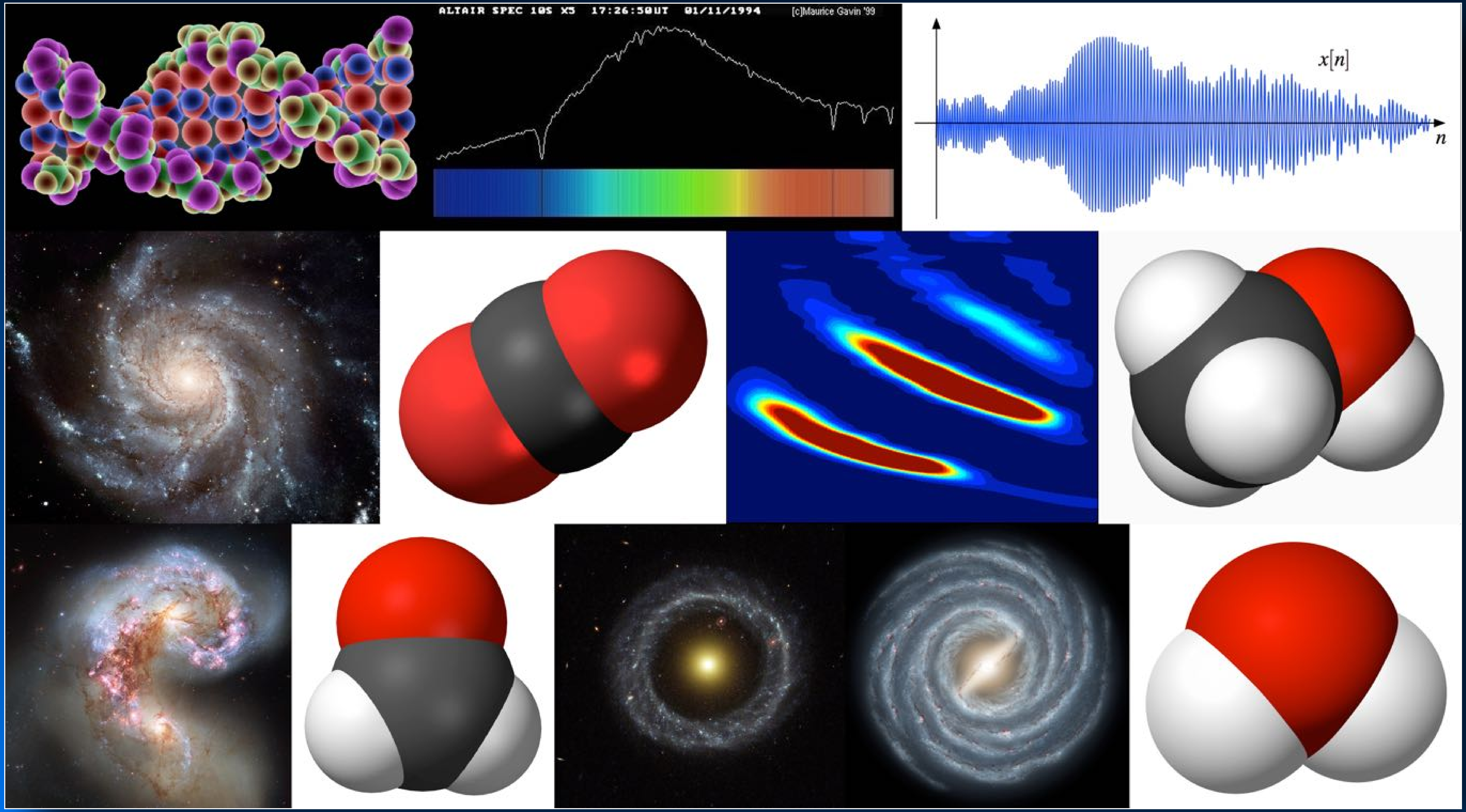


Form, Size & Shape

What is Morphology?

Form, Size & Shape

What is Morphology?



Description & Analysis of Morphology

Analysis: Morphometrics

Central tendency (mean)?
Modal Shape(s)?
Distribution of modes?
Continuous or discontinuous variation?

Covariance with environment?
Covariance with ecology?
Covariance with geography?
Covariance with genotype?

Description & Analysis of Morphology

Morphometrics

Zoological Systematics, 42(1): 4–33 (January 2017), DOI: 10.11865/zs.201702

REVIEW

Morphometrics: History, development, methods and prospects

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Abstract Morphometrics has been pursued by graphical and computational means since the European Renaissance, drawing on core geometric principles first discovered in China and Classical Greece. Through the late 1800s, two distinct approaches to such analyses were pursued: a deformationist approach, epitomized by D'Arcy Thompson's graphical trans-formation grids and the statistical approach popularized by Francis Galton, Karl Pearson, and Julian Huxley in which Cartesian spaces were employed to summarize patterns of variation in size and/or shape variables. Unification of these approaches was an oft-stated goal throughout the 20th century, but proved elusive until the mid-1980s when David Kendall, Fred Bookstein, and Colin Goodall proposed a radically new way of understanding form — as the locations of configurations of landmarks on the surfaces of a nested series of hyperdimensional manifolds. Once this new mathematics of form was understood development of basic concepts, procedures, graphical tools, and statistical tests followed quickly such that the core of the long-hoped for synthesis took less than a decade to achieve. The result — geometric morphometrics — continues to develop into an ever-more extensive toolkit that can be used by researchers to describe and understand a wide range of problems involving the characterization of morphological similarities and differences in all of their many and varied contexts. In particular, the new approaches involving the direct analysis of image pixels and new tools such as machine learning and artificial intelligence are set to reinvigorate (and possibly to revolutionize) the field once again.

Key words Morphometrics, form, size, shape, biology, geometry.

1 Introduction

Morphology is a critical source of information throughout the natural sciences, materials sciences, and engineering. Across all these fields an object's form consists of two primary components: size and shape. Size is the physical scale of an object, usually determined by comparing one or more of its spatial dimensions (e.g., diameter, height, length, width, perimeter, area, volume, mass) to a reference which serves as a basis for measurement. Shape is also usually conceptualized via comparison to some reference (e.g., circle, triangle, mean distance, mean point configuration) and has been defined operationally as that component of form which remains after differences in size, position and rotation between two or more objects have been discarded (Kendall, 1977). Morphometrics, then, is the quantitative analysis of form and covariances with form (Bookstein, 1991). While this domain overlaps strongly with that of geometry, to date morphometrics has been pursued in descriptive biological contexts of far more restricted scope than those of geometry, biometry and/or spatial analysis, despite the fact that all these fields share a common origin.

The Chinese philosopher Mozi (470-390 BC) authored the earliest known description of a mathematical point which he defined as the part of a line which cannot be divided into smaller parts (Needham, 1959). Mozi, like Plato and Euclid who established geometry as the foundation of mathematics in western Europe, noted that a line segment is a one-dimensional figure joining two mathematical points in space and that the length of a line segment records the separation between its defining endpoints. These concepts of point and separation are fundamental to morphometrics where point

Special Issue: Geometric morphometrics: Current shape and future directions
Received 6 October 2016, accepted 6 January 2017
Executive editor: Fuqiang Chen; guess editor: Ming Bai

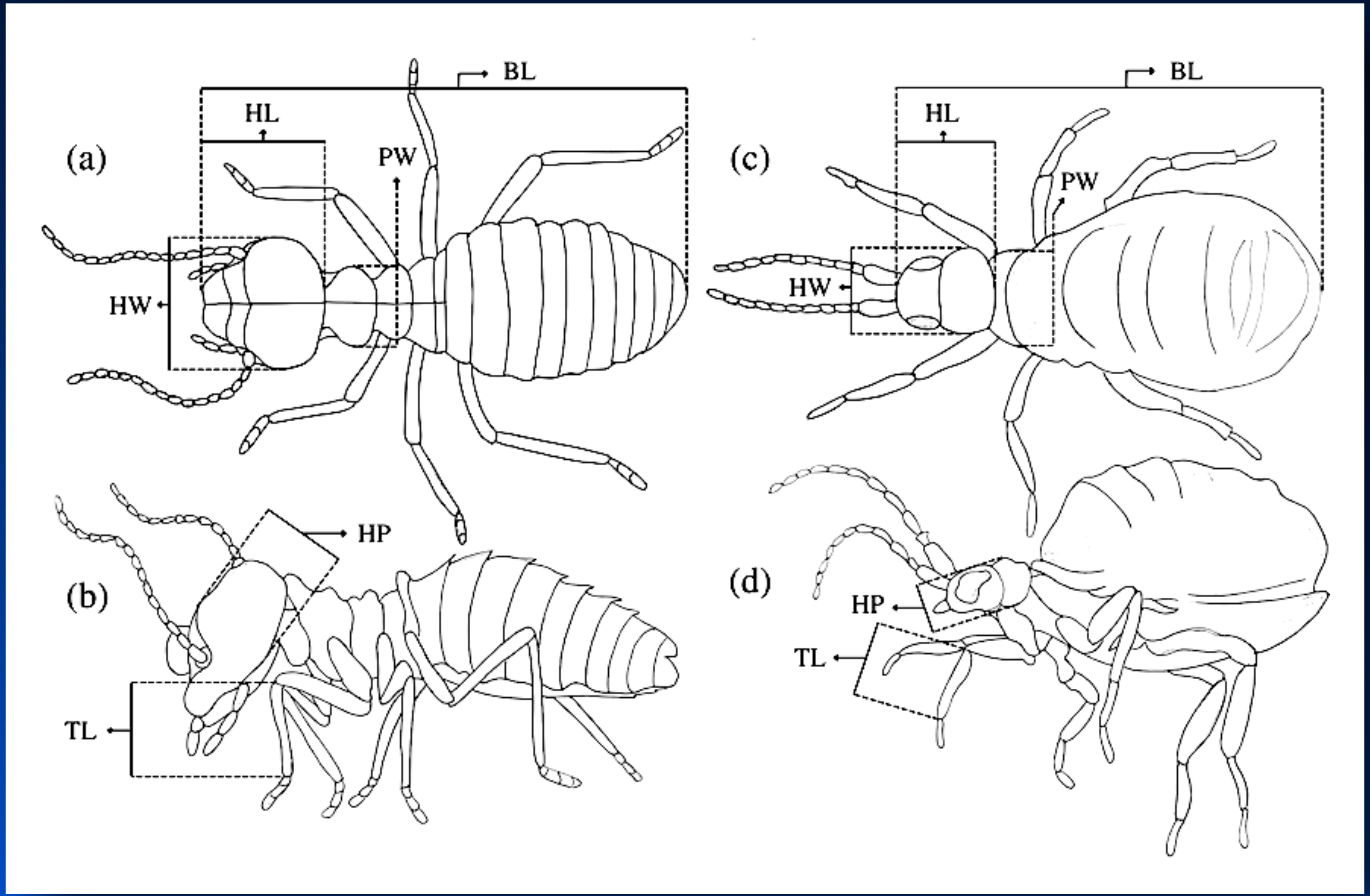
MacLeod, N., 2017, Morphometrics: history, development methods and prospects: Zoological Systematics, v. 42, p. 4–33, doi:10.11865/zs.201702.

Topics

- Traditional approaches to morphometric data collection
  - Linear Distances
  - Landmarks
  - Boundary Outlines
- The geometric morphometric synthesis
- Example analyses
- Post-synthesis developments
- Future directions

Form, Size & Shape

Linear Distances



Set of corresponding linear distances used to compare the forms of termite worker morphologies (*Constrictotermes cyphergaster*) and *Corotoca* beetles that inhabit termite colonies.

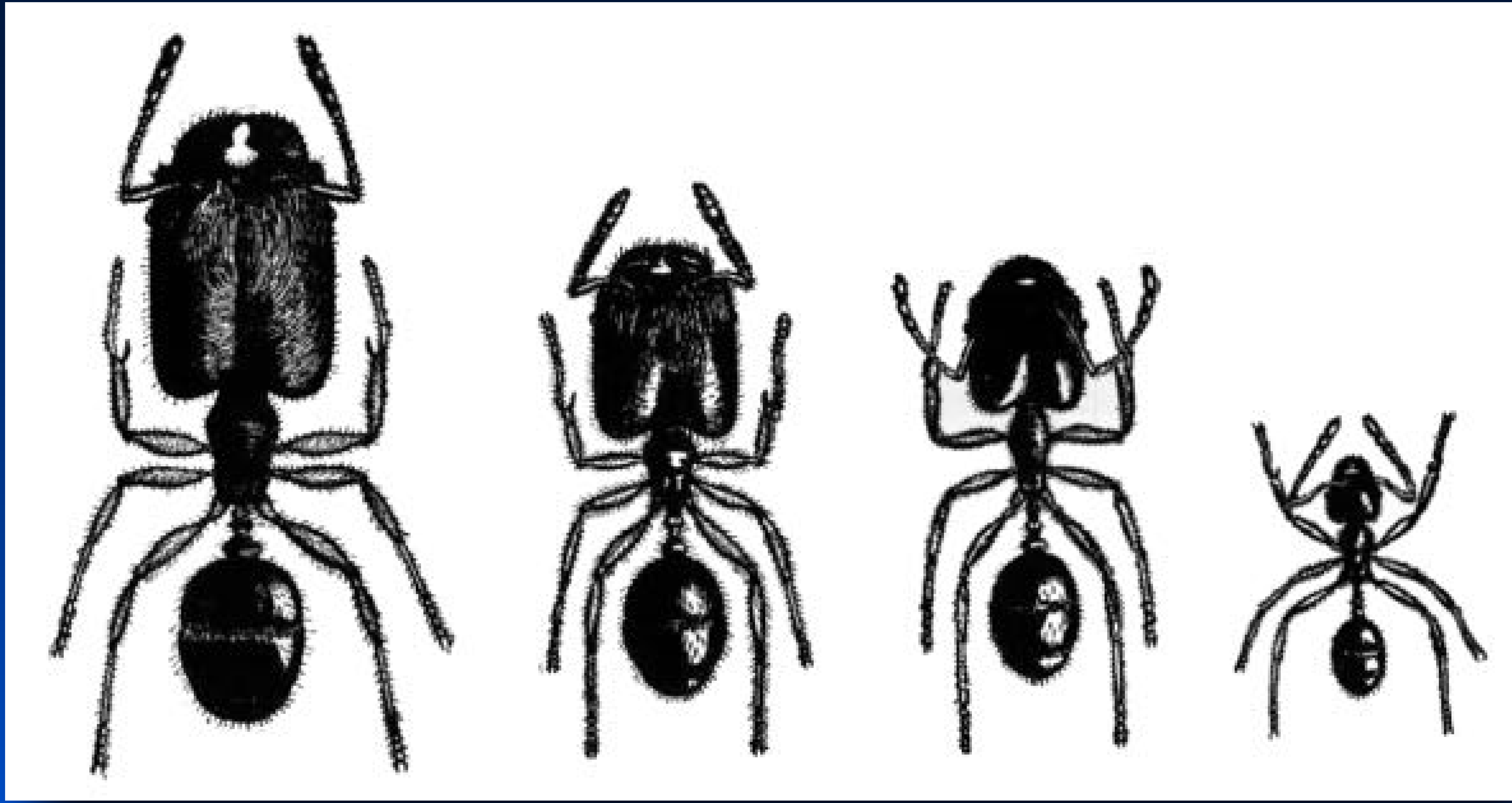
- In morphometrics, linear distance data refers to sets of straight-line distance measures between corresponding parts of the body or body part (e.g., body length, body width, leg length, antenna length).
- In essence, such data quantify the size of the body and/or distribution of sizes of the constituent body parts.
- Such characterizations represent metric derivations of non-metric, qualitative taxonomic descriptions (e.g., large, medium, small).

Image from Lira et al. (2022)

Scalar Morphometrics

Allometry

Julian Huxley
(1887 – 1975)



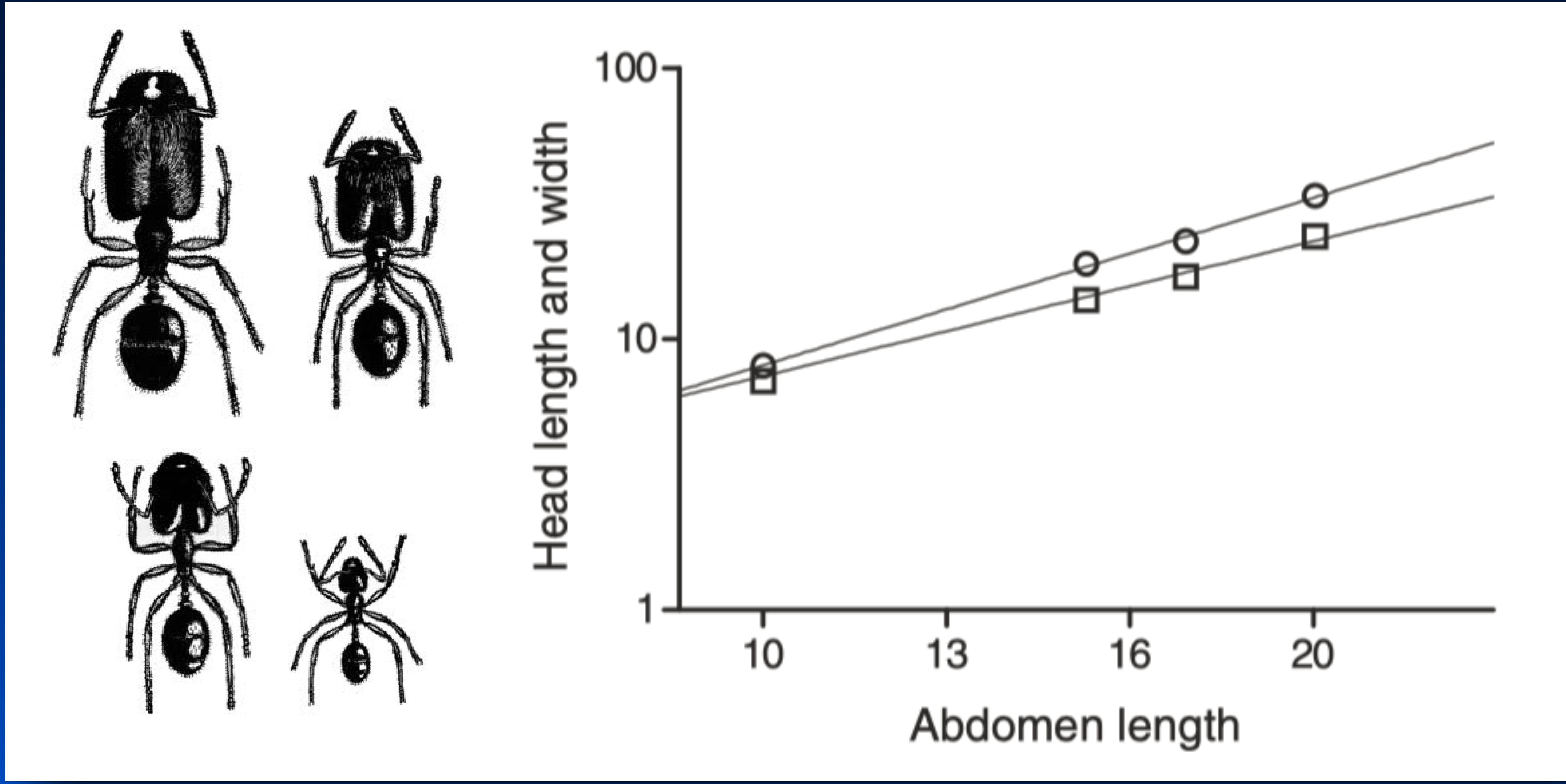
Four neuter castes of the ant *Pheidole instabilis* illustrating differential growth rates in the head verses the body. This observation was used by Huxley and his French counterpart, Georges Teisser, to establish allometry as a common biological phenomenon.

Image from Huxley (1936)

Scalar Morphometrics

Allometry

Julian Huxley
(1887 – 1975)

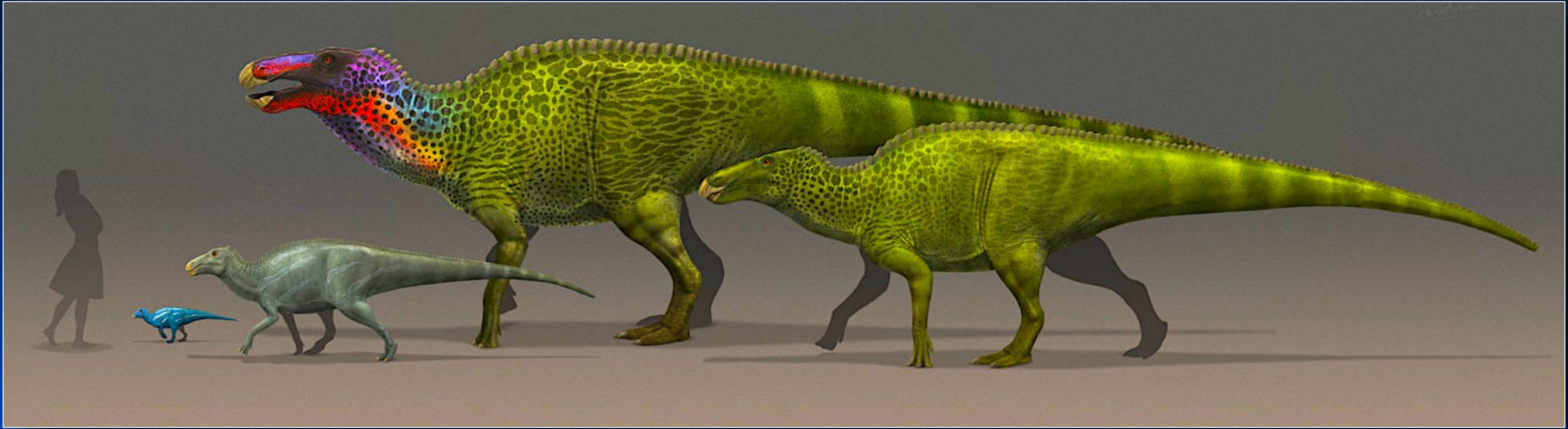


Quantification of differential rates of size change in *Pheidole instabilis* head length and head width relative to abdominal length using linear regression analysis after logarithmic transformation of the head data. Logarithmic transformation in effect linearizes the exponential pattern of variation.

Images from Stevens (2009)

Scalar Morphometrics

Allometry - The study of the influence of body size on form and function.



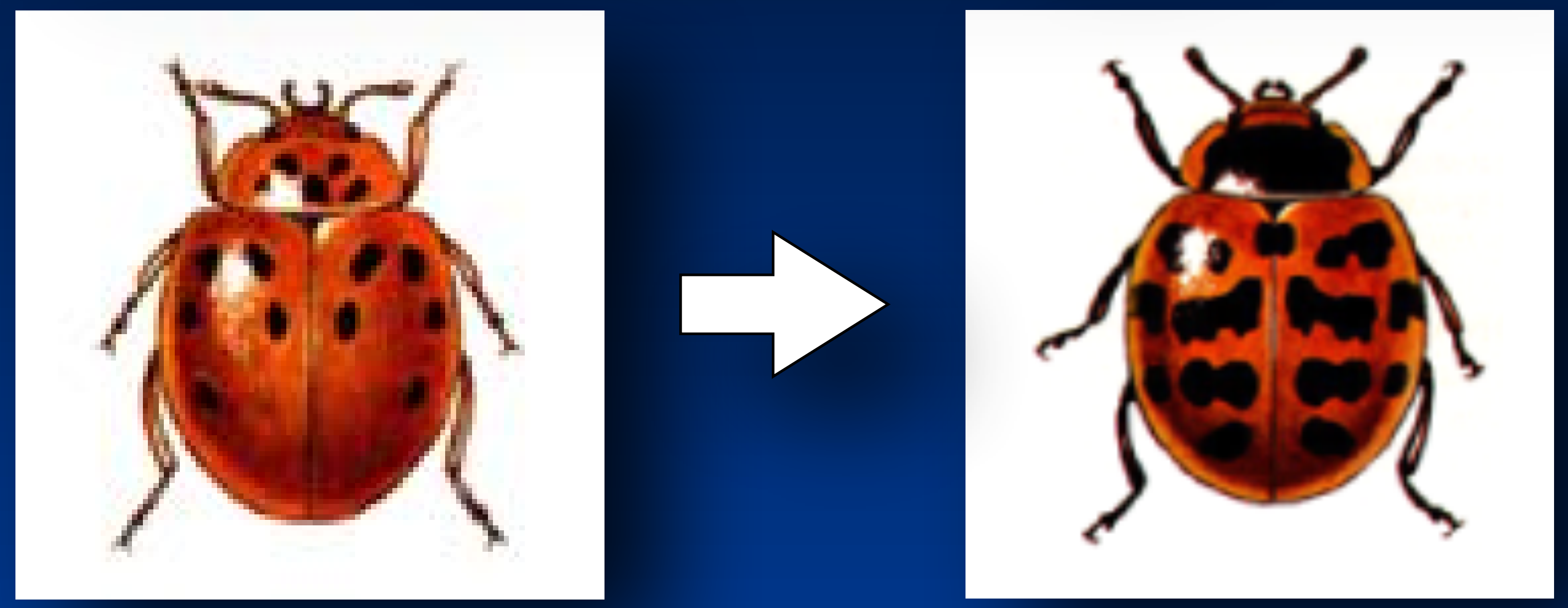
- Ontogenetic - allometry between different individuals of the same species at different stages of development.
- Static - allometry between different individuals at comparable developmental stages.
- Phylogenetic - allometry between individuals of different species, within an evolutionary lineage at comparable developmental stages.

Scalar Morphometrics

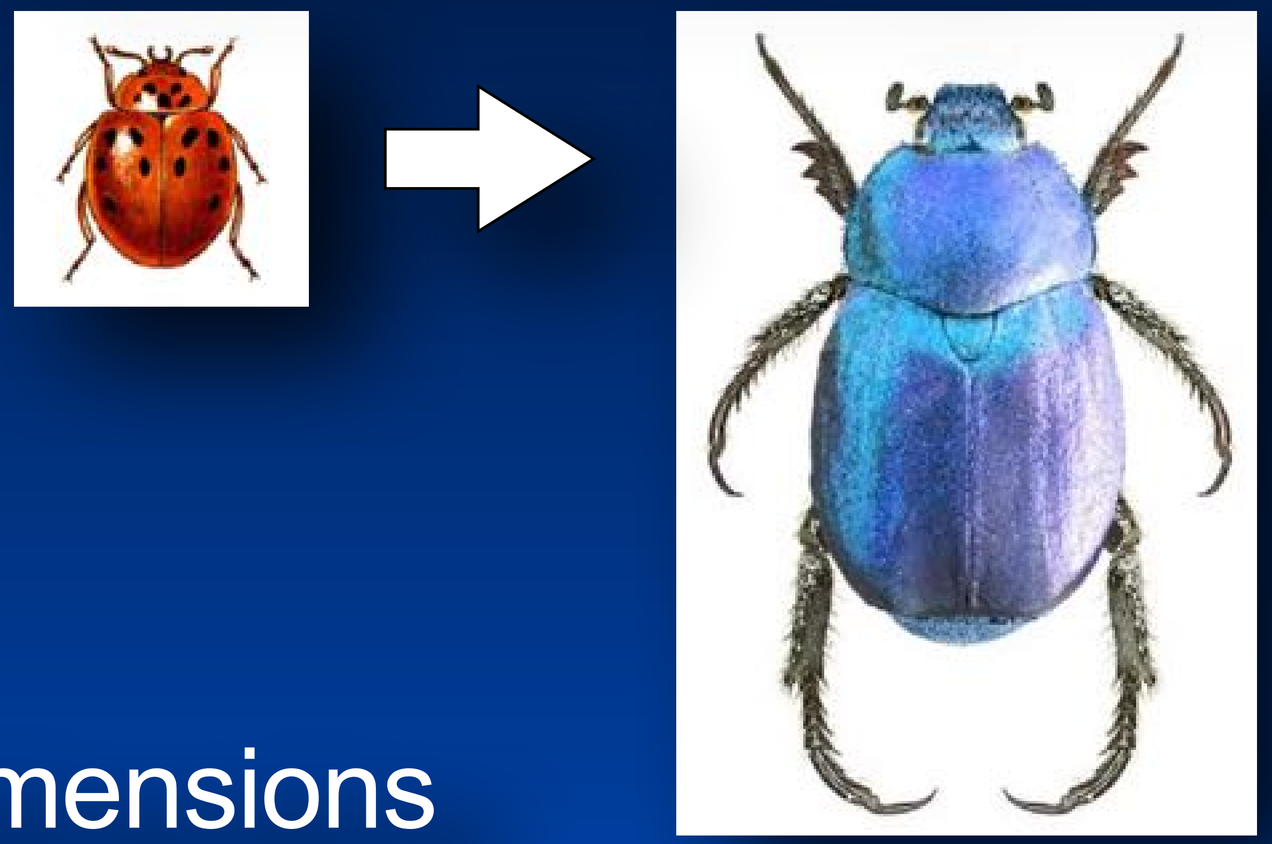
Size, Shape & Form

Size Change

Shape Change



Form Change

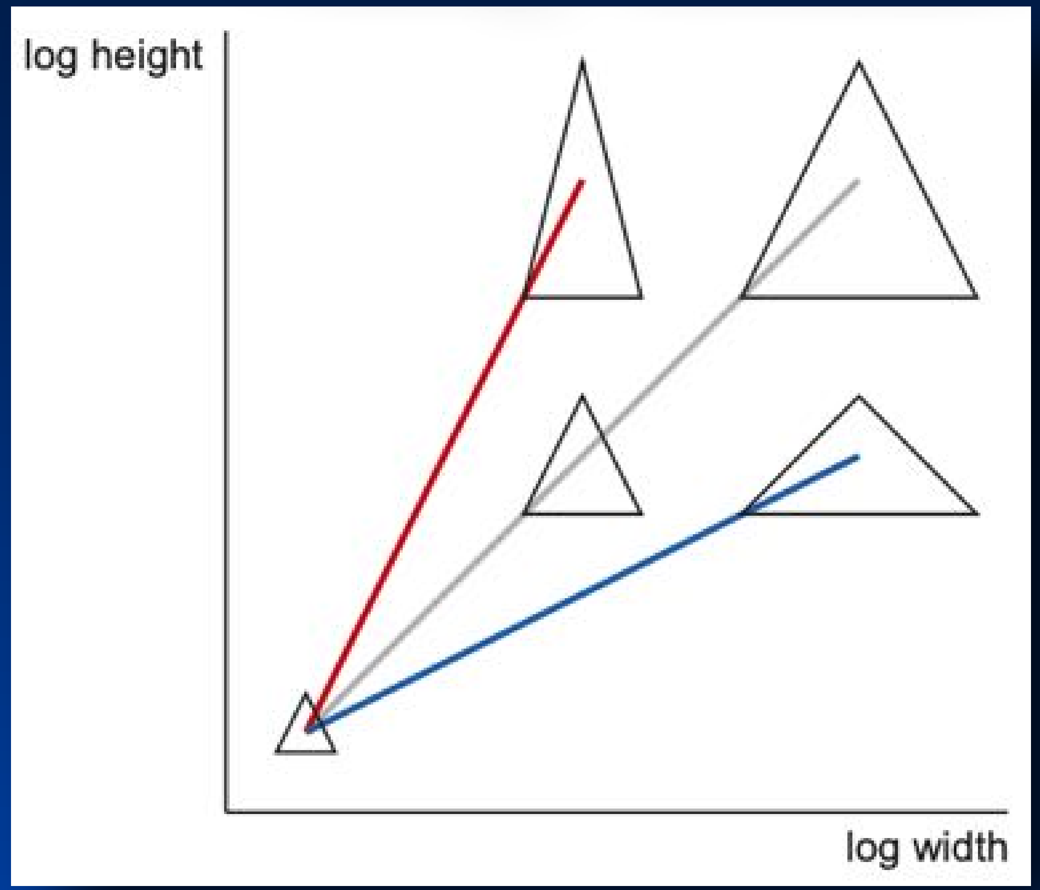


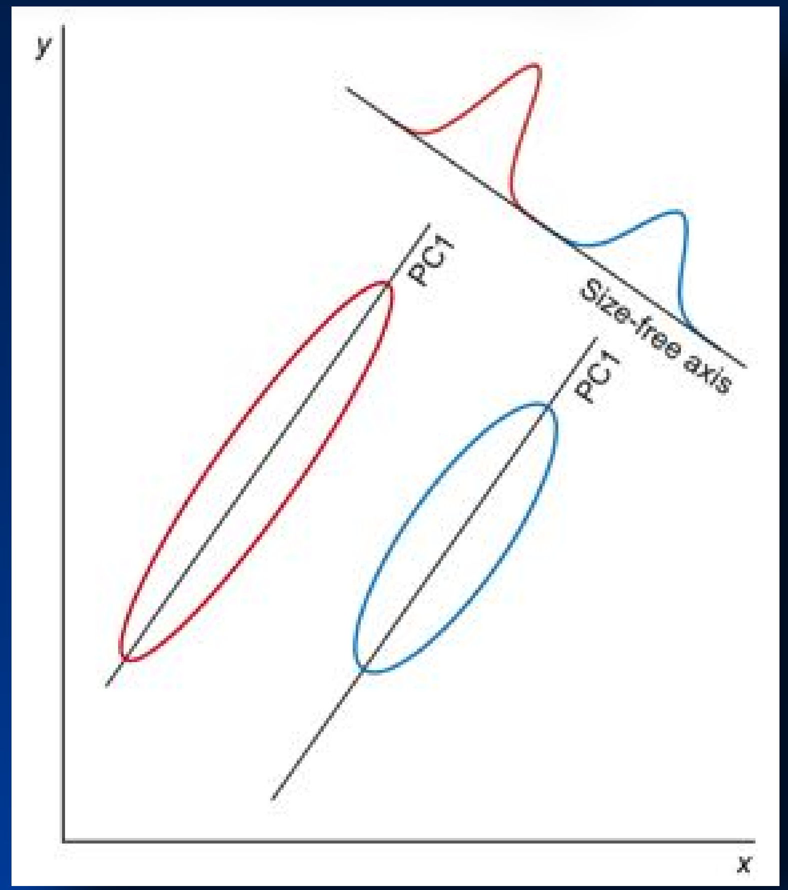
- Size Change - a generalized metric increase or decrease in all dimensions of an organ or body.
- Shape Change - a localized metric increase or decrease in the dimensions of an organ or body that takes place at different rates in different regions.
- Form Change - the sum total of size and shape changes across all parts of an organ or body.

Scalar Morphometrics

Size, Shape & Allometry

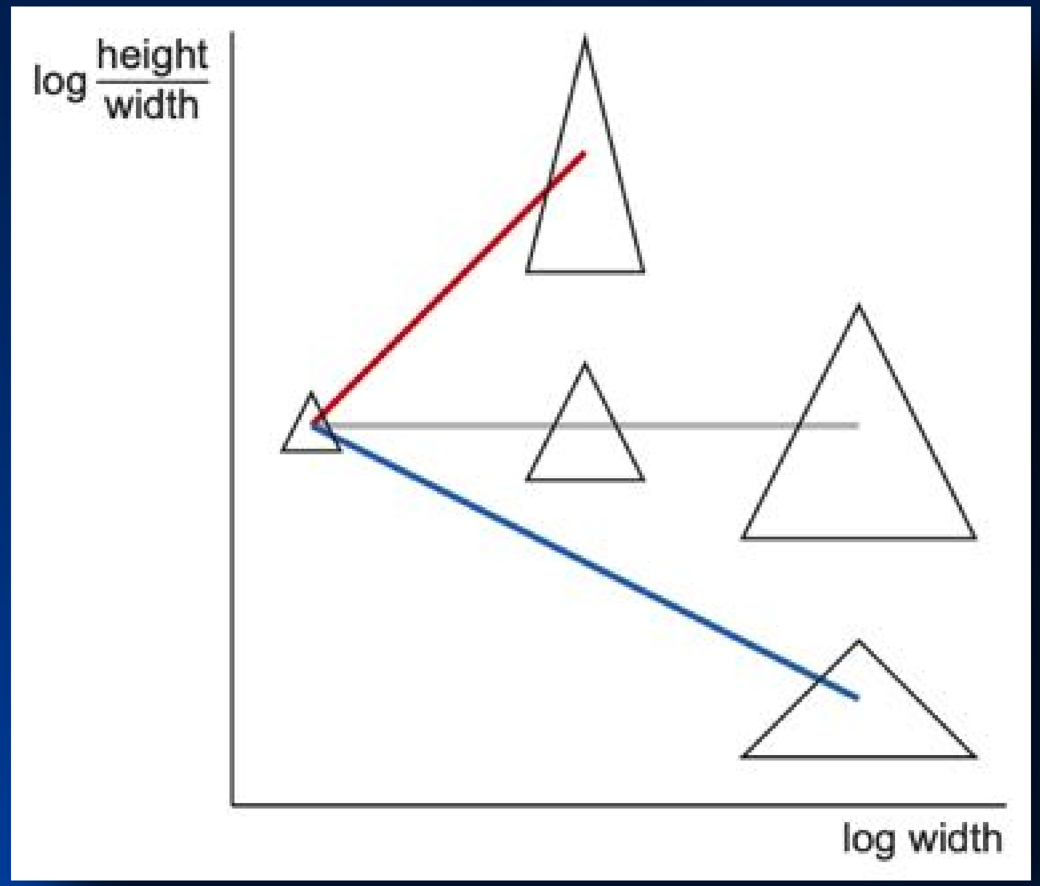
Huxley - Jolicouer View of Allometry

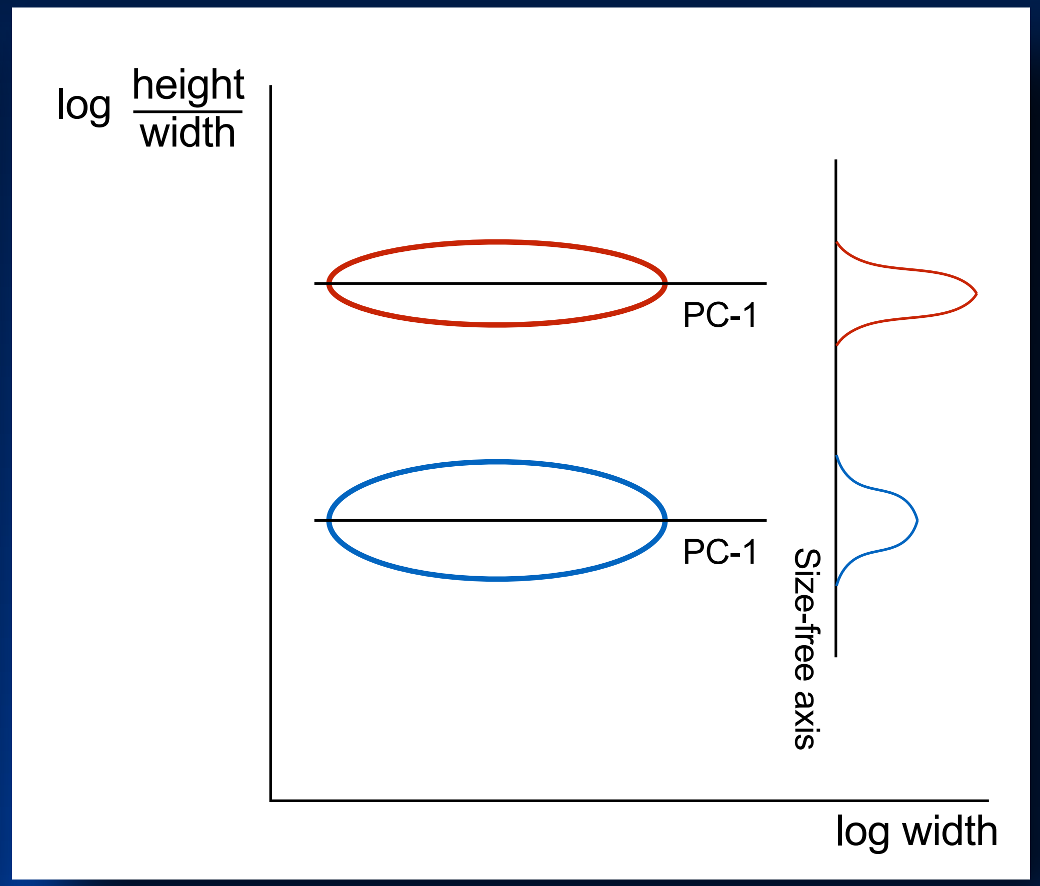




- Size represents the dominant contributor to variation and covariance structure.
- Shape can be represented as a vector, or space, oriented perpendicular to the allometric size vector(s).

Mosimann - Gould View of Allometry





- Size and shape can be separated cleanly as a result of definitional differences.
- Shape is quantified as a vector of ratios (= perpendicular to the vector of isometry).

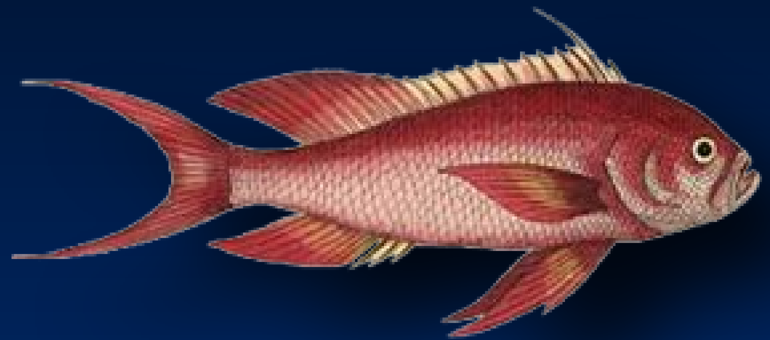
It is useful for analysis to create shape spaces that include the effects of allometry.

Diagrams based on Klingenberg (2016)

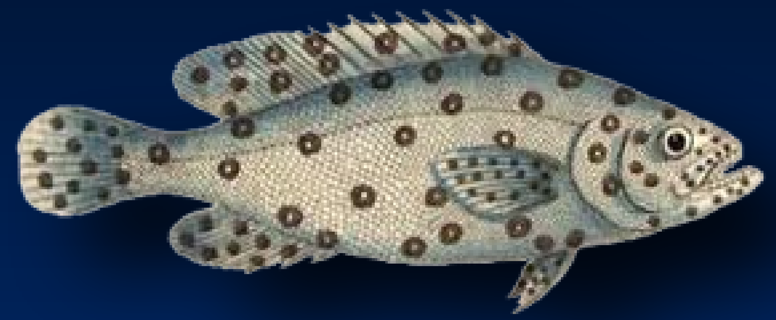
Scalar Morphometrics

Scalar (Linear Distance) Characterization of Morphology

Anisotremus virginicus



Anthias anthias



Anthias argus

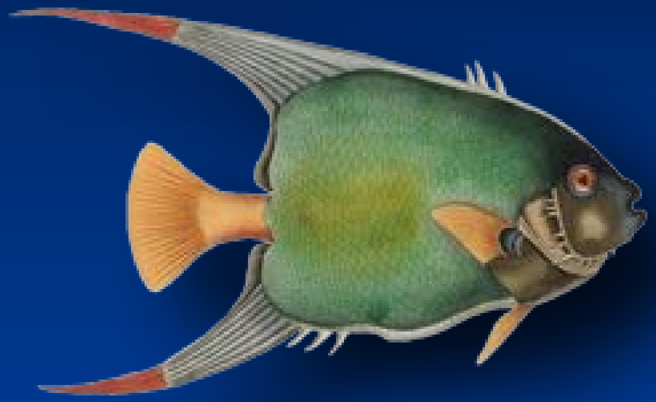
Archosargus rhomboidalis

Chelmon rostratus

Conodon nobilis

Epinephelus merra

Gerres erythrourus



Holacanthus ciliaris

Holocentrus ascensionis

Johnius carutta

Lonchurus lanceolatus

Oligoplites saliens

Ophiocara macrolepidota

Perca

Pomacentrus pavo

Pomadasys furcatus

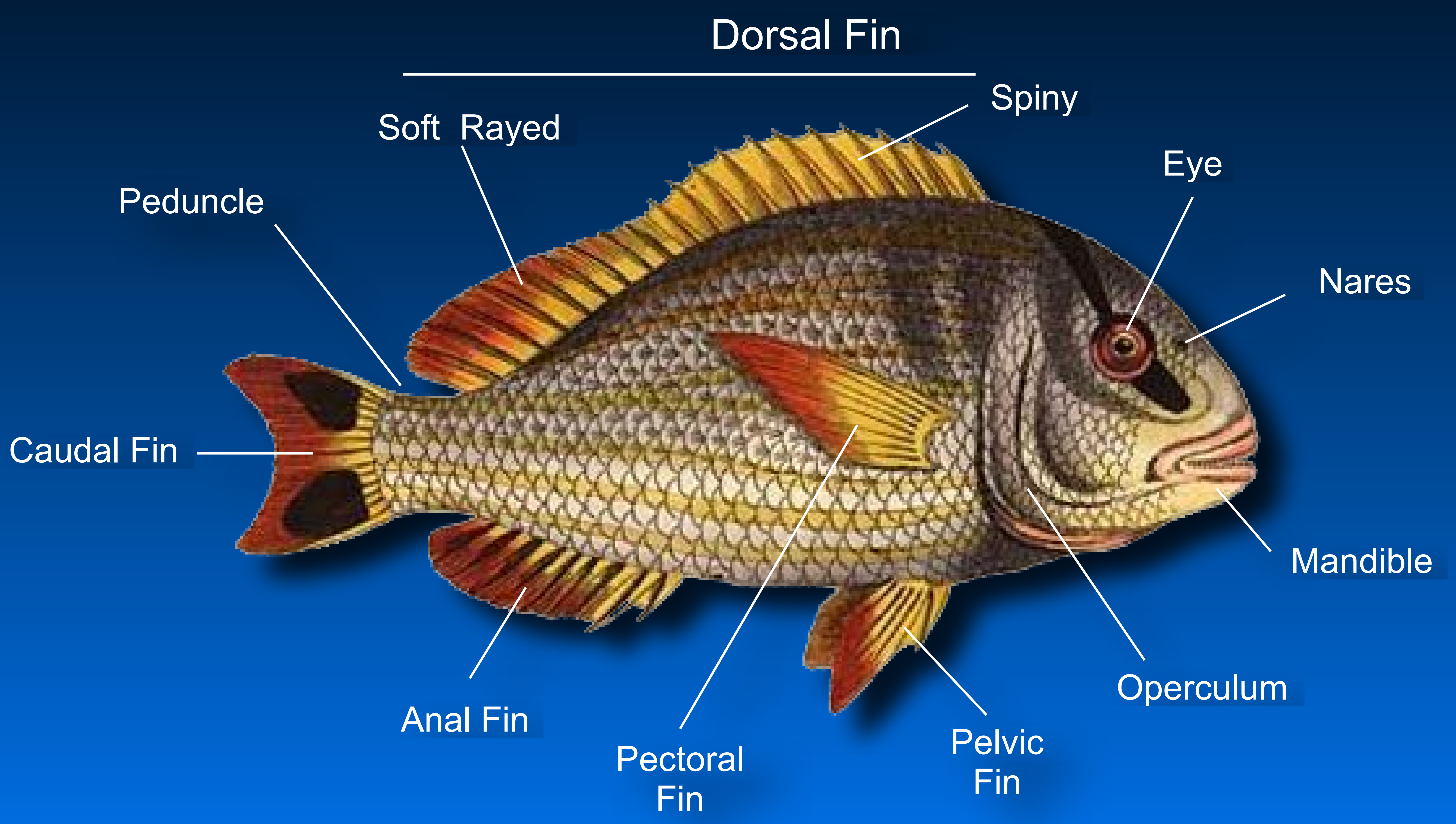
Sparisoma chrysopterum

Trachichthys australis

Umbrina cirrosa

Morphometric Data

Anisotremus virginicus

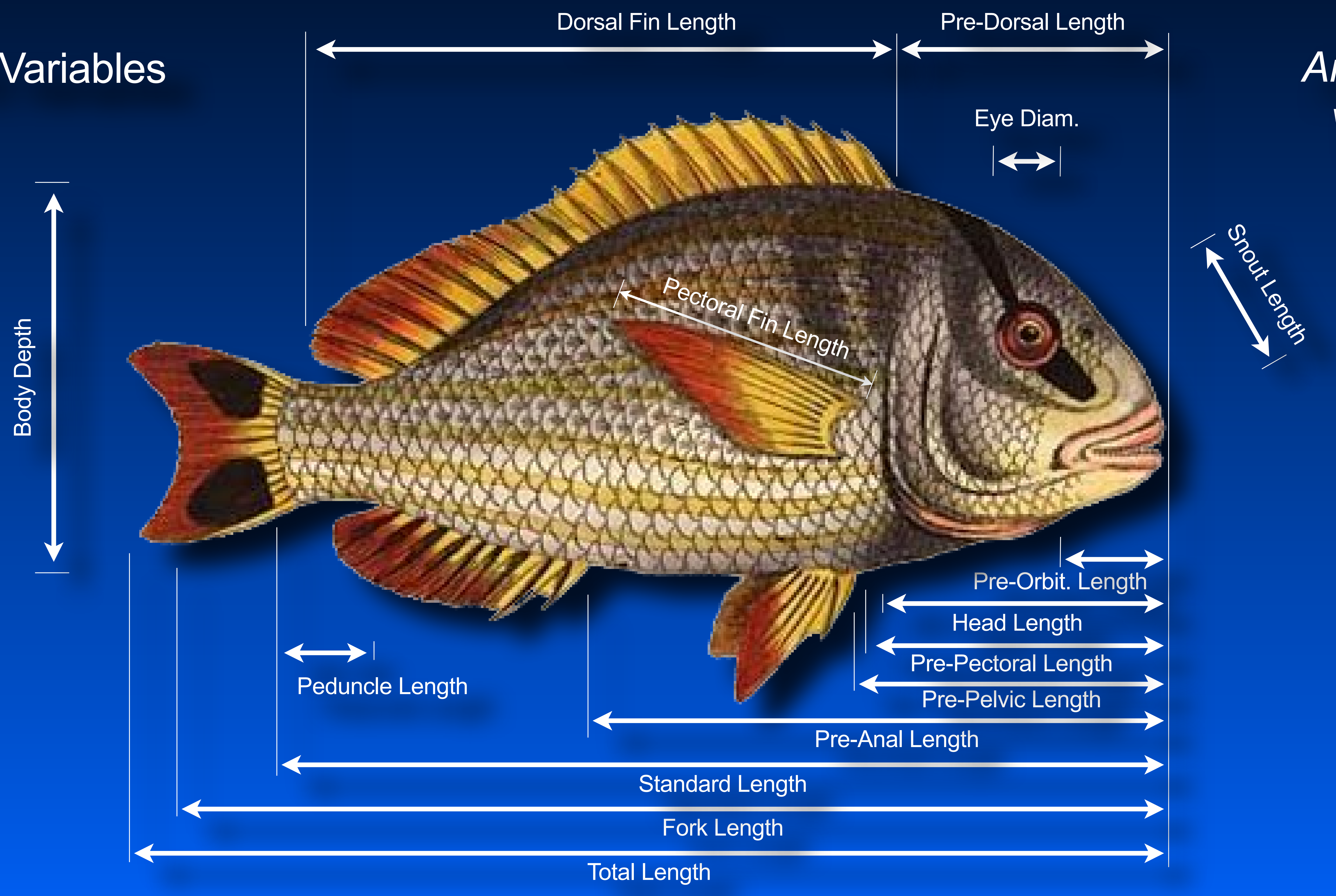


Morphometric Data

Scalar (Linear Distance) Characterization of Morphology

15 Variables

Anisotremus virginicus



Linear Distance Data

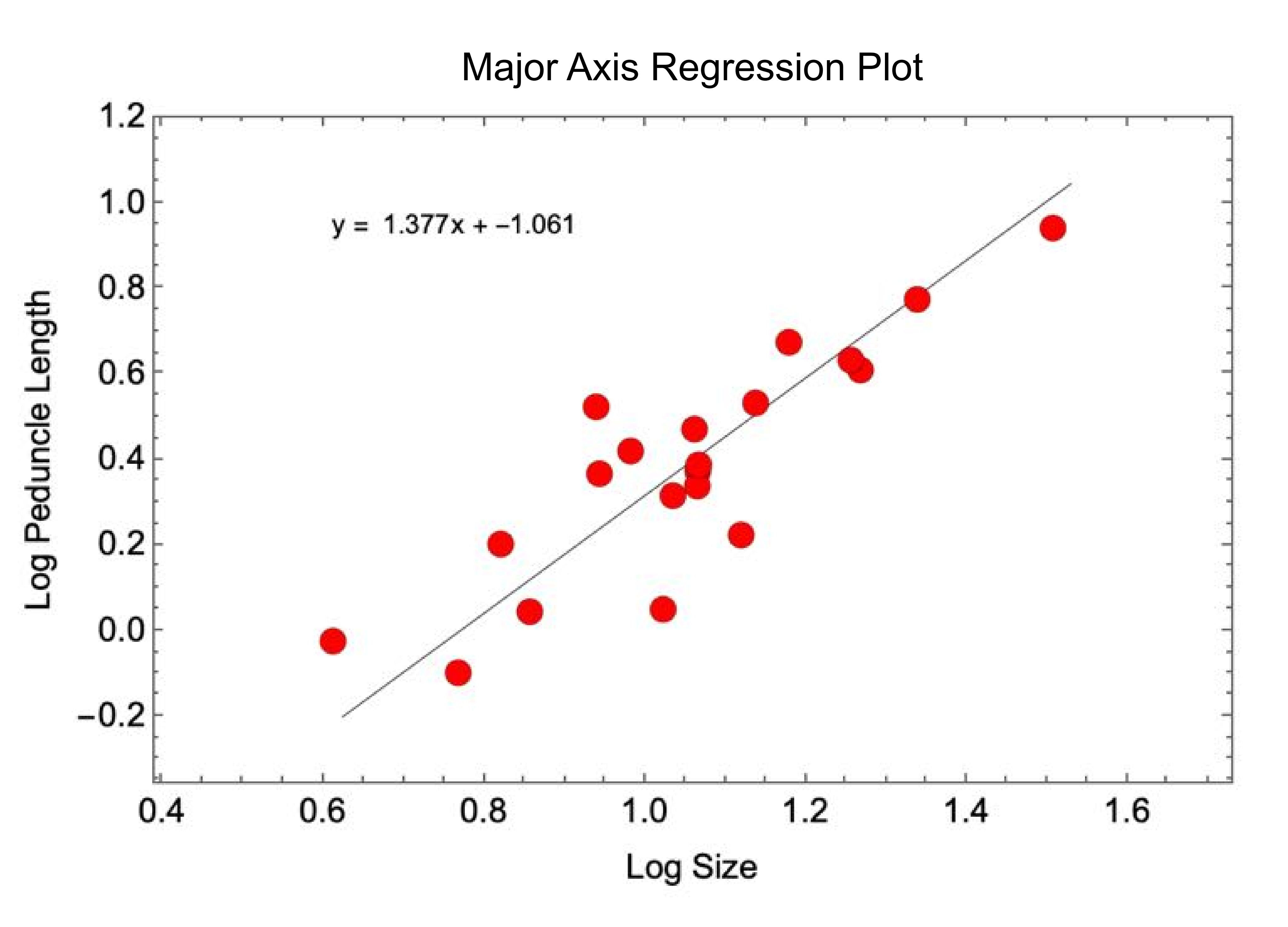
Fish Dataset: Linear Distances

Species	Pre-Orbit Length	Head Length	Pre-Dorsal Length	Pre-Pectoral Length	Pre-Pelvic Length	Pre-Anal Length	Std. Length	Fork Length	Total Length	Snout Length	Eye Diam.	Pectoral Fin Length	Dorsal Fin Length	Peduncle Length	Body Depth
Anisotremus virginicus	2.835	7.769	7.572	8.505	8.702	16.213	25.000	27.848	29.320	3.533	1.669	7.453	14.875	2.356	10.800
Anthias anthias	2.439	11.057	9.350	11.220	10.244	25.772	40.000	44.553	57.073	2.989	1.707	12.709	32.276	4.065	12.439
Anthias argus	1.207	4.135	4.803	4.315	4.007	9.786	15.000	17.491	17.491	1.383	0.745	3.288	8.964	1.592	4.983
Archosargus rhomboidalis	1.662	4.864	5.982	5.831	4.683	12.689	20.000	21.813	23.082	2.473	1.027	4.108	12.266	2.326	8.822
Chelmon rostratus	3.320	5.547	4.984	6.279	5.585	8.848	15.000	16.969	16.613	3.381	0.938	2.782	9.415	1.106	7.052
Conodon nobilis	2.106	6.853	7.236	7.198	6.815	16.539	25.000	27.335	29.479	2.528	1.110	4.238	16.233	2.067	7.695
Epinephelus merra	2.545	6.655	9.491	8.582	8.618	21.055	30.000	36.182	34.582	3.055	1.127	5.966	19.781	1.672	8.182
Gerres erythrourus	3.063	7.047	8.578	7.108	6.893	16.636	25.000	27.206	32.629	3.574	1.317	8.477	14.308	2.176	10.172
Holacanthus ciliaris	3.066	8.748	3.358	8.310	7.775	20.819	27.000	36.783	35.323	3.184	1.655	5.851	41.663	4.721	18.155
Holocentrus adscensionis	2.058	8.232	8.155	8.918	8.841	16.615	25.000	28.925	29.420	2.943	1.601	8.052	15.206	2.439	8.460
Johnius carutta	0.928	3.301	3.817	3.390	3.242	9.402	14.000	15.945	15.945	1.093	0.472	2.698	9.491	0.796	3.301
Lonchurus lanceolatus	1.560	6.120	6.920	6.600	6.640	14.040	22.000	30.000	30.000	1.826	0.600	9.890	15.320	1.120	5.280
Oligoplites saliens	2.541	7.719	11.507	8.055	7.527	13.712	35.000	36.055	39.986	2.833	1.151	5.323	20.760	3.404	10.212
Ophiocara macrolepidota	1.383	5.390	6.667	5.284	5.142	10.461	20.000	24.362	24.362	1.636	1.028	5.032	10.284	3.333	6.028
Perca fluviatilis	3.594	16.212	12.247	12.330	14.808	34.470	50.000	55.700	59.253	4.405	2.395	9.902	29.409	5.948	16.274
Pomacentrus pavo	0.667	2.272	2.541	2.353	2.609	5.197	8.500	9.956	12.531	0.791	0.458	2.105	6.714	0.944	3.734
Pomadasys furcatus	6.762	22.779	21.428	24.130	26.211	46.494	70.000	73.954	88.516	9.049	5.096	13.028	42.125	8.737	23.818
Sparisoma chrysopterum	3.303	7.988	8.257	7.873	7.565	16.667	25.000	28.226	30.338	4.382	1.190	5.806	14.324	2.957	8.909
Trachichthys australis	1.602	6.817	10.144	6.283	5.339	13.306	20.000	22.587	25.791	2.945	3.450	5.252	6.858	2.628	10.883
Umbrina cirrosa	3.311	12.157	11.886	11.886	11.886	26.594	40.000	44.125	46.459	4.145	1.682	12.144	25.834	4.288	13.894

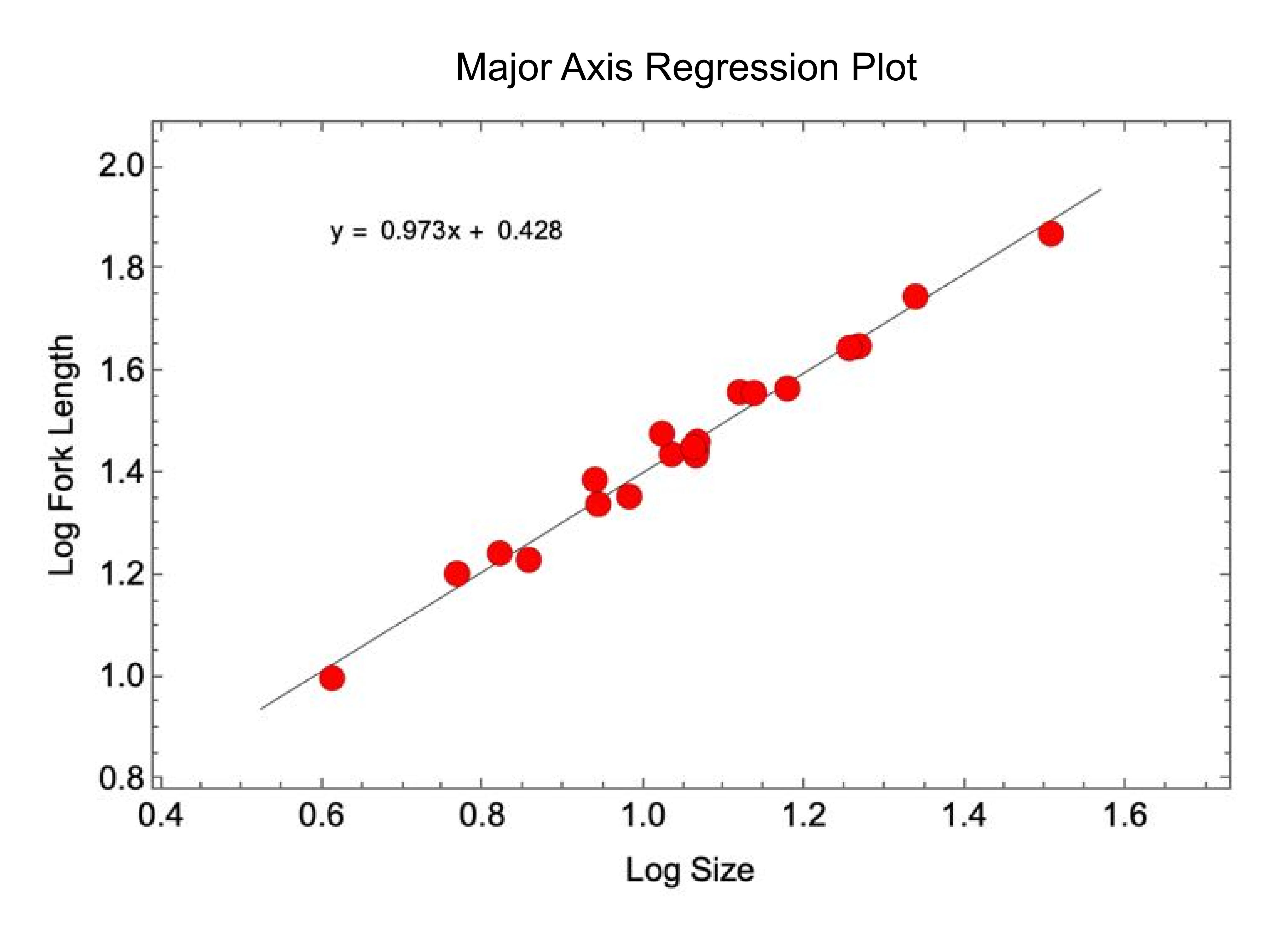
Scalar Morphometrics

Scalar (Linear Distance) Characterization of a Fish

Bivariate Allometric Analysis



Positive Allometry



Negative Allometry

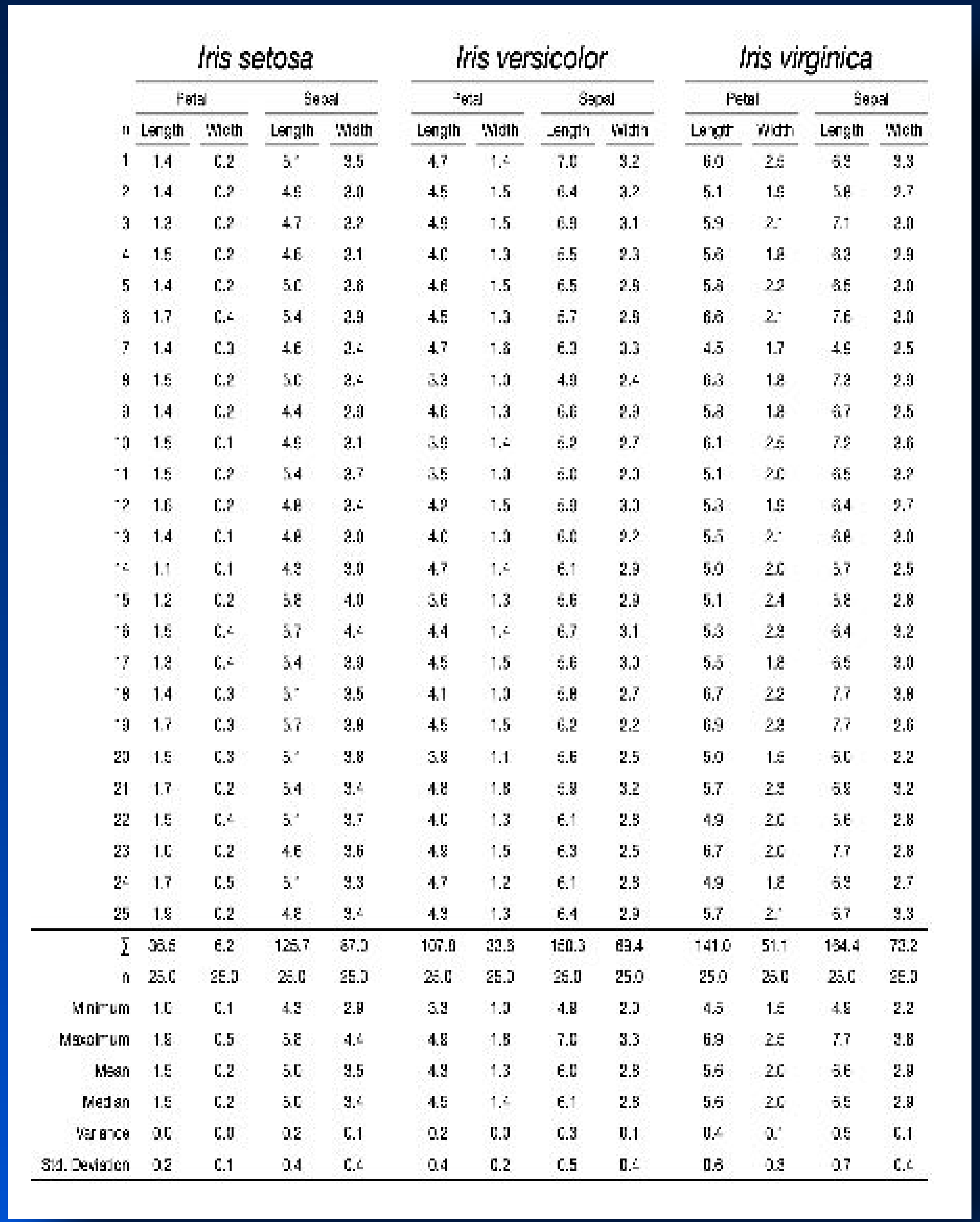
Scalar Morphometrics

Principal Components Analysis (PCA)

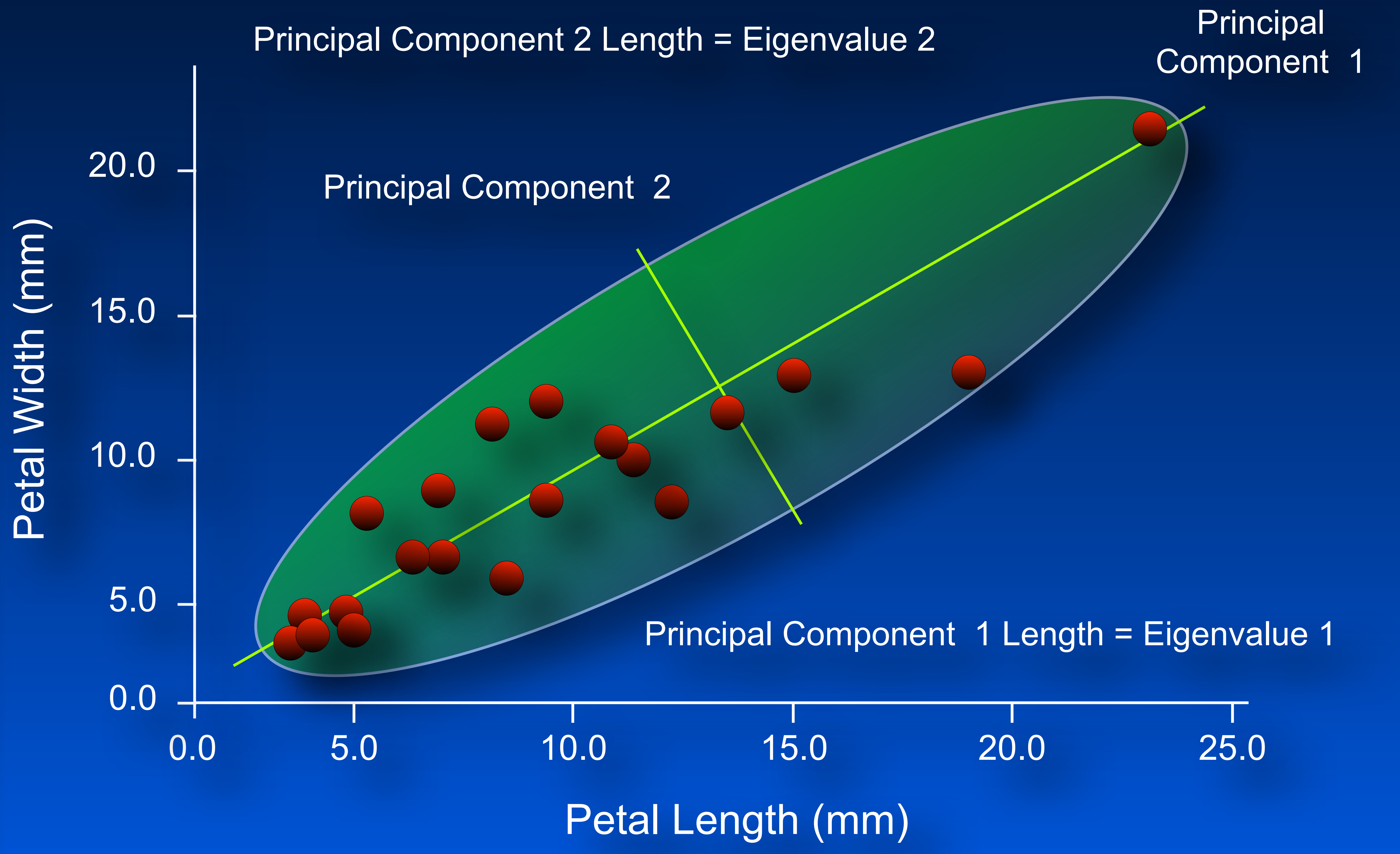
Iris setosa

Iris versicolor

Iris virginica



	Iris setosa				*Iris versicolor*				*Iris virginica*			
	Petal		Sepal		Petal		Sepal		Petal		Sepal	
n	Length	Width	Length	Width	Length	Width	Length	Width	Length	Width	Length	Width
1	1.4	0.2	5.1	3.5	4.7	1.4	7.0	3.2	6.0	2.5	6.3	3.3
2	1.4	0.2	4.9	3.0	4.5	1.5	6.4	3.2	5.1	1.9	5.8	2.7
3	1.3	0.2	4.7	3.2	4.9	1.5	6.9	3.1	5.9	2.1	7.1	3.0
4	1.5	0.2	4.6	3.1	4.0	1.3	5.5	2.3	5.6	1.8	6.3	2.9
5	1.4	0.2	5.0	3.6	4.6	1.5	6.5	2.8	5.8	2.2	6.5	3.0
6	1.7	0.4	5.4	3.9	4.5	1.3	5.7	2.8	6.6	2.1	7.6	3.0
7	1.4	0.3	4.6	3.4	4.7	1.6	6.3	3.3	4.5	1.7	4.9	2.5
8	1.5	0.2	5.0	3.4	3.3	1.0	4.9	2.4	6.3	1.8	7.3	2.9
9	1.4	0.2	4.4	2.9	4.6	1.3	6.6	2.9	5.8	1.8	6.7	2.5
10	1.5	0.1	4.9	3.1	3.9	1.4	5.2	2.7	6.1	2.5	7.2	3.6
11	1.5	0.2	5.4	3.7	3.5	1.0	5.0	2.0	5.1	2.0	6.5	3.2
12	1.6	0.2	4.8	3.4	4.2	1.5	5.9	3.0	5.3	1.9	6.4	2.7
13	1.4	0.1	4.8	3.0	4.0	1.0	6.0	2.2	5.5	2.1	6.8	3.0
14	1.1	0.1	4.3	3.0	4.7	1.4	6.1	2.9	5.0	2.0	5.7	2.5
15	1.2	0.2	5.8	4.0	3.6	1.3	5.6	2.9	5.1	2.4	5.8	2.8
16	1.5	0.4	5.7	4.4	4.4	1.4	6.7	3.1	5.3	2.3	6.4	3.2
17	1.3	0.4	5.4	3.9	4.5	1.5	5.6	3.0	5.5	1.8	6.5	3.0
18	1.4	0.3	5.1	3.5	4.1	1.0	5.8	2.7	6.7	2.2	7.7	3.8
19	1.7	0.3	5.7	3.8	4.5	1.5	6.2	2.2	6.9	2.3	7.7	2.6
20	1.5	0.3	5.1	3.8	3.9	1.1	5.6	2.5	5.0	1.5	6.0	2.2
21	1.7	0.2	5.4	3.4	4.8	1.8	5.9	3.2	5.7	2.3	6.9	3.2
22	1.5	0.4	5.1	3.7	4.0	1.3	6.1	2.8	4.9	2.0	5.6	2.8
23	1.0	0.2	4.6	3.6	4.9	1.5	6.3	2.5	6.7	2.0	7.7	2.8
24	1.7	0.5	5.1	3.3	4.7	1.2	6.1	2.8	4.9	1.8	6.3	2.7
25	1.9	0.2	4.8	3.4	4.3	1.3	6.4	2.9	5.7	2.1	6.7	3.3
∑	36.5	6.2	125.7	87.0	107.0	32.8	150.3	69.4	141.0	51.1	164.4	73.2
n	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
Minimum	1.0	0.1	4.3	2.9	3.3	1.0	4.9	2.0	4.5	1.5	4.9	2.2
Maximum	1.9	0.5	5.8	4.4	4.9	1.8	7.0	3.3	6.9	2.5	7.7	3.8
Mean	1.5	0.2	5.0	3.5	4.3	1.3	6.0	2.8	5.6	2.0	6.6	2.9
Median	1.5	0.2	5.0	3.4	4.5	1.4	6.1	2.8	5.6	2.0	6.5	2.8
Variance	0.0	0.0	0.2	0.1	0.2	0.0	0.3	0.1	0.4	0.1	0.5	0.1
Std. Deviation	0.2	0.1	0.4	0.4	0.4	0.2	0.5	0.4	0.6	0.3	0.7	0.4



Scalar Morphometrics

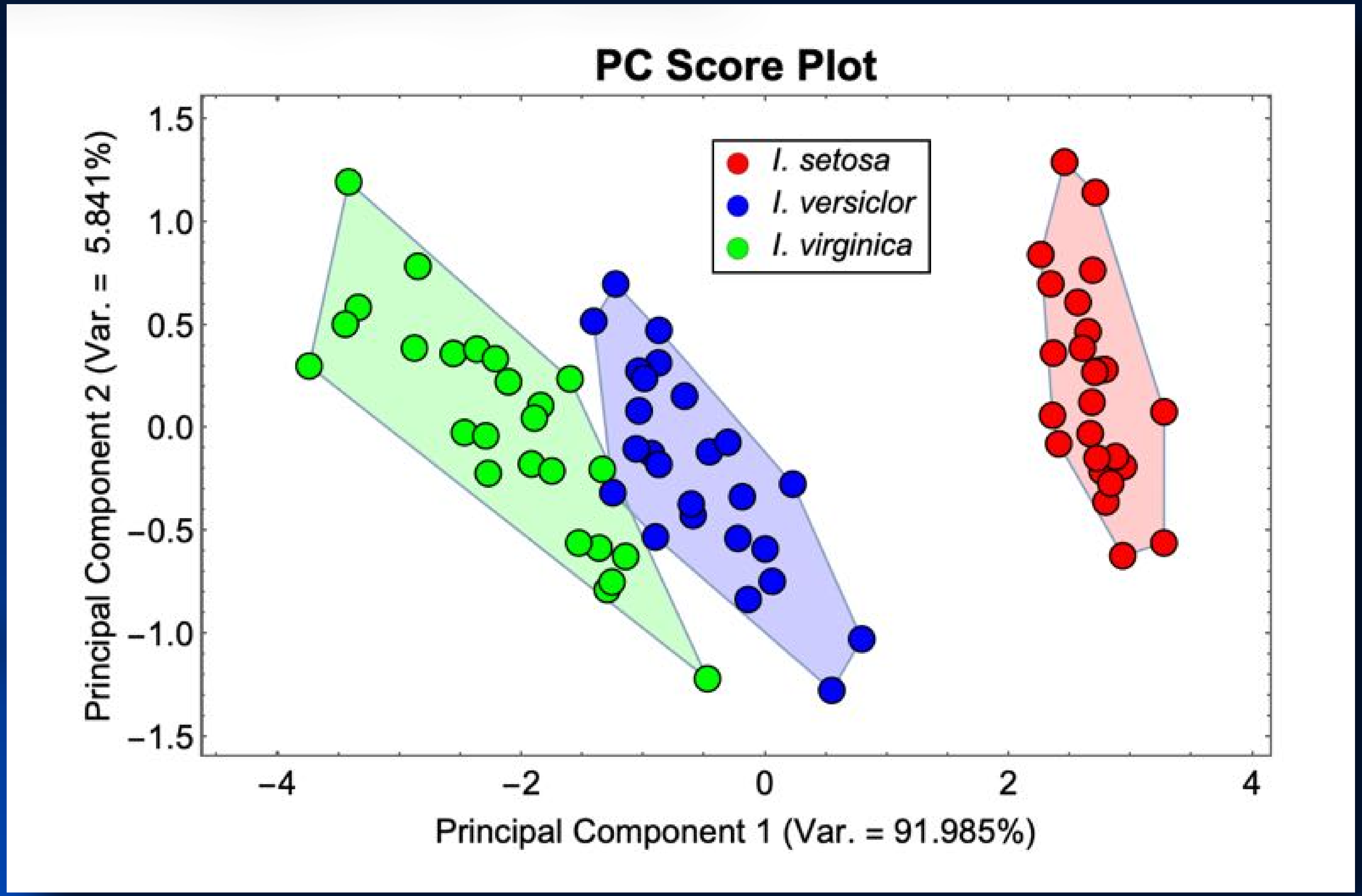
Principal Components Analysis (PCA)

Iris setosa

Iris versicolor

Iris virginica

- A data transformation (not a statistical procedure).
- Eigenvector decomposition of a square, non-singular matrix that expresses covariance relations between variables.
  - Perhaps the most commonly applied multivariate data-analysis procedure.
  - Involves no distributional assumptions.
  - Assumes all observations comprise a single group (makes no between-group distinctions or optimizations).
  - Can be used to reduce data dimensionality such that information loss is minimized.



Scalar Morphometrics

Scalar (Linear Distance) Characterization of a Fish

A Multivariate Approach to the Characterization of Allometry via PCA

Eigen-values	Eigen-value	% Variance	Cum. % Variance
1	0.659	84.850	84.850
2	0.040	5.095	89.945
3	0.029	3.699	93.643
4	0.025	3.273	96.917
5	0.010	1.262	98.179
6	0.006	0.795	98.974
7	0.003	0.393	99.367
8	0.002	0.257	99.624
9	0.001	0.109	99.733
10	0.001	0.087	99.820
11	0.001	0.069	99.889
12	0.000	0.056	99.945
13	0.000	0.031	99.977
14	0.000	0.016	99.992
15	0.000	0.008	100.000

Variables	Eigen-vector 1	Eigen-vector 2	Eigen-vector 3	Eigen-vector 4
Pre-Orbit Length	0.254	-0.162	0.081	0.592
Head Length	0.275	-0.004	0.038	0.008
Pre-Dorsal Length	0.223	-0.137	0.547	-0.293
Pre-Pectoral Length	0.259	0.014	0.125	0.112
Pre-Pelvic Length	0.272	0.076	0.139	0.098
Pre-Anal Length	0.259	0.166	0.024	-0.008
Standard Length	0.249	0.155	0.065	-0.074
Fork Length	0.238	0.216	0.023	-0.061
Total Length	0.247	0.192	0.020	-0.146
Snout Length	0.265	-0.311	0.153	0.395
Eye Diameter	0.276	-0.567	-0.109	-0.276
Pectoral Fin Length	0.247	0.299	0.284	-0.284
Dorsal Fin Length	0.250	0.498	-0.374	0.236
Peduncle Length	0.298	-0.140	-0.538	-0.368
Body Depth	0.252	-0.198	-0.328	0.081

Variables	Eigen-vector 1	Isometric Vector	Difference Vector	Allometry
Pre-Orbit Length	0.254	0.258	0.985	Negative
Head Length	0.275	0.258	1.063	Positive
Pre-Dorsal Length	0.223	0.258	0.862	Negative
Pre-Pectoral Length	0.259	0.258	1.004	Isometry
Pre-Pelvic Length	0.272	0.258	1.053	Positive
Pre-Anal Length	0.259	0.258	1.004	Isometry
Standard Length	0.249	0.258	0.963	Negative
Fork Length	0.238	0.258	0.924	Negative
Total Length	0.247	0.258	0.956	Negative
Snout Length	0.265	0.258	1.028	Isometry
Eye Diameter	0.276	0.258	1.069	Positive
Pectoral Fin Length	0.247	0.258	0.957	Negative
Dorsal Fin Length	0.250	0.258	0.966	Negative
Peduncle Length	0.298	0.258	1.156	Positive
Body Depth	0.252	0.258	0.976	Negative

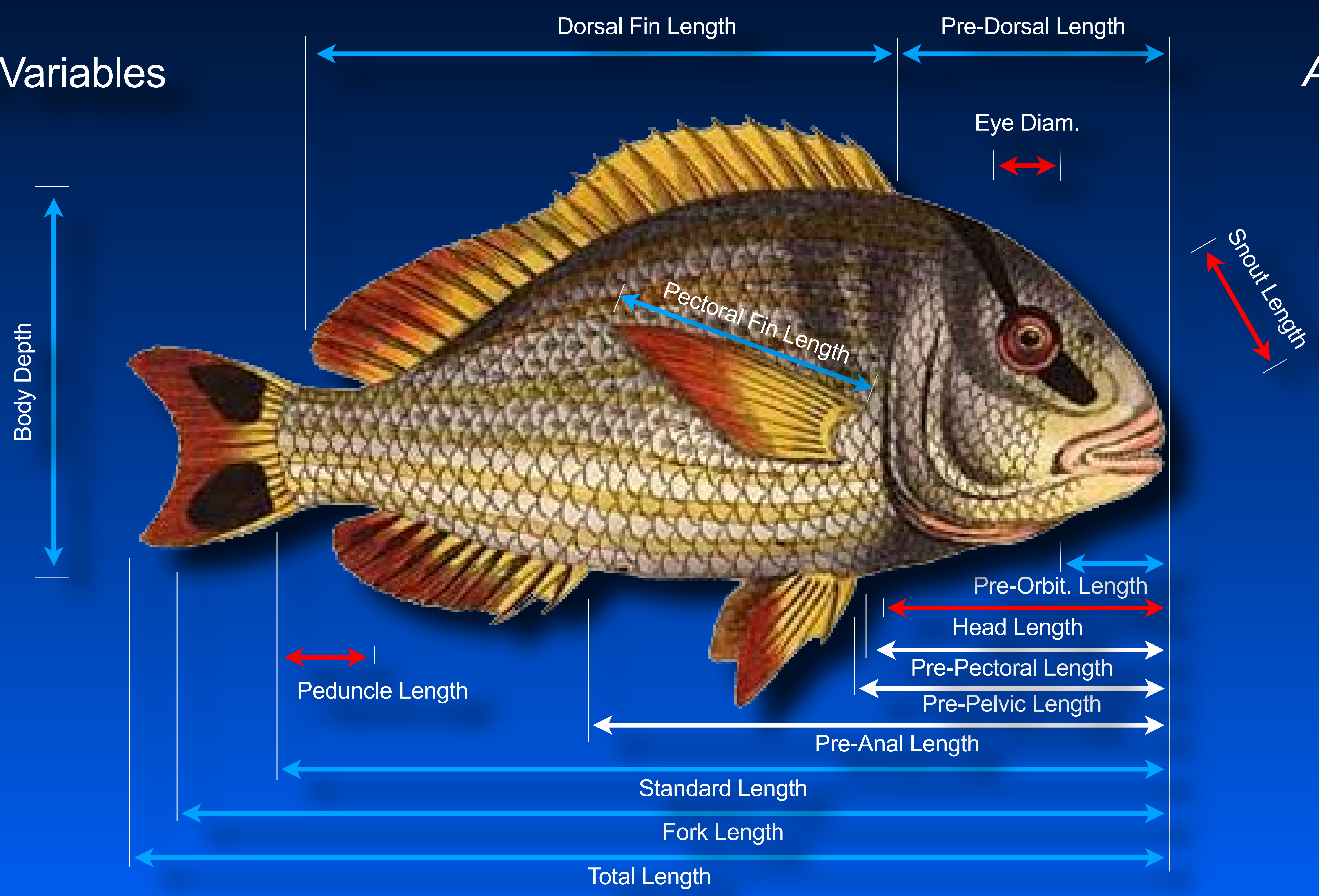
Isometric Axis = $(1/n)^{0.5}$

Scalar Morphometrics

Allometry in *Anisotremus virginicus*

15 Variables

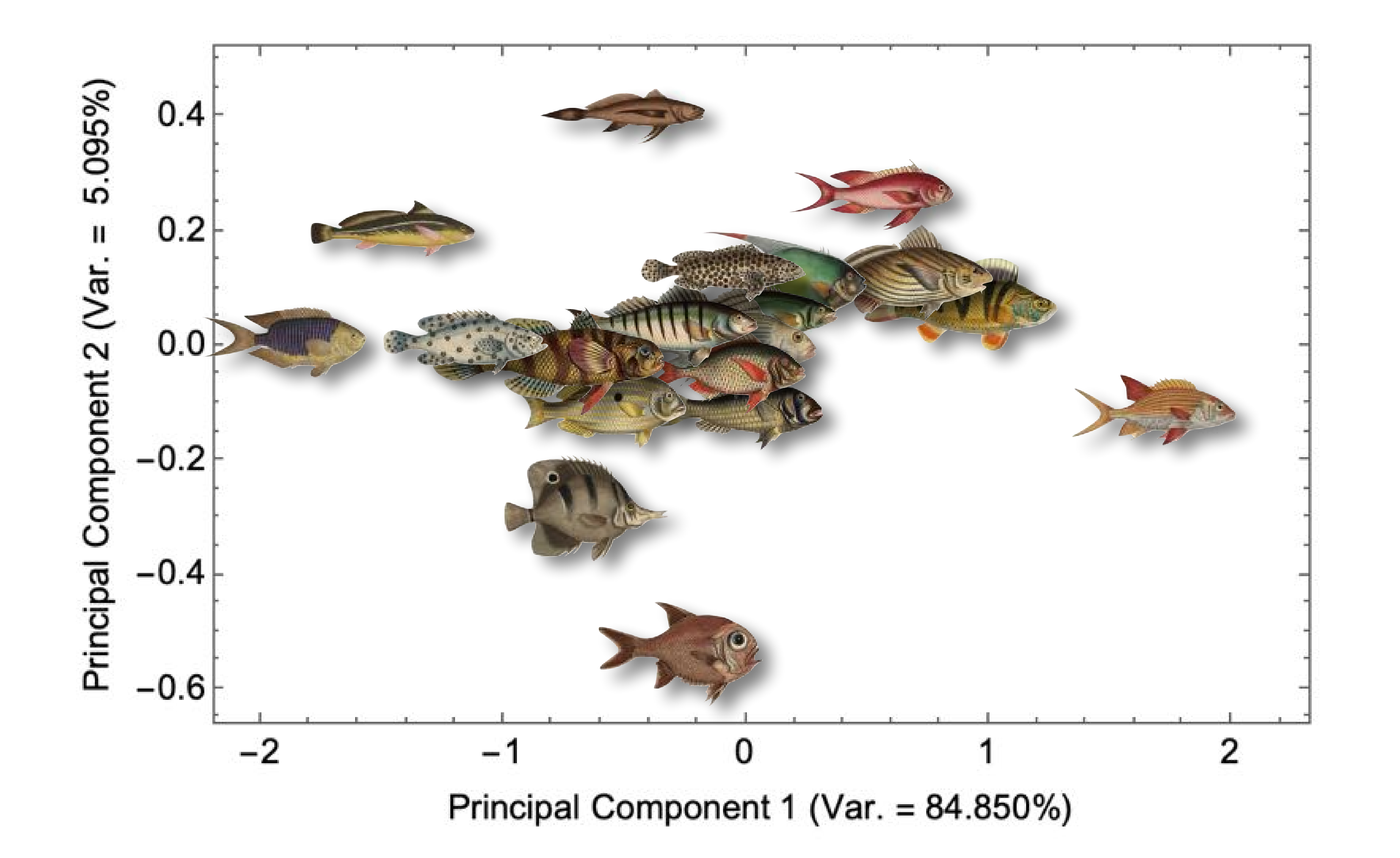
Anisotremus virginicus

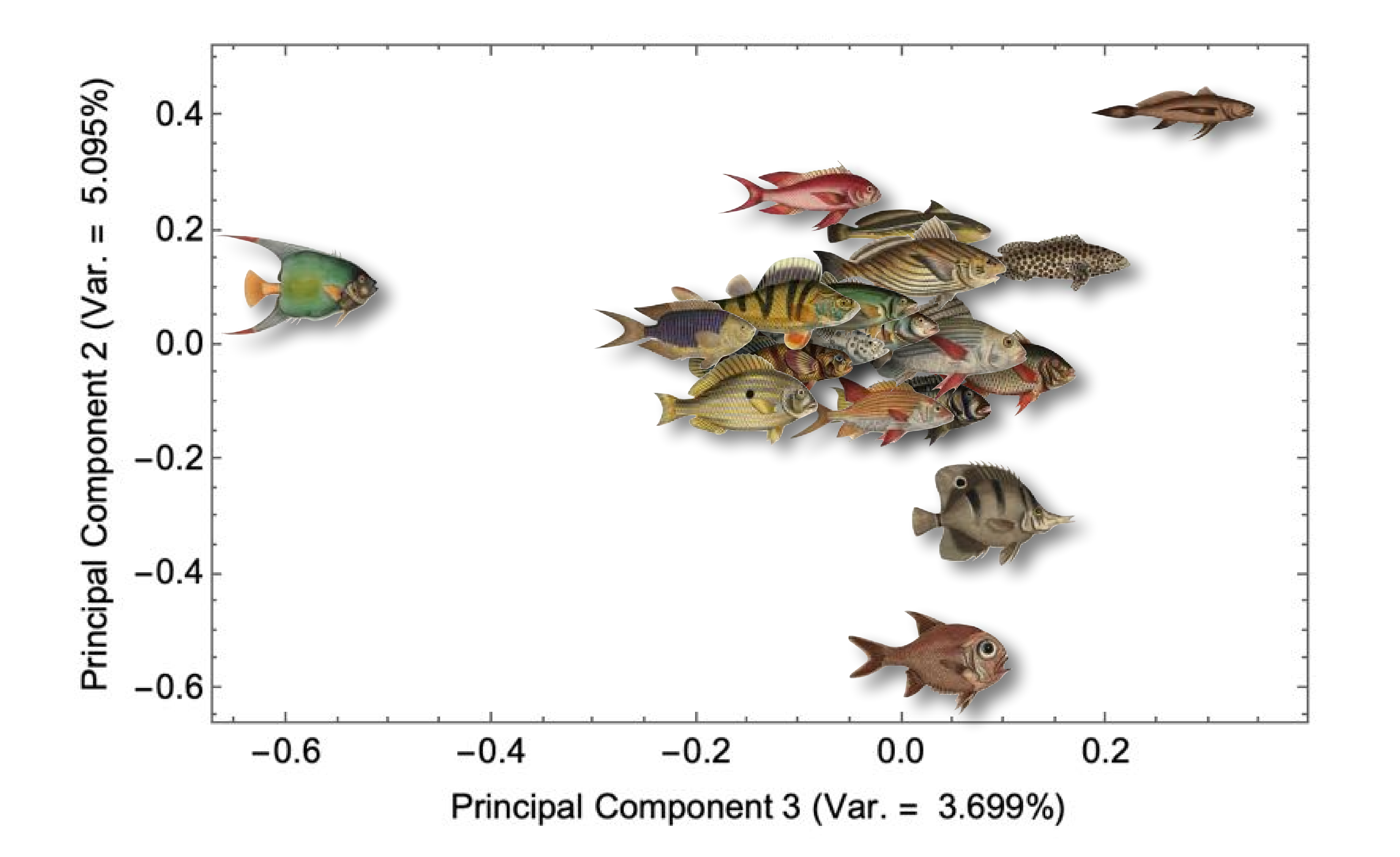


Scalar Morphometrics

Scalar (Linear Distance) Characterization of Fish Shape Variation

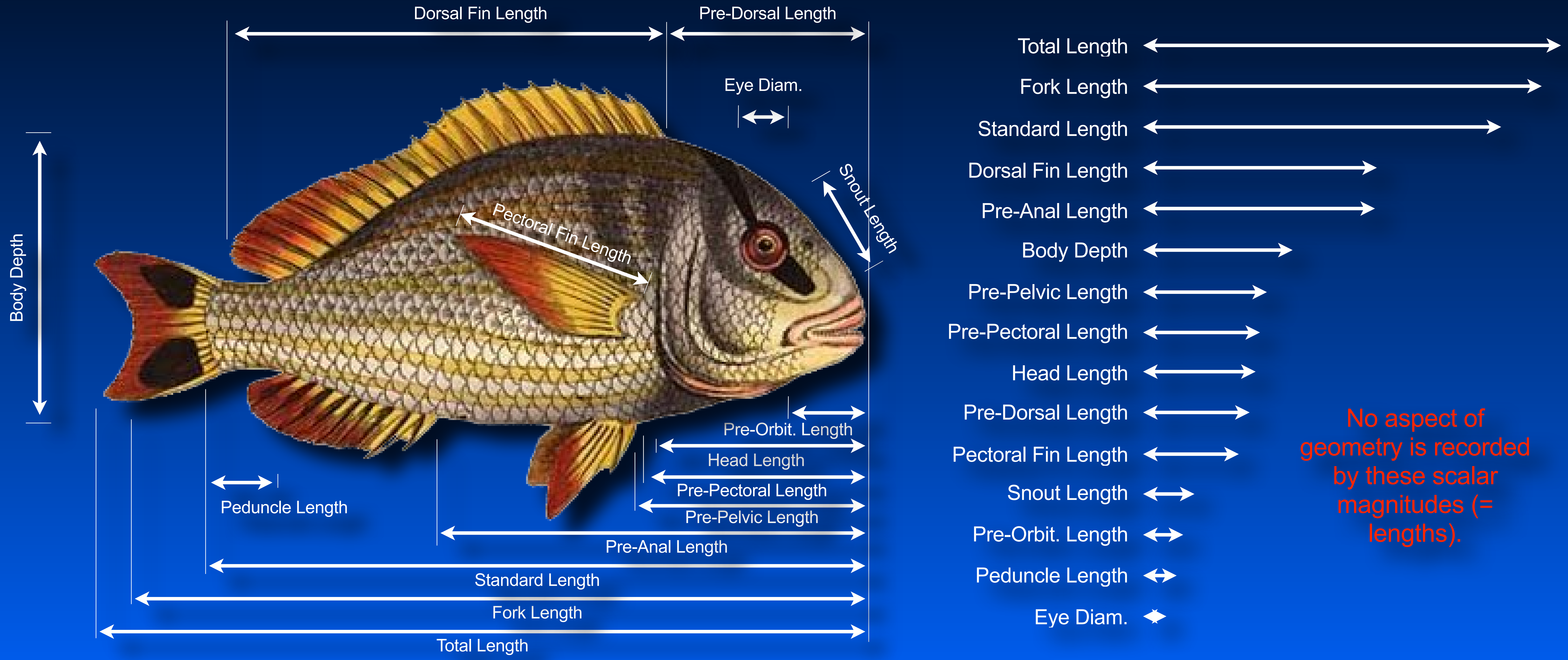
A Multivariate Approach to the Visualization of Form/Shape Variation via PCA





Scalar Morphometrics

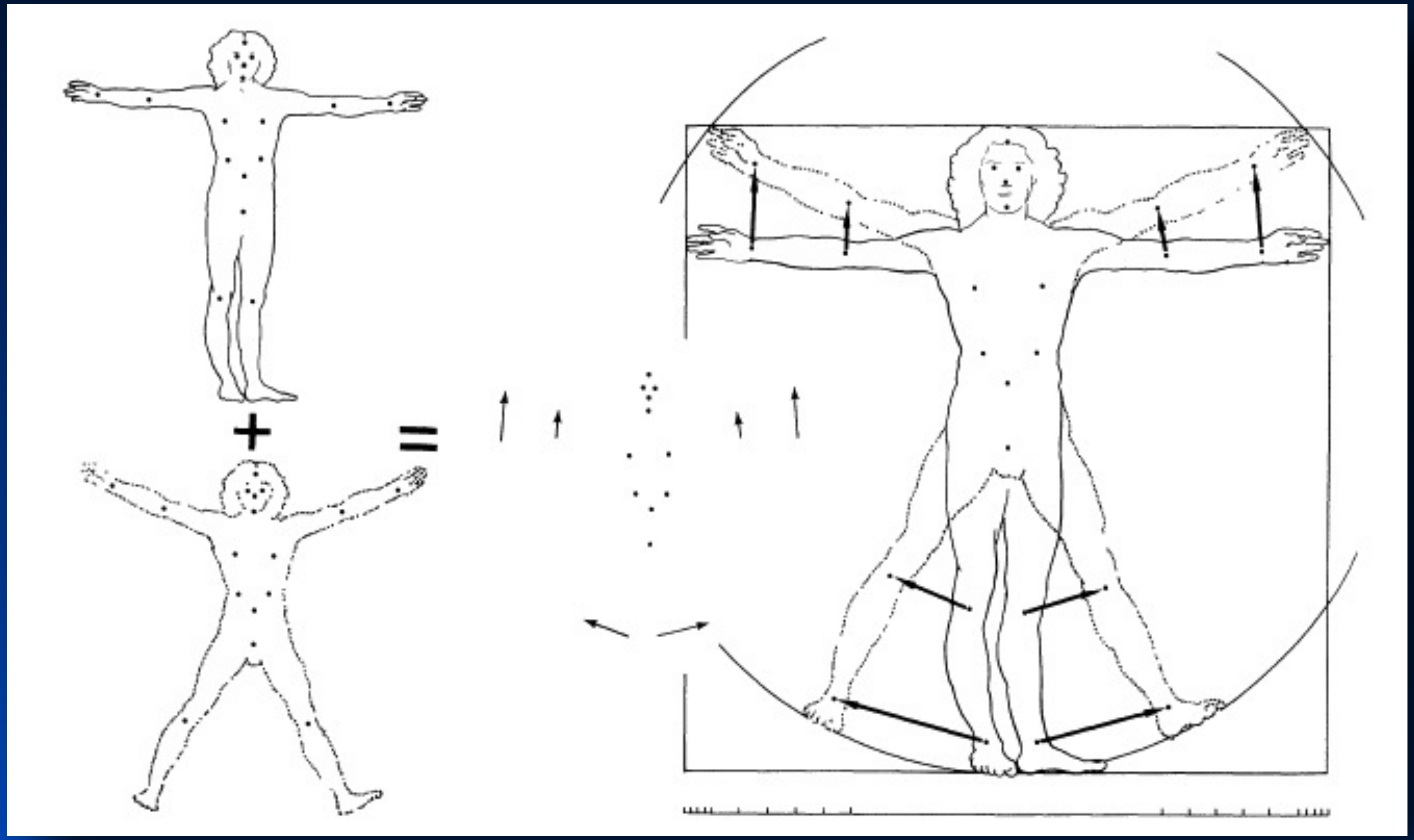
Problems with the Scalar (Linear Distance) Morpholometrics

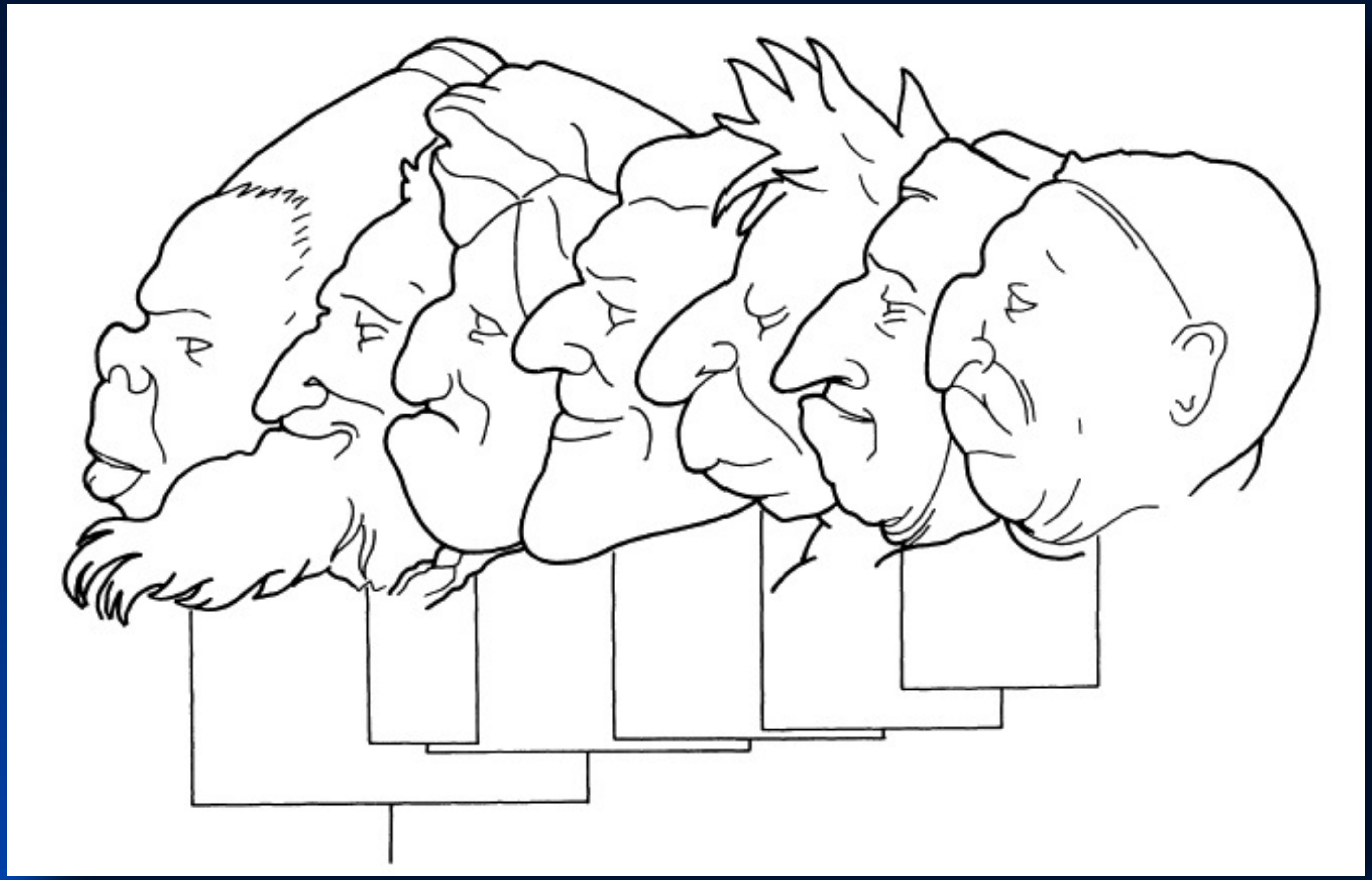


No aspect of geometry is recorded by these scalar magnitudes (= lengths).

Superposition Morphometrics

An Alternative Approach to the Characterization of Morphology





- Represents shapes using sets of landmarks.
- Adjusts translation, rotation and scaling to achieve configurations of maximum correspondence.
- Constructs an estimate of shape similarity by summing differences between landmark locations over all pairs of corresponding landmarks.

Diagrams from Benson et al. (1982)

Superposition Morphometrics

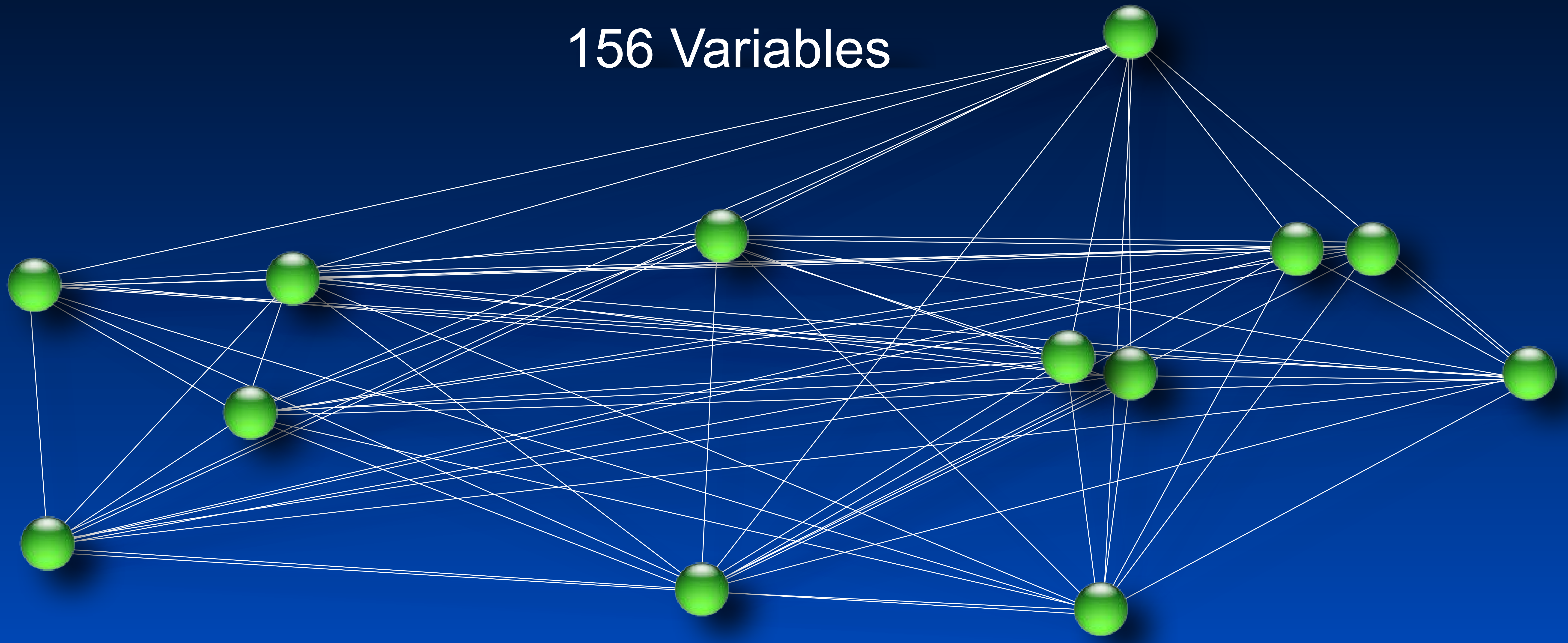
Morphometric Landmarks

- Landmarks are a specific points on a biological form (or image of a form) located according to some specific location rule.
- Landmarks are presumed to correspond in some sensible way over the forms included in a data set.
- There’s a good deal of confusion regarding whether landmarks constitute biologically homologous aspects of the forms.

Scalar Morphometrics

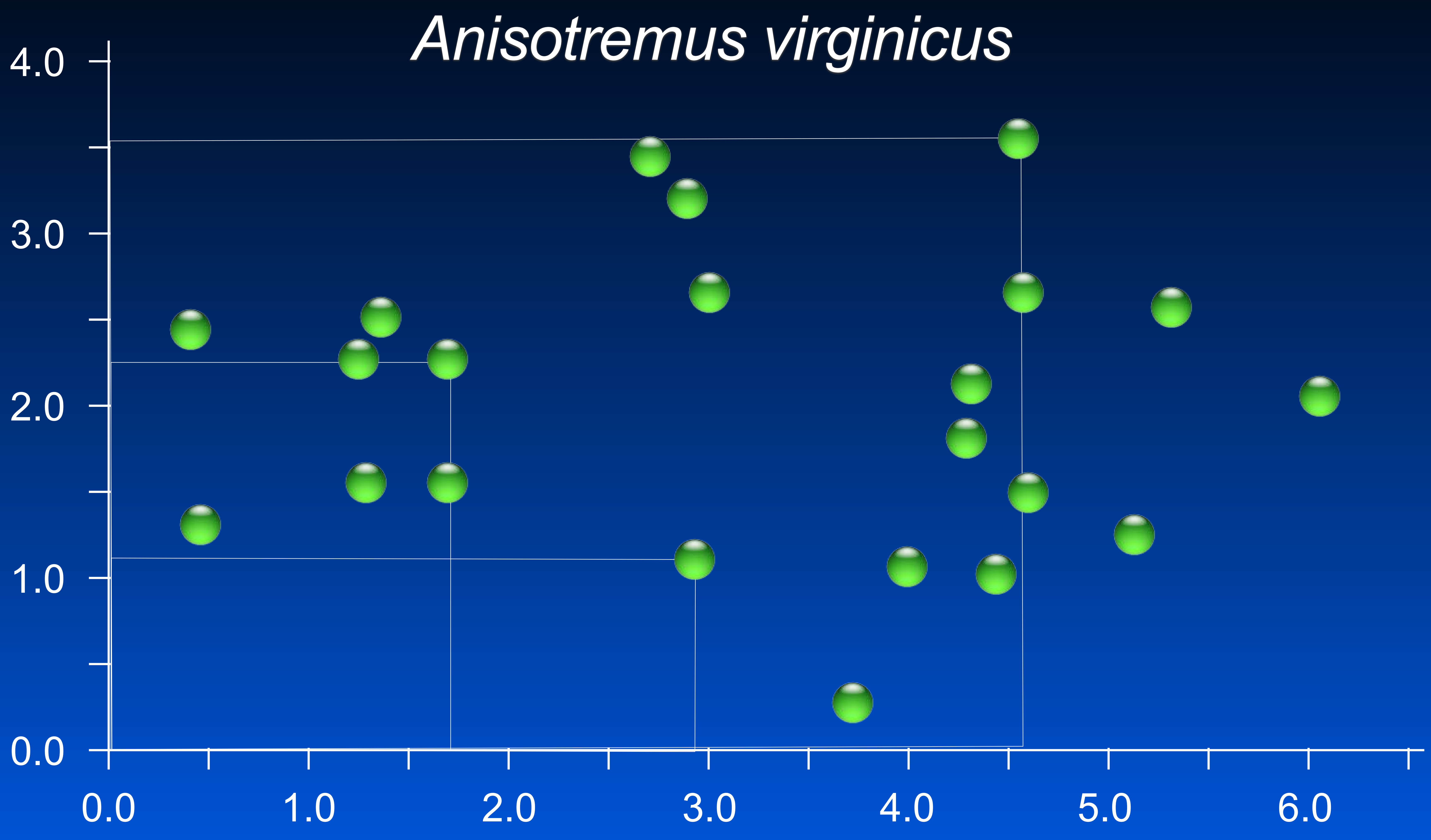
Anisotremus virginicus

156 Variables



Since the endpoints of the linear distances constitute landmarks, specification of a landmark configuration is implicit in the collection of distance data. However, traditional linear distance networks almost always constitute a much reduced subsample of the total inter-landmark distances that could be based on the same landmark configuration.

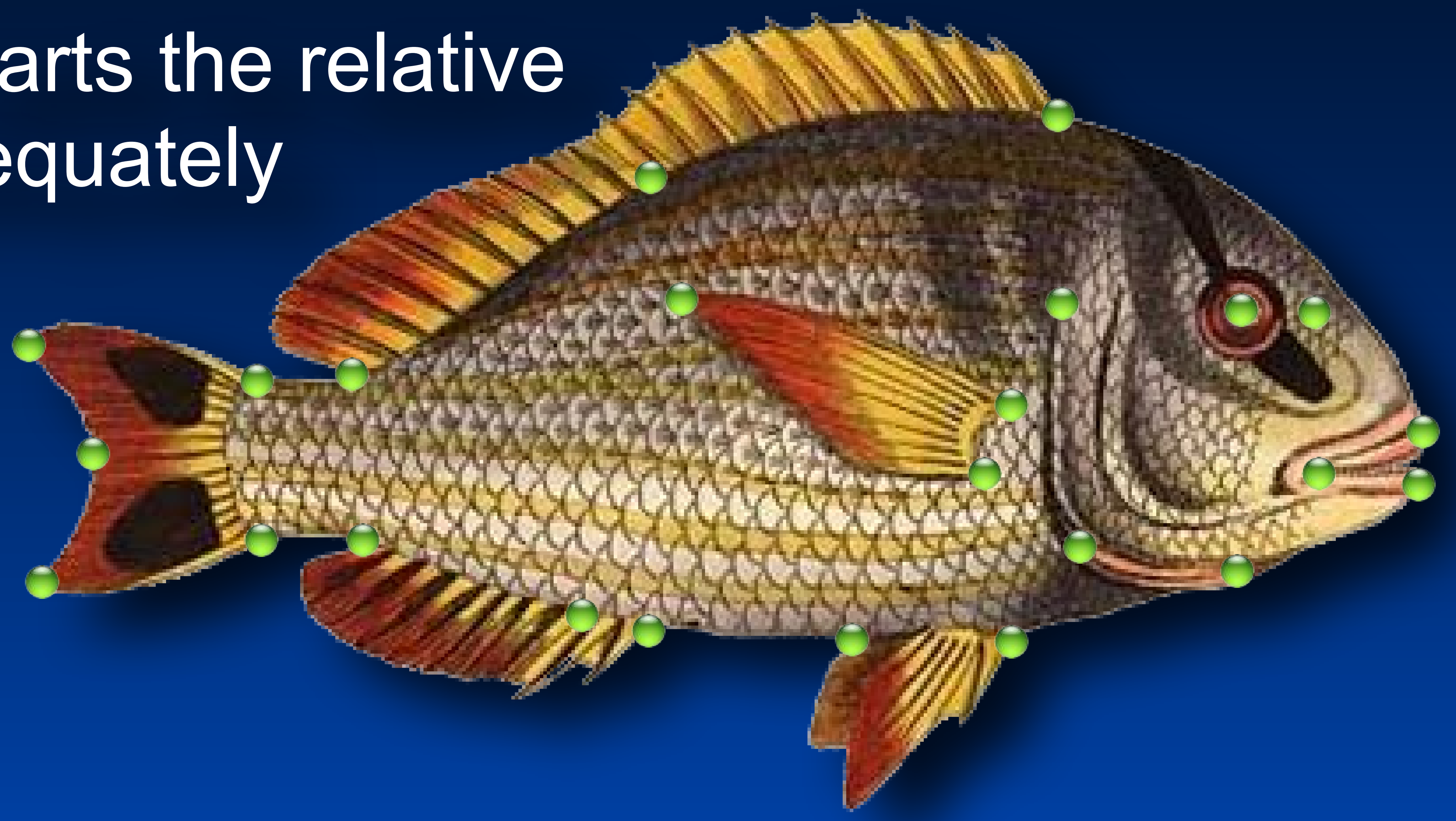
Scalar Morphometrics



But there is also a deep congruence between the linear distance mode of data collection and the landmark mode of data collection.

Superposition Morphometrics

When To Use Morphometric Landmarks



- When an object is composed of multiple parts the relative locations of which can be represented adequately by a single point.
- When the object is rigid (*e.g.*, bone, shell) such that the relative positions of its parts don't change.
- When all landmarks can be recognized unambiguously on all objects within the population/sample under study.
- When the hypothesis of interest pertains to, or can be reasonably supposed to involve, positions that can be represented by landmark points.

Superposition Morphometrics

How to Choose Morphometric Landmarks

- You must be able to give precise instructions regarding how each landmark is located.
- Landmarks should represent structures and be independent of the location of other landmarks and/or structures.
- Each landmark represents part of a corres-pondence hypothesis.
- The landmark configuration as a whole should encompass all relevant aspects of the form required for evaluating the hypothesis under consideration.

Superposition Morphometrics

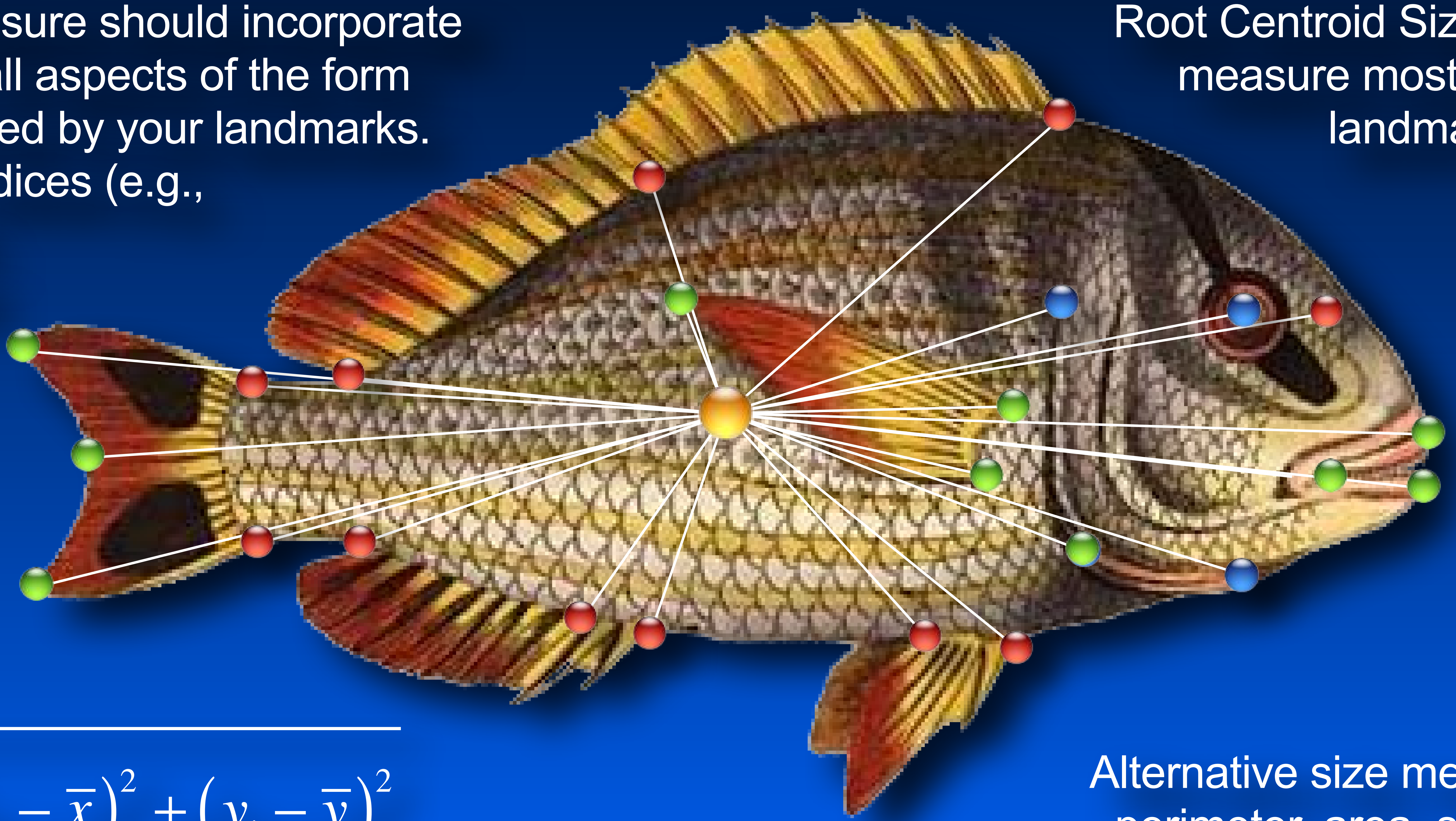
Types of Morphometric Landmarks

- Type 1 - a mathematical point that represents the juxtaposition of different tissues, organs, bones, sutures, or other unified component part of the form.
- Type 2 - a mathematical point that represents some geometric aspect of the form (e.g., maxima of curvature).
- Type 3 - a mathematical point whose location contains at least one 'deficient' or 'dependent' coordinate (e.g., points that are 'farthest' from other points, extremal points, centroids, endpoints of diameters, etc.).

Superposition Morphometrics

Size in the Context of Morphometric Landmarks

Ideally a size measure should incorporate information from all aspects of the form that are represented by your landmarks. Traditional size indices (e.g., maximum length, Feret ratio) rarely do this and so should be avoided.



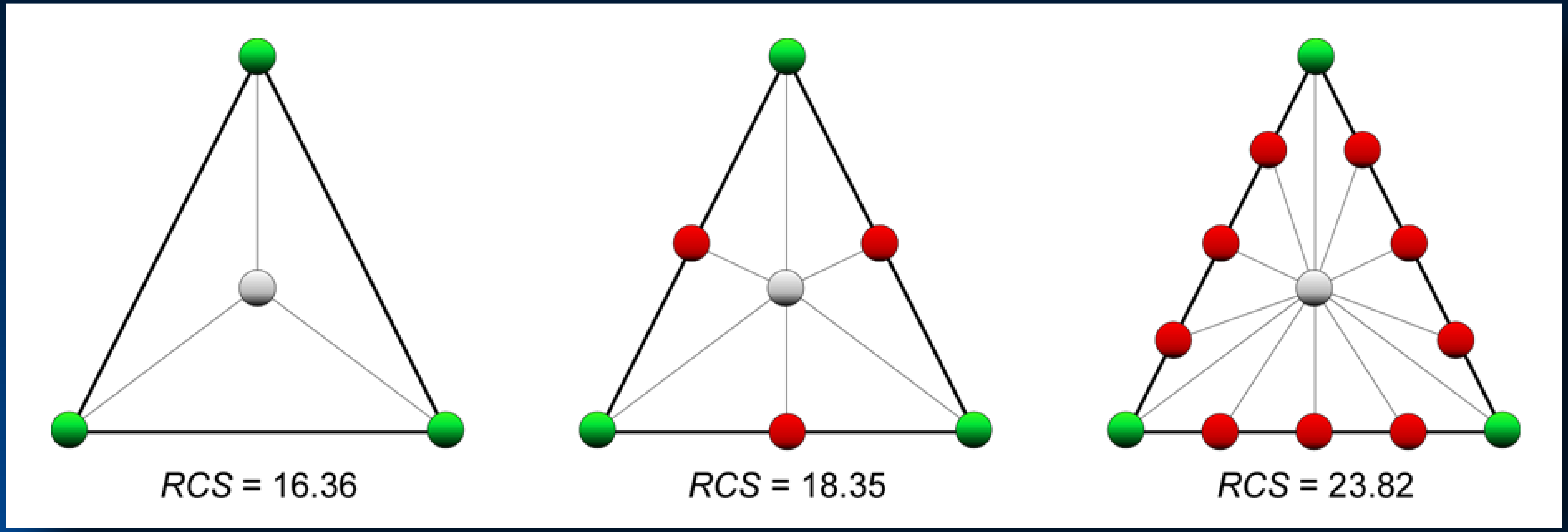
Root Centroid Size (RCS) is the size measure most commonly used in landmark morphometrics.

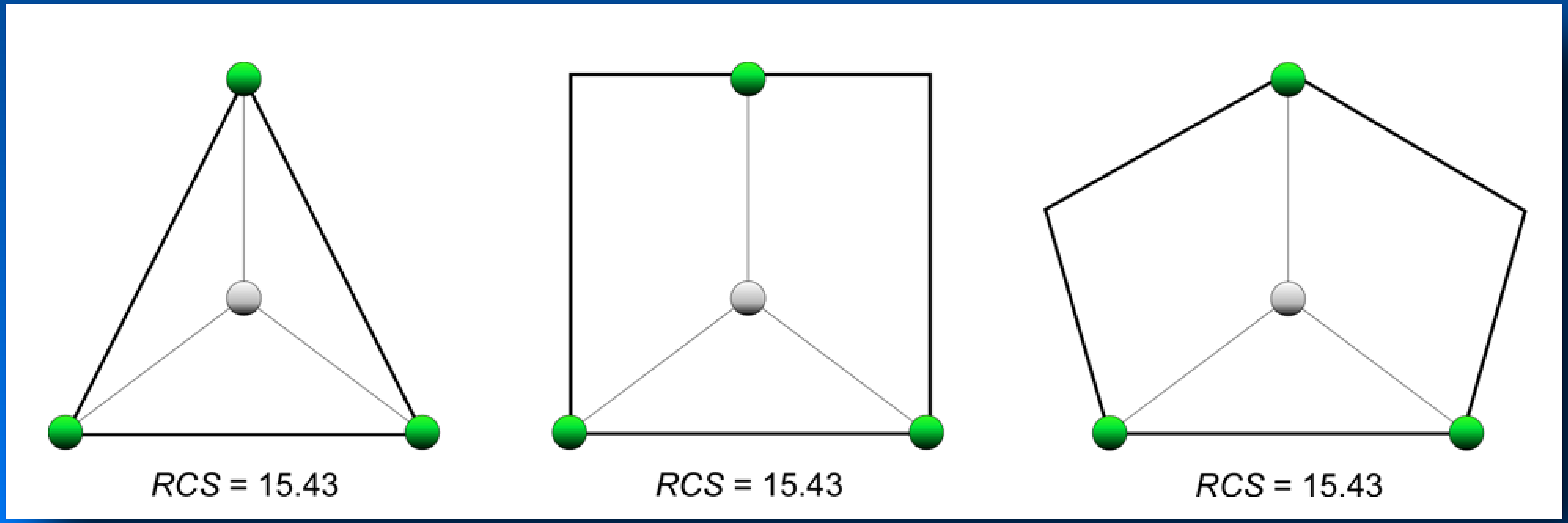
$$RCS = \sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 + (y_i - \bar{y})^2}$$

Alternative size measures include the perimeter, area, square root of area, volume, cube root of volume, weight/mass

Superposition Morphometrics

Lingering Problems With Centroid Size





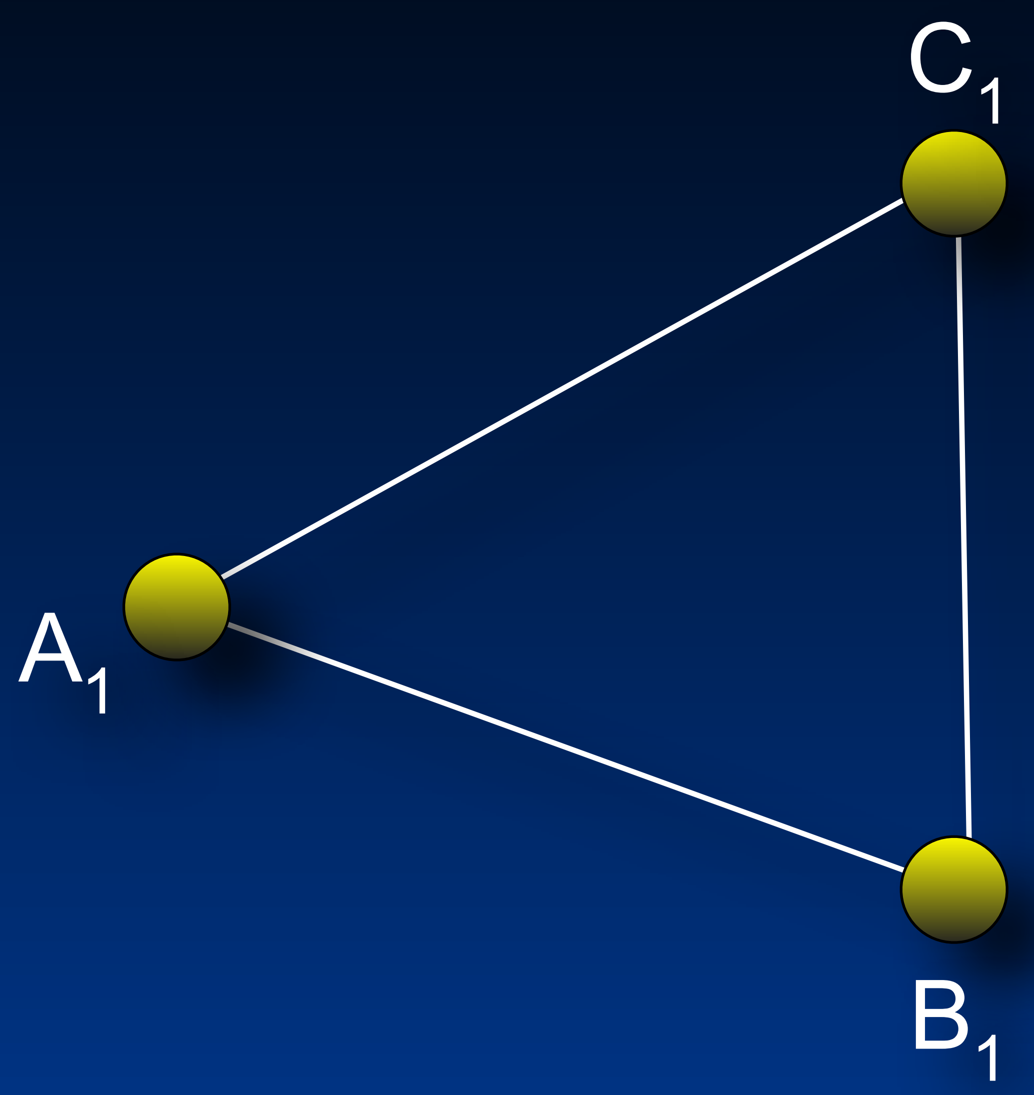
Superposition Morphometrics

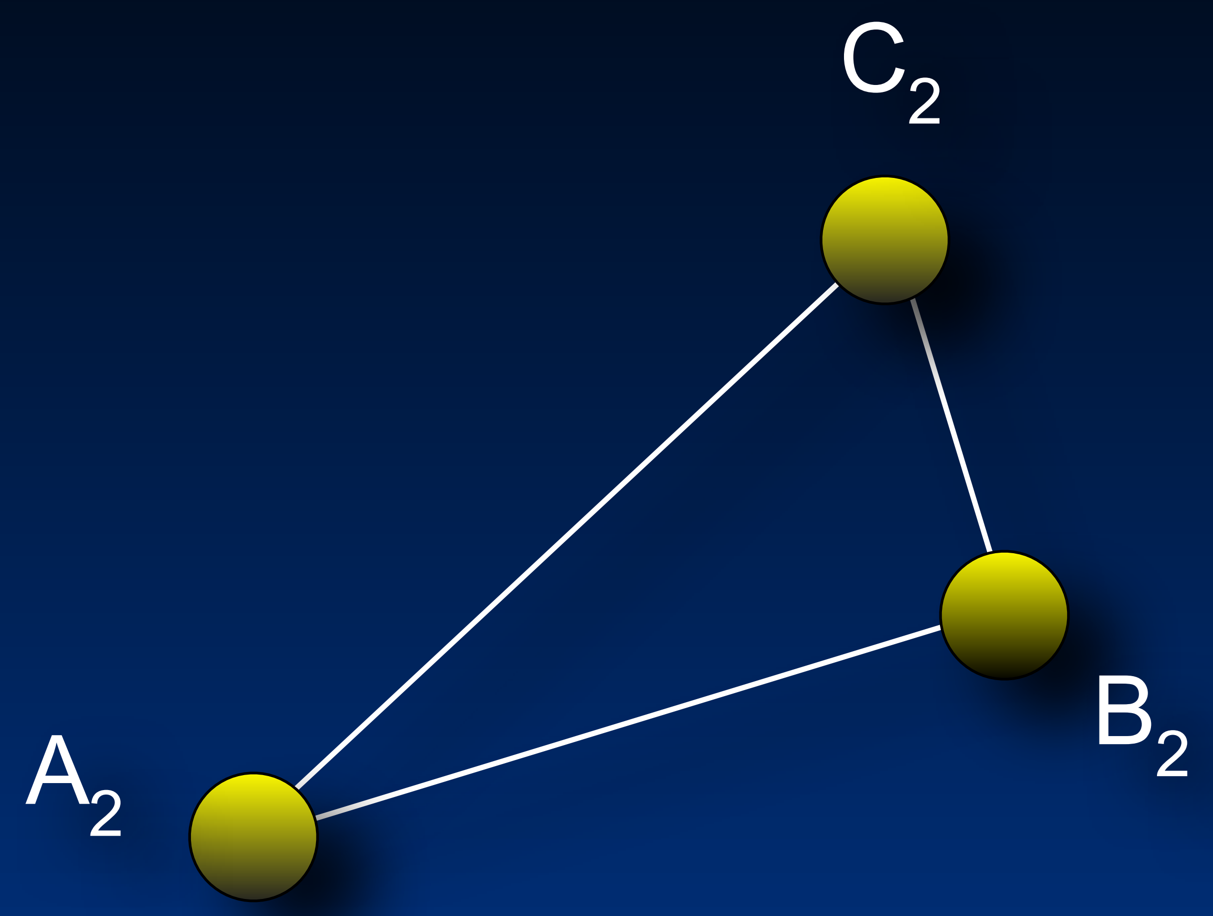
Alternatives to Centroid Size

- Perimeter
- Area or square root of area
- Volume or cube root of volume
- Weight or mass
- Square-root of mean of sum of all squared distances from centroid to each landmark

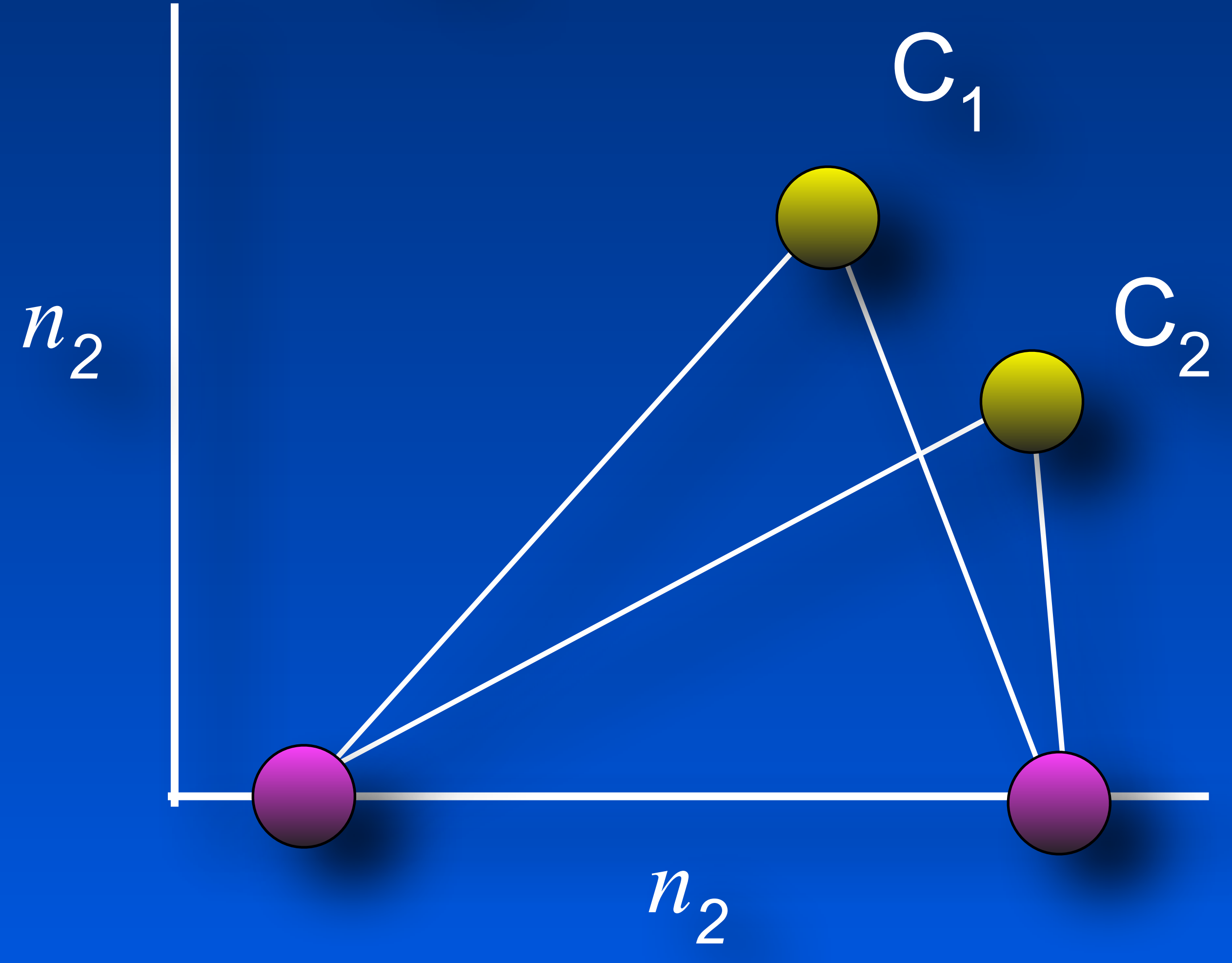
Superposition Morphometrics

Landmark Alignment

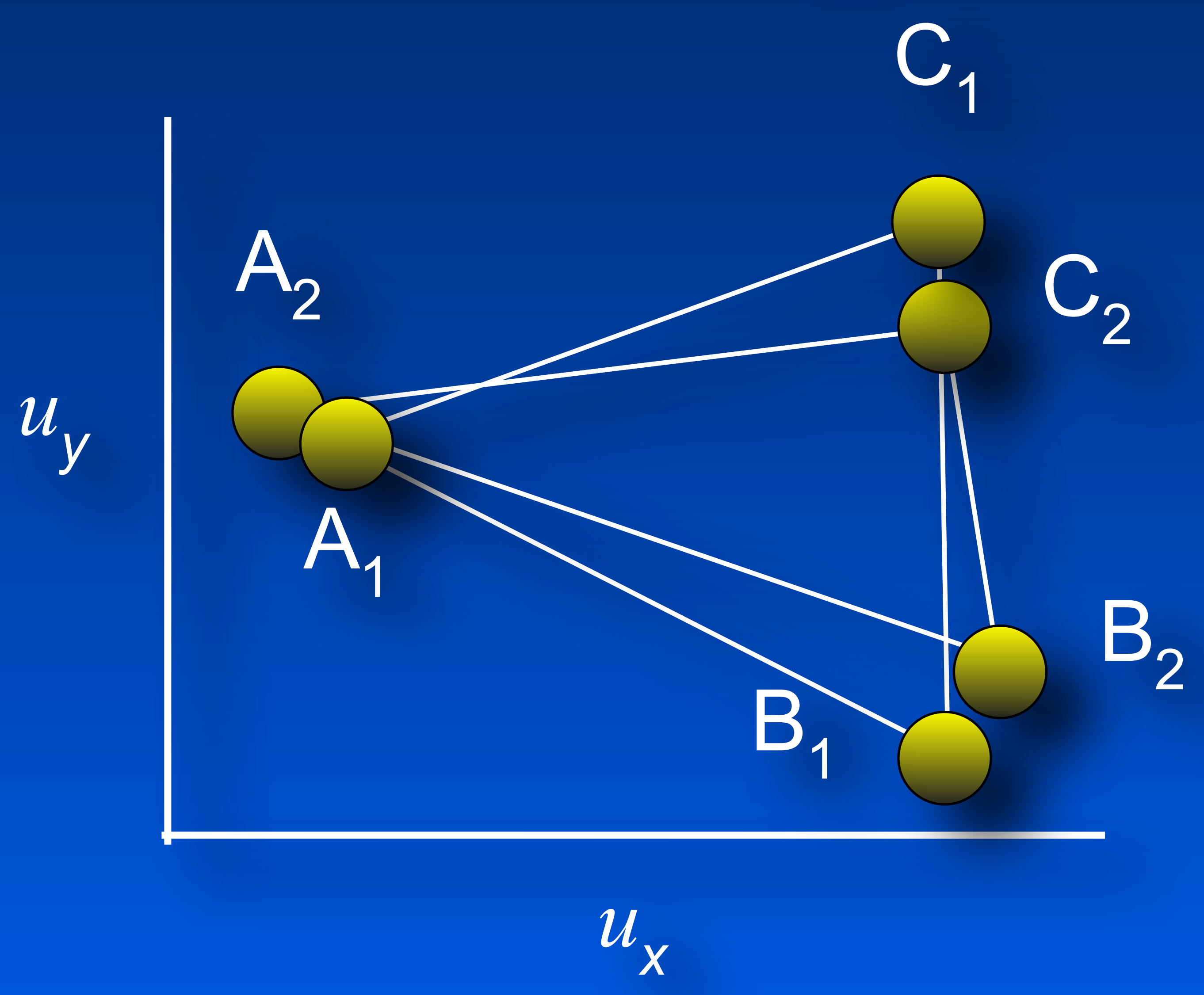




- Position - positions differ.
- Scale - sizes differ.
- Rotation - orientations differ.



Bookstein
Shape Coordinates



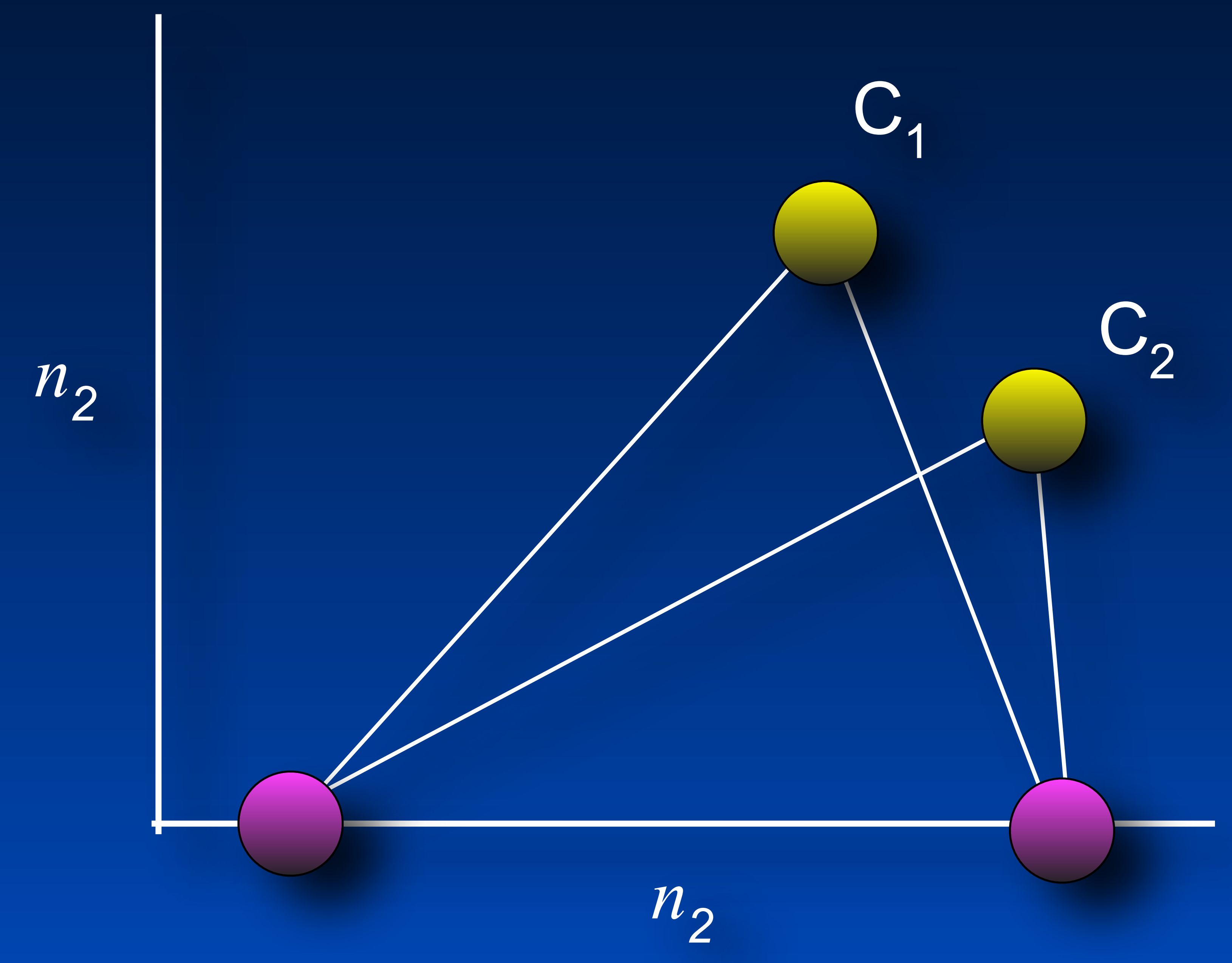
Procrustes
Shape Coordinates

Superposition Morphometrics

Bookstein Shape Coordinates

$$\eta_1 = \frac{\left(x_B - x_A\right)\left(x_C - x_A\right) + \left(y_B - y_A\right)\left(y_C - y_A\right)}{\left(x_B - x_A\right)^2 + \left(y_B - y_A\right)^2}$$

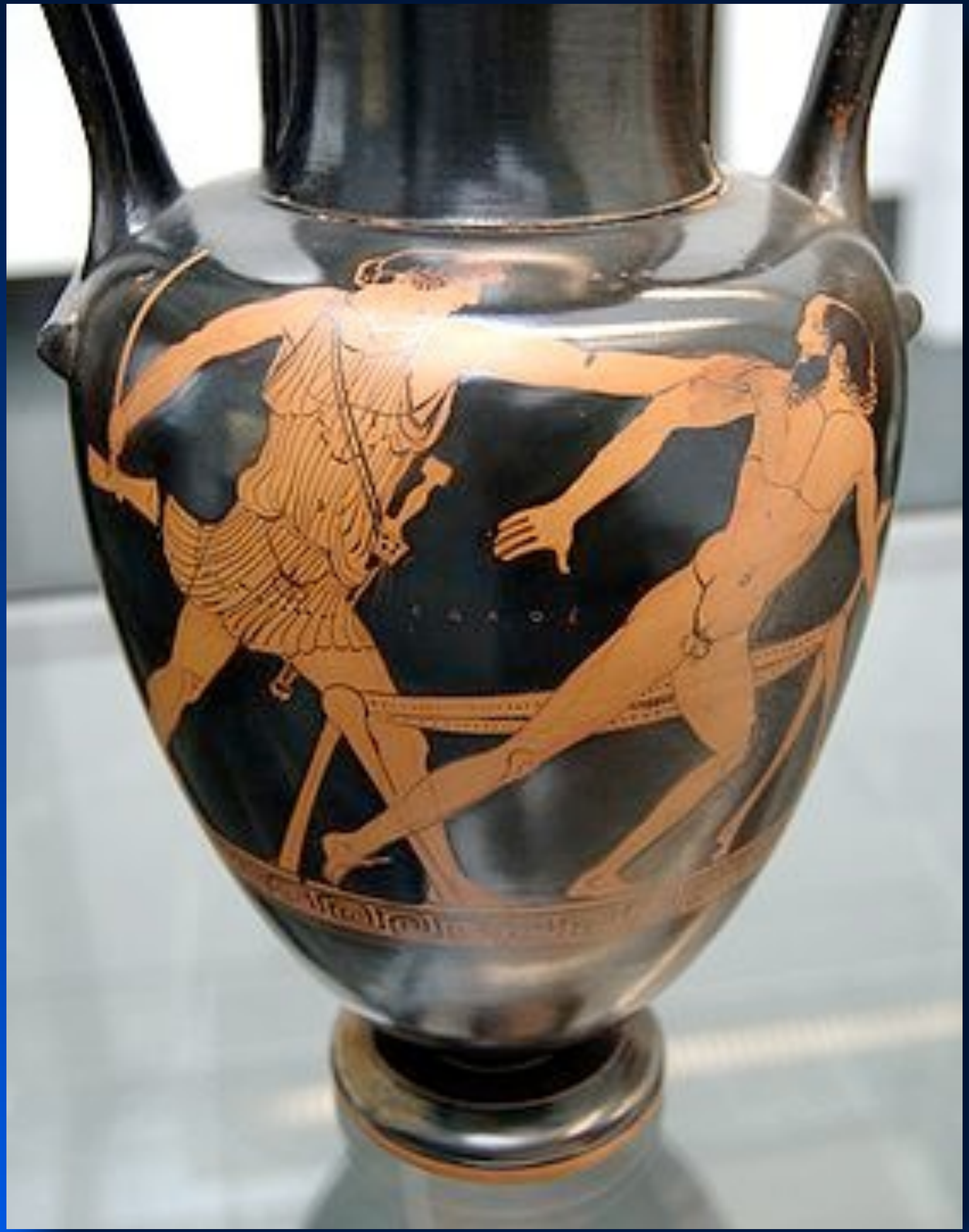
$$\eta_2 = \frac{\left(x_B - x_A\right)\left(y_C - y_A\right) - \left(y_B - y_A\right)\left(x_C - x_A\right)}{\left(x_B - x_A\right)^2 + \left(y_B - y_A\right)^2}$$

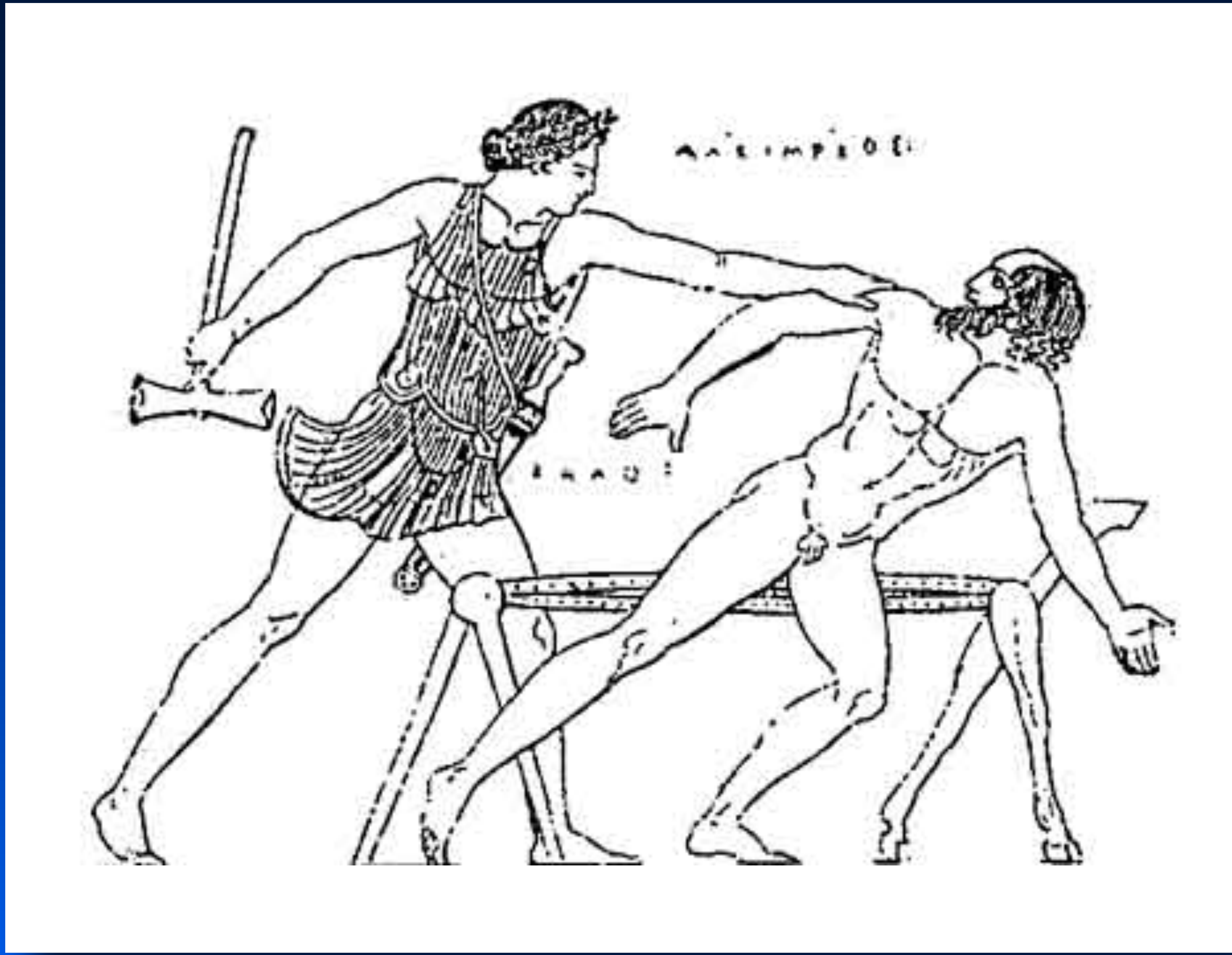


Bookstein
Shape Coordinates

Superposition Morphometrics

Procrustes Alignment

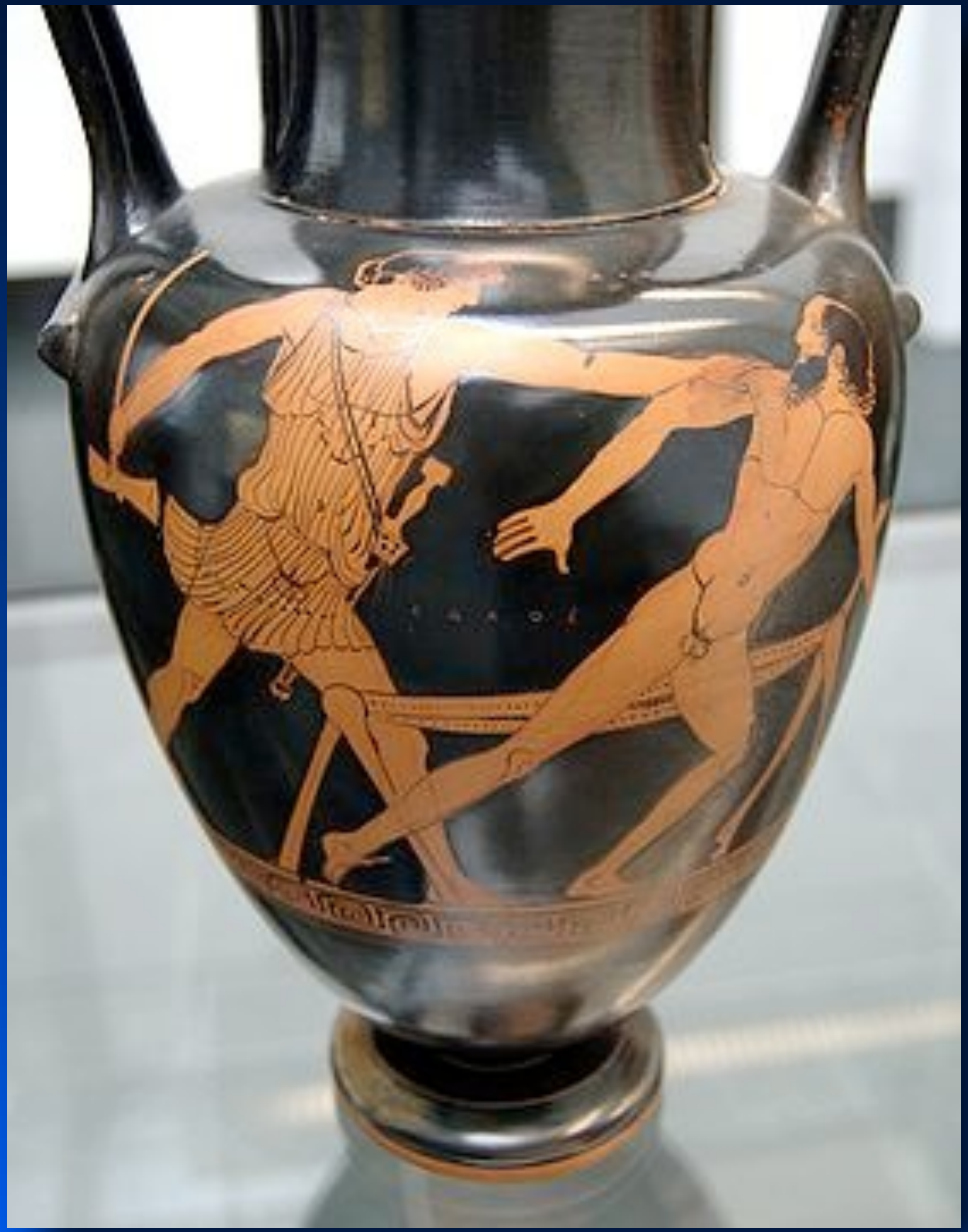




Superposition Morphometrics

Procrustes Alignment

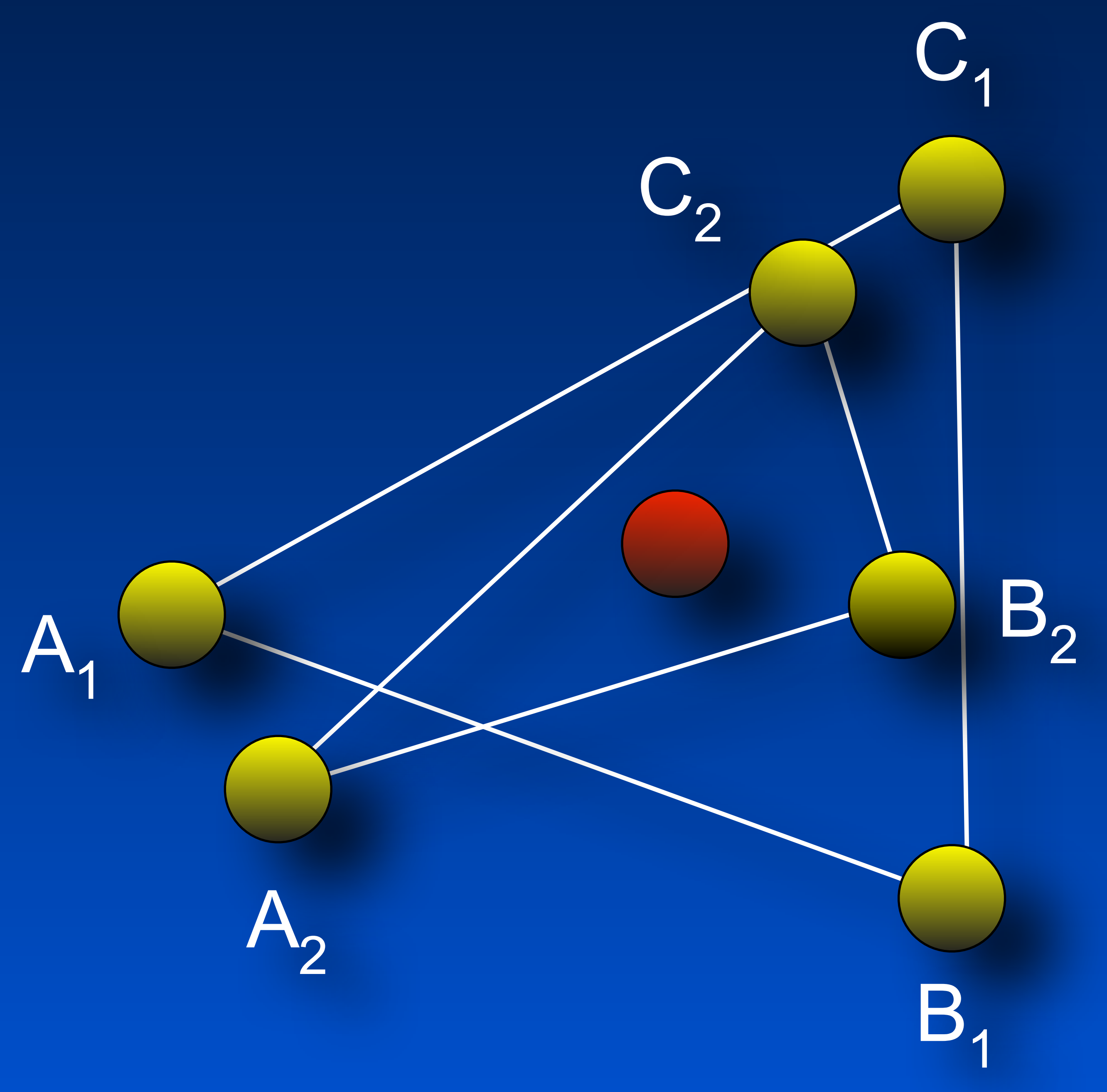
Superimposition method that minimizes the sum of squared deviations of landmark locations from corresponding landmarks. Least-squares methods are somewhat sensitive to the inclusion of shape outliers in the sample, but this model has well-known statistical properties and a well-understood distributional theory that supports the probabilistic inference of shape differences.



Superposition Morphometrics

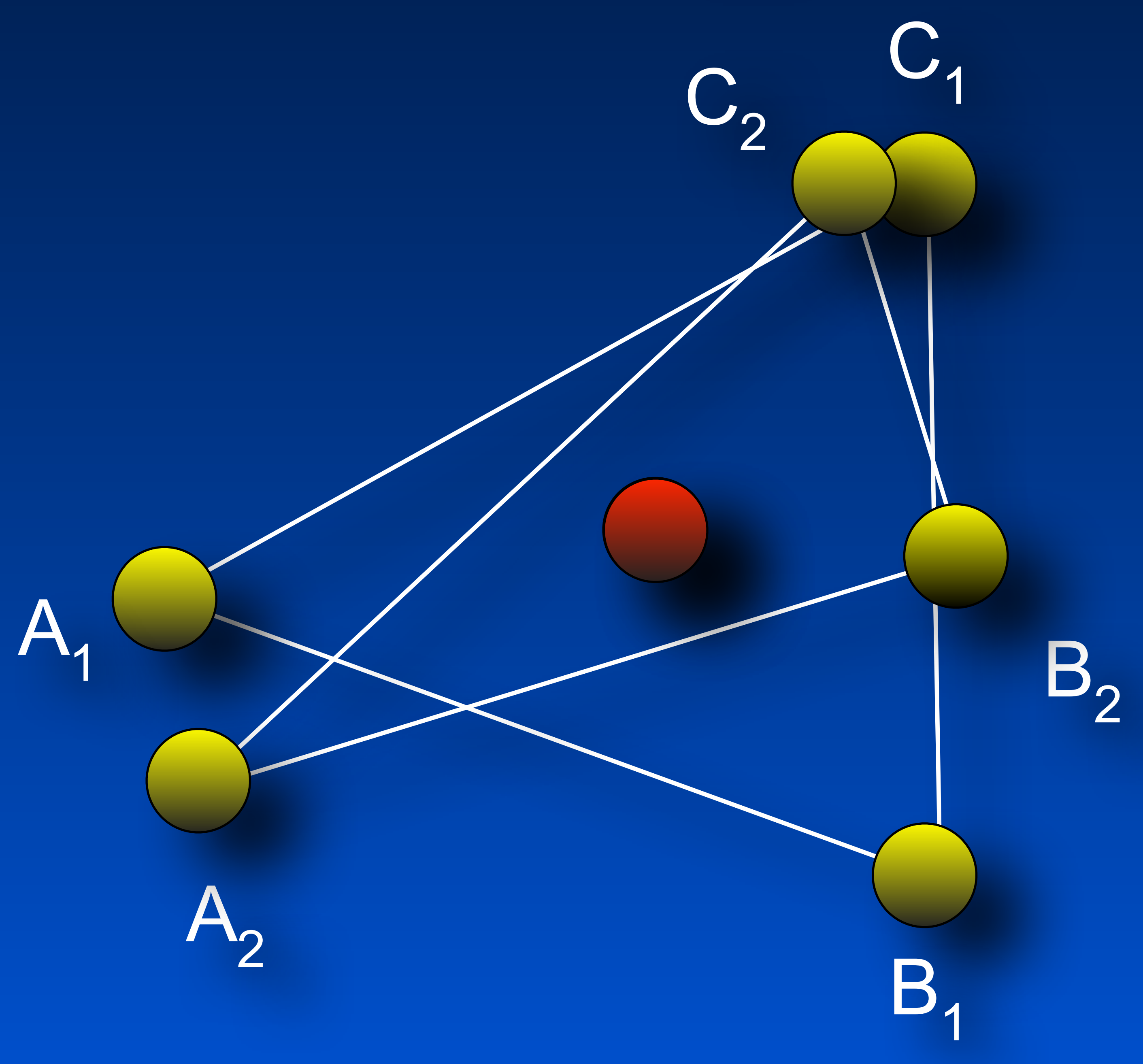
Procrustes Alignment

Translation



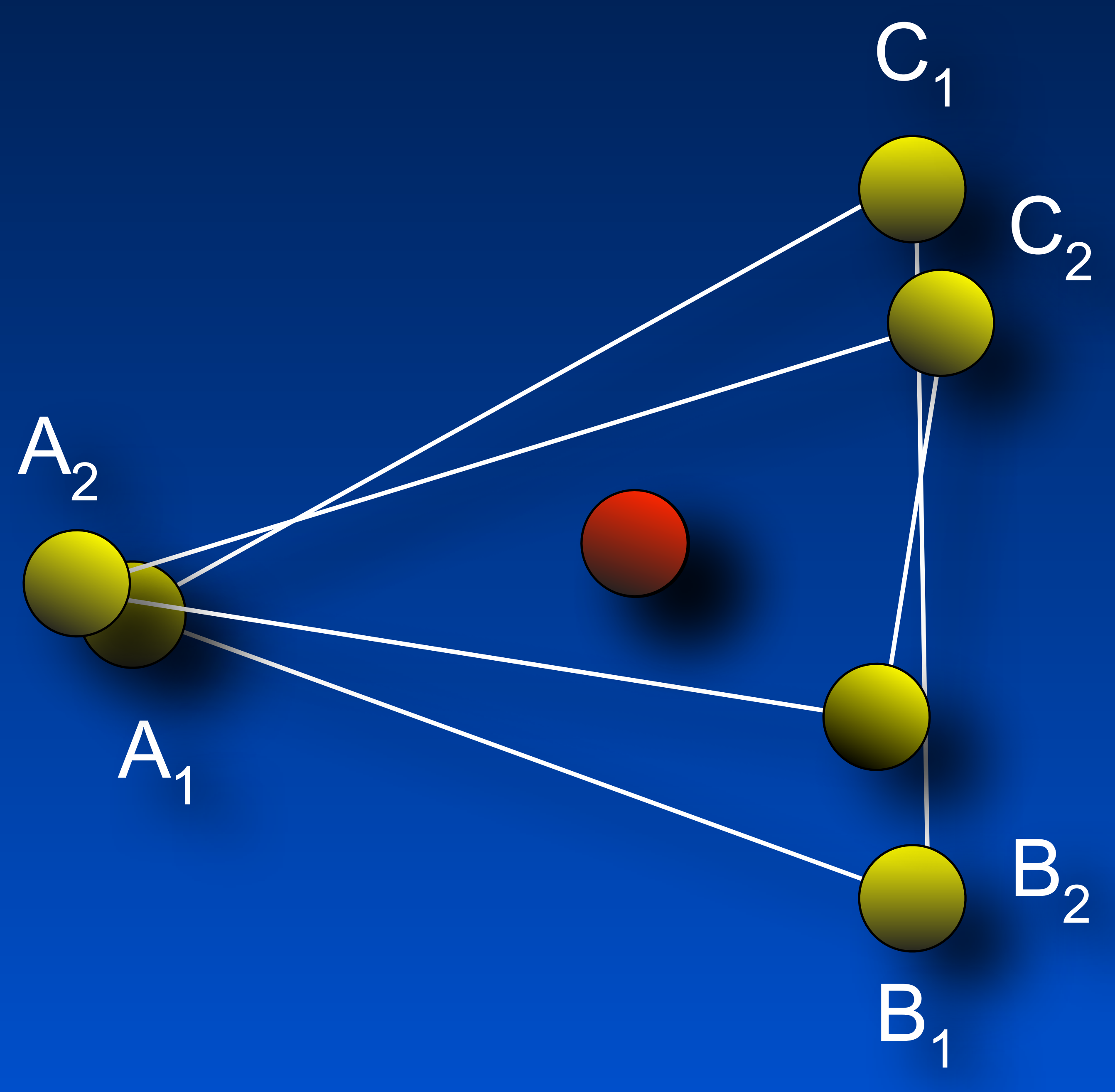
$$\hat{x}_i = x_i - \bar{x}; \quad \hat{y}_i = y_i - \bar{y}$$

Scaling



$$\hat{x}_i = x_i / RCS; \quad \hat{y}_i = y_i / RCS$$

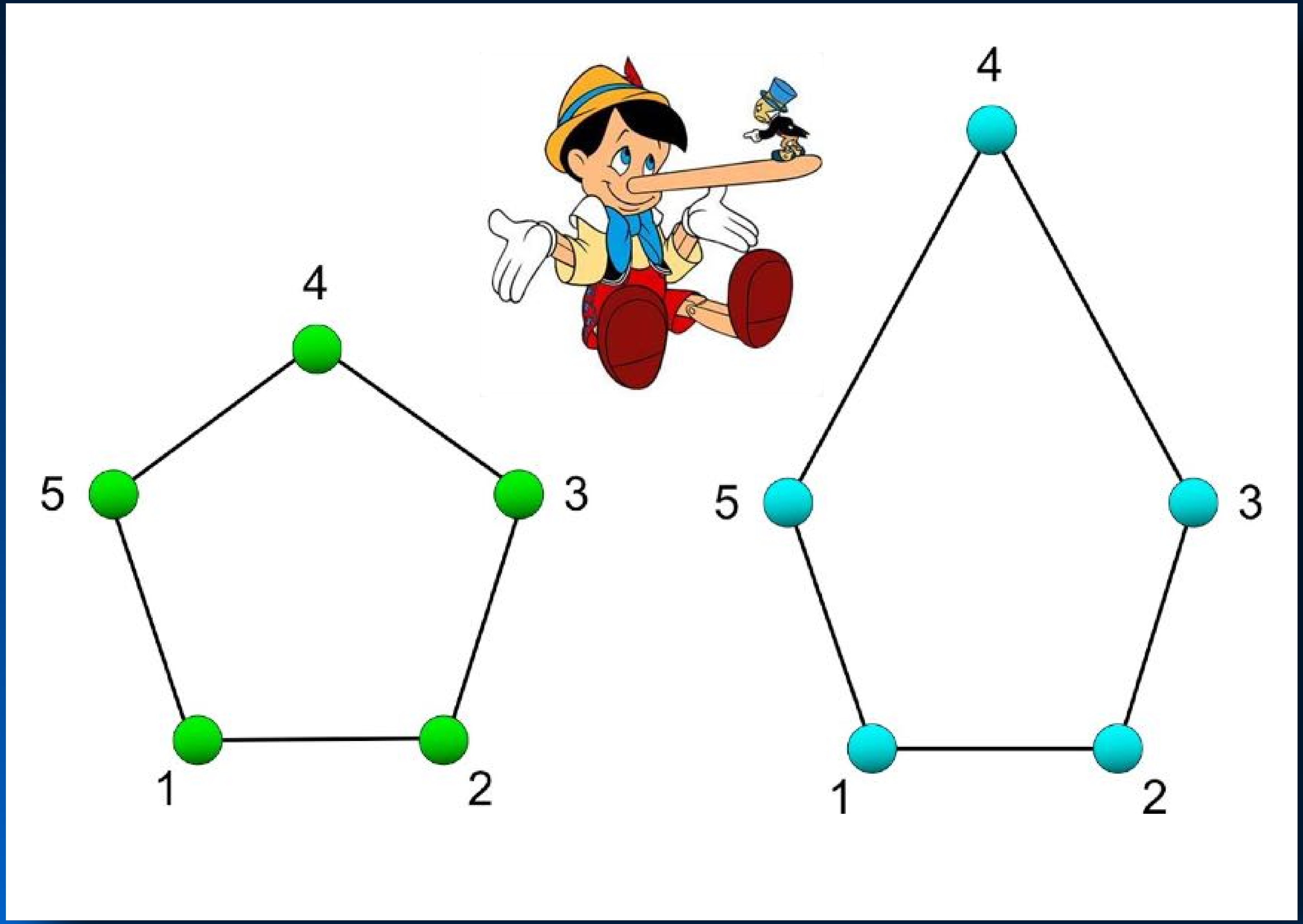
Rotation

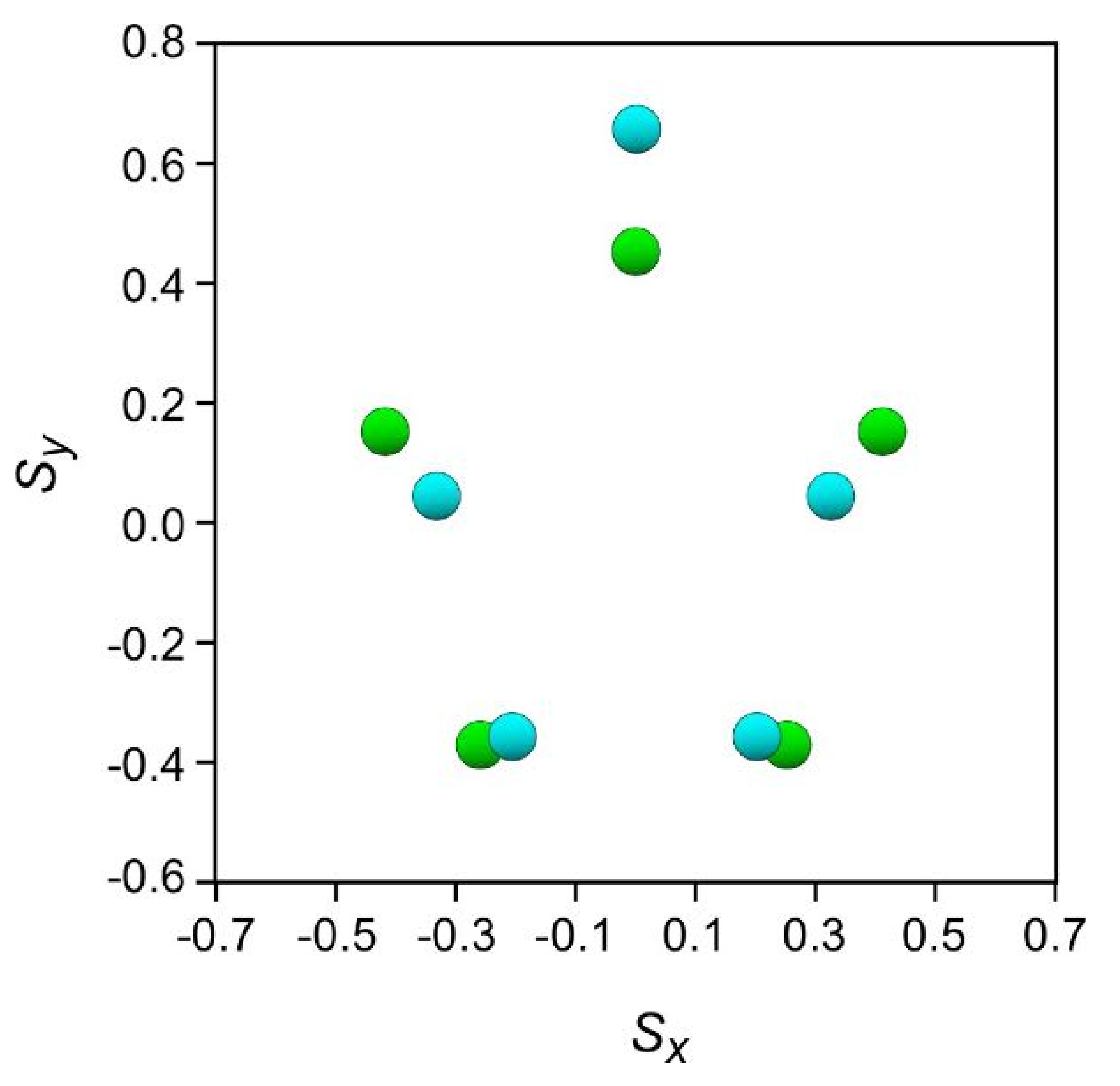


$$\theta = \arctan\left(\sum_{i=1}^{m} y_{Ti}x_{Ri} - x_{Ti}y_{Ri} \middle/ \sum_{i=1}^{m} x_{Ti}x_{Ri} - y_{Ti}y_{Ri} \right)$$

Superposition Morphometrics

Pinocchio Effect



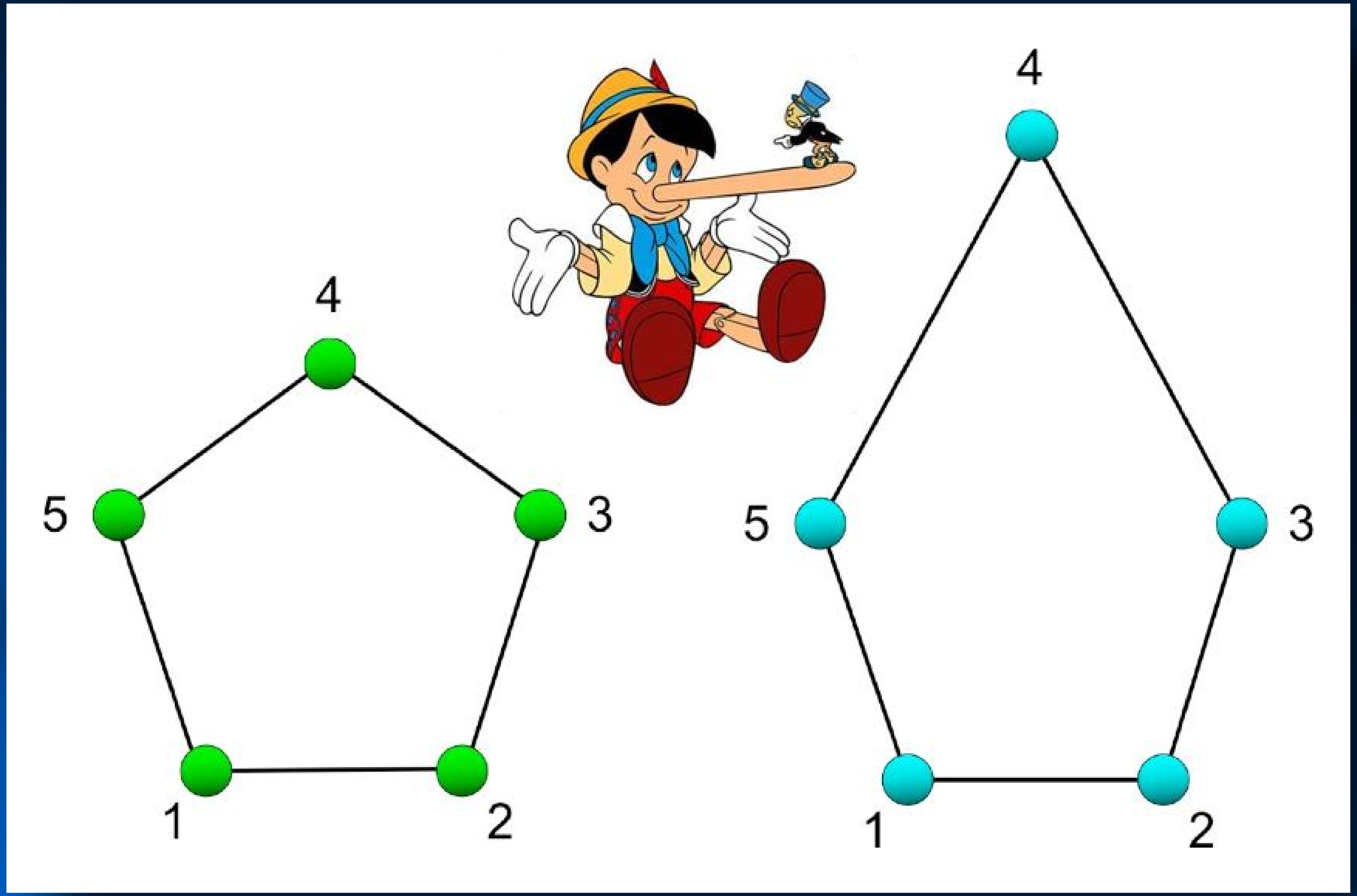


The Pinocchio Effect arises when shape change is focused on just one, or a small number, of landmarks located close to one another. Procrustes alignment will attempt to spread the deformation over all landmarks, leading to erroneous interpretations.

Superposition Morphometrics

Resistant-Fit Procrustes Superposition

Superimposition method that uses median (standard) and repeated-median (generalized) based estimates of fitting parameters rather than least-squares estimates. Resistant-fit procedures are less sensitive to subsets of extreme values than those of strictly least-squares methods. As such, their results may provide a more accurate description of differences in shape that are due to changes in the positions of just a few landmarks.



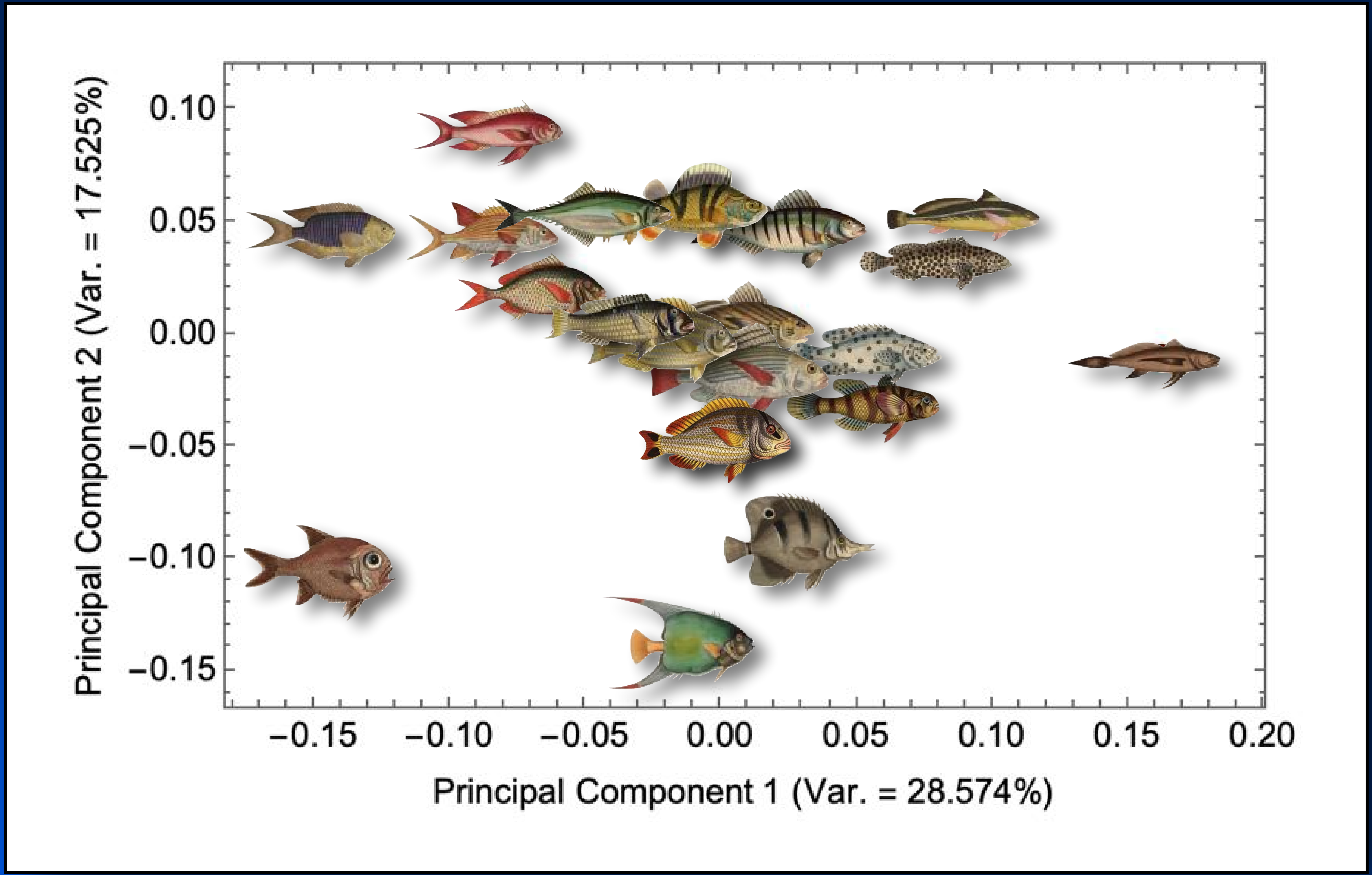
Scalar Morphometrics

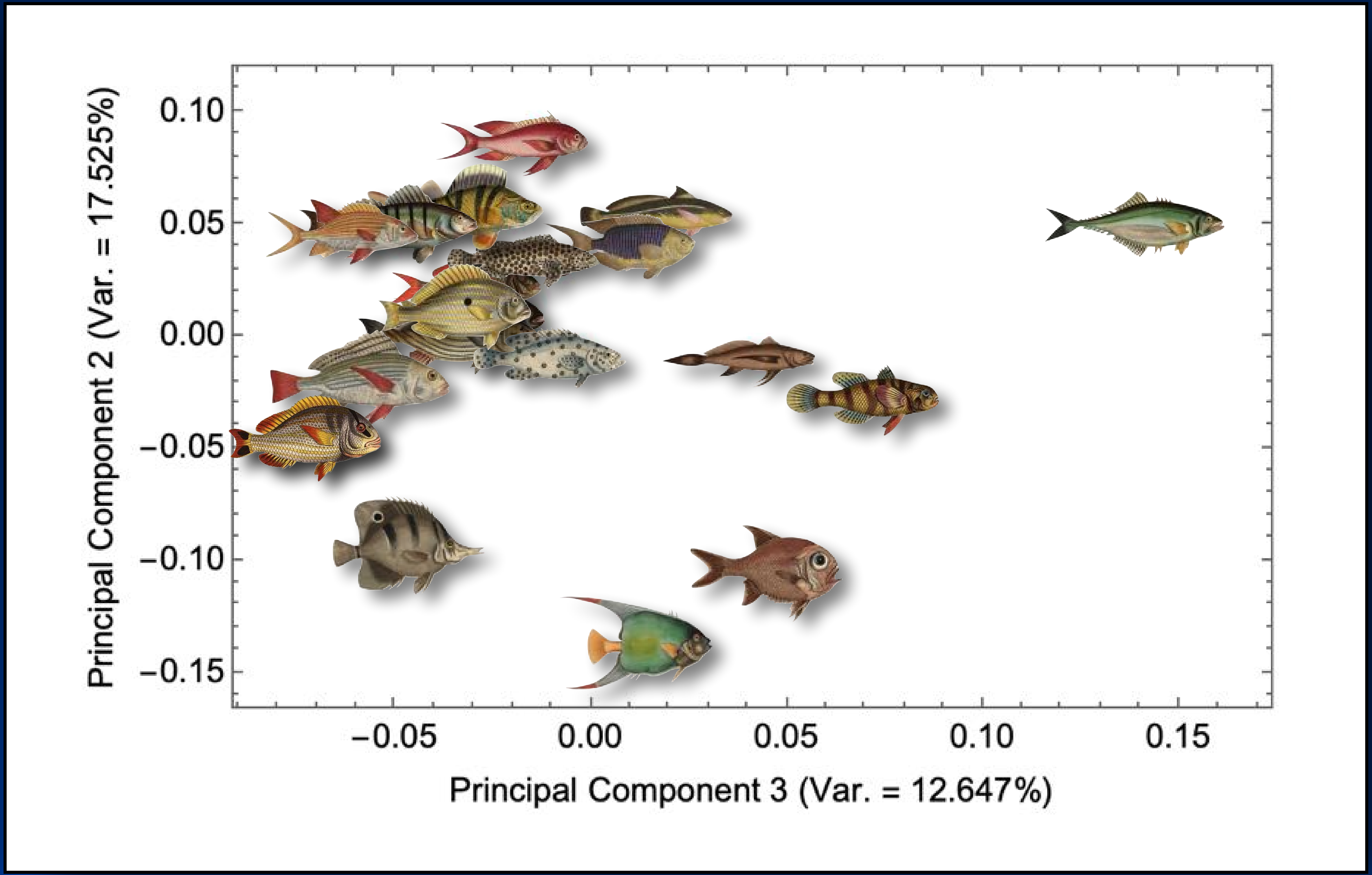
Scalar (Linear Distance) Characterization of Morphology

Superposition Morphometrics

"Geometric" (Landmark) Characterization of a Fish Shape Variation

A Multivariate Approach to the Visualization of Form/Shape Variation via PCA

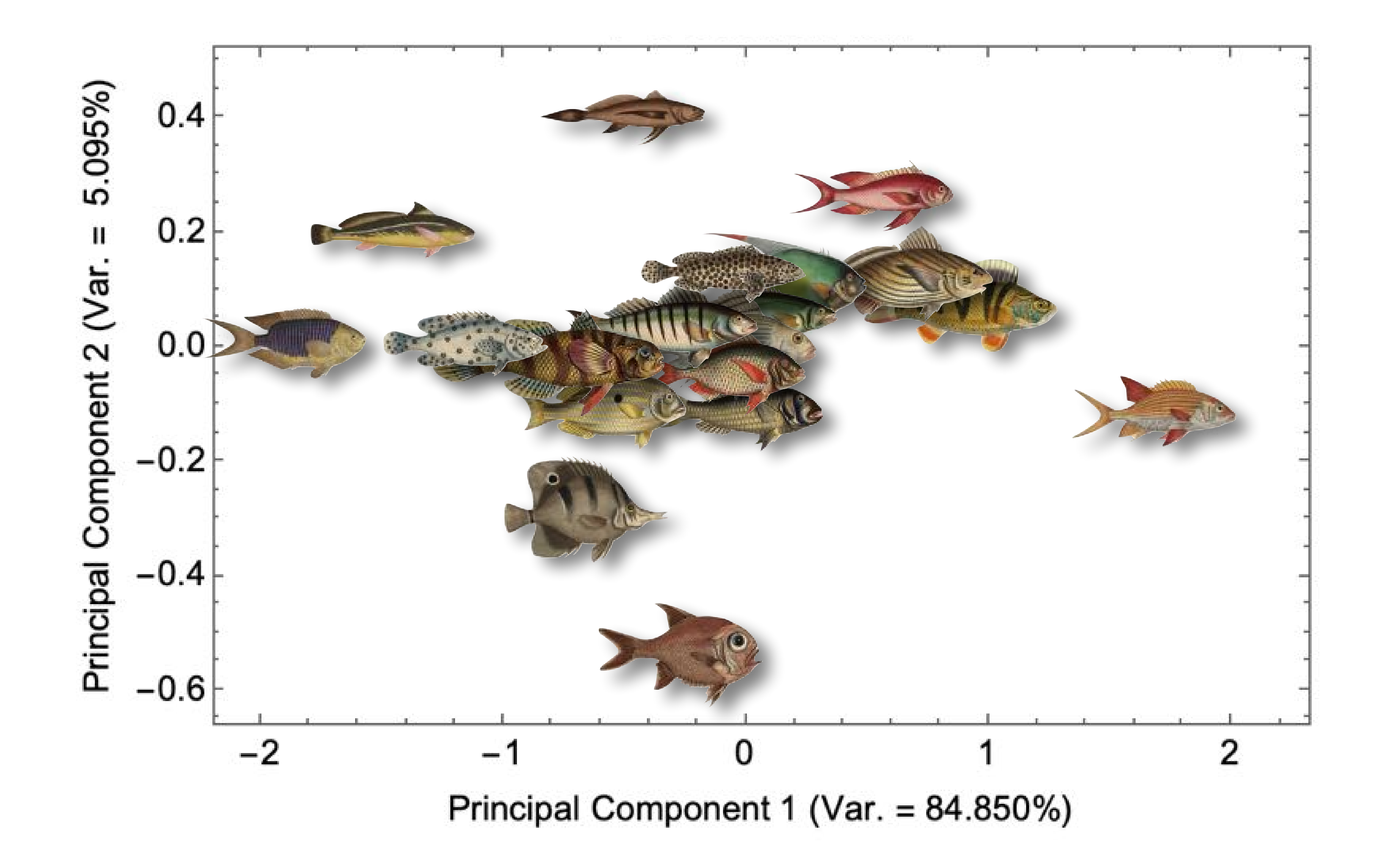


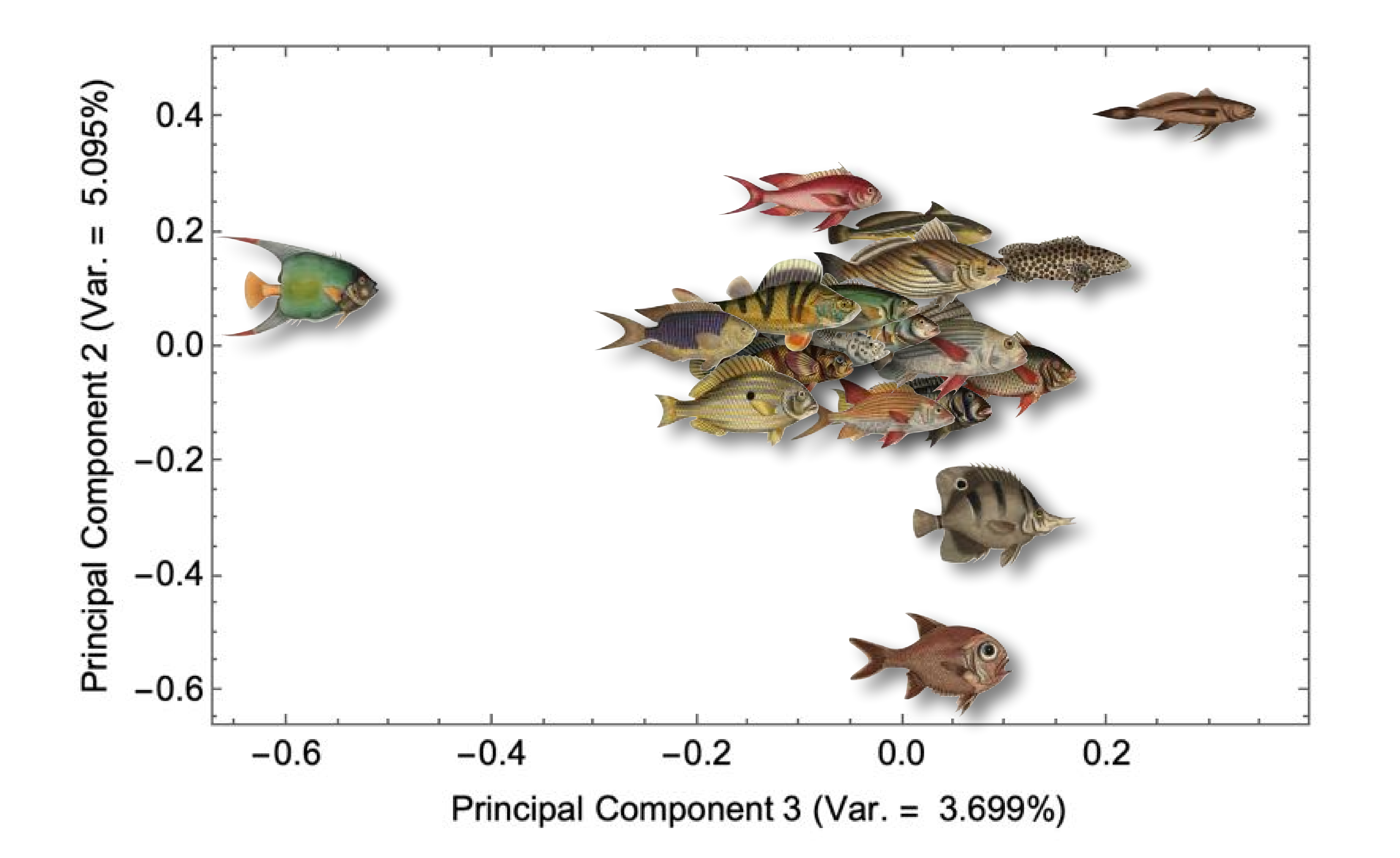


Scalar Morphometrics

Linear Distance-Based Characterization of a Fish Shape Variation

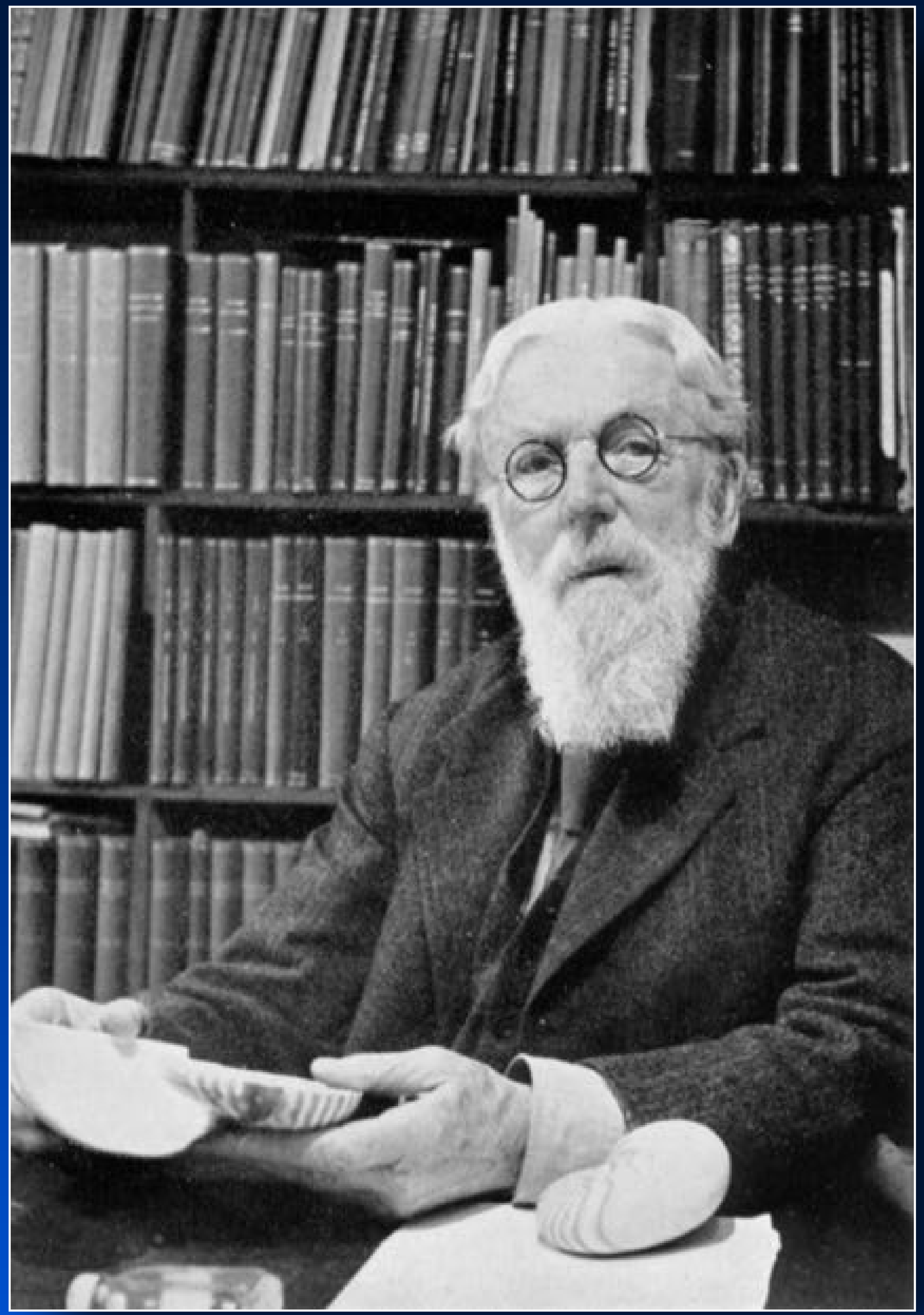
A Multivariate Approach to the Visualization of Form/Shape Variation via PCA



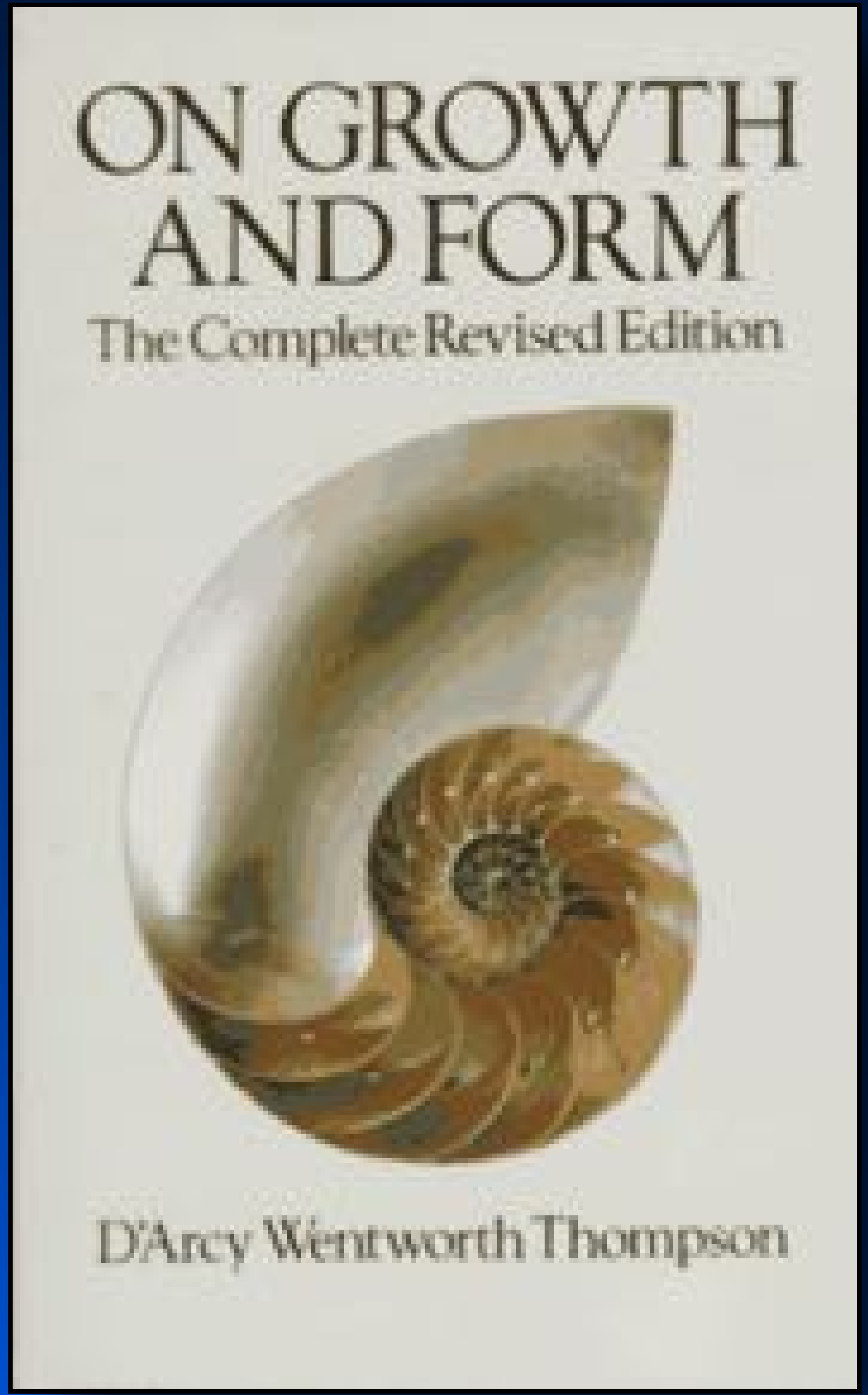


Deformation Morphometrics

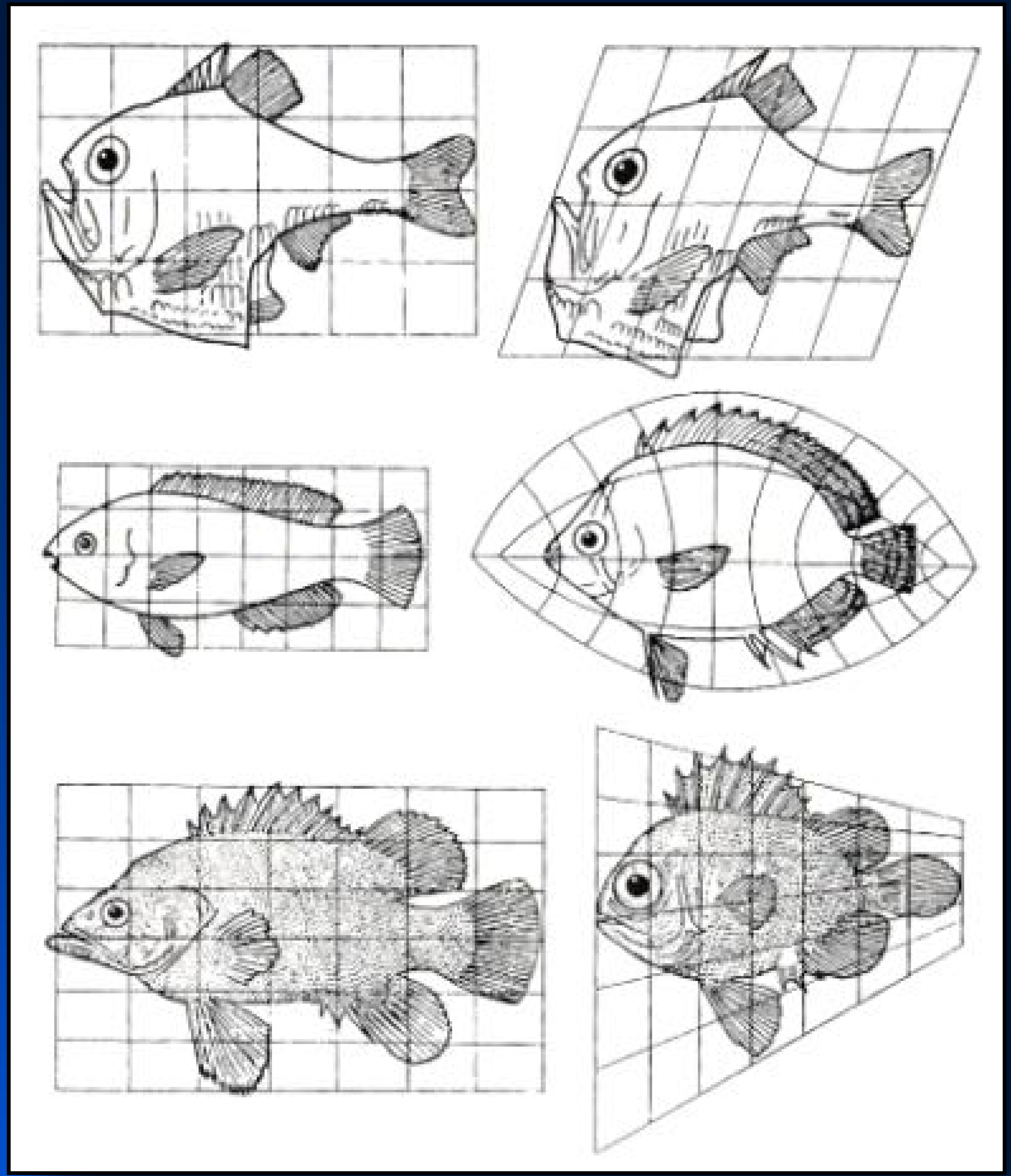
Shape Change As a Deformation



D'Arcy Wentworth Thompson
(1860 – 1948)



Thompson (1913)

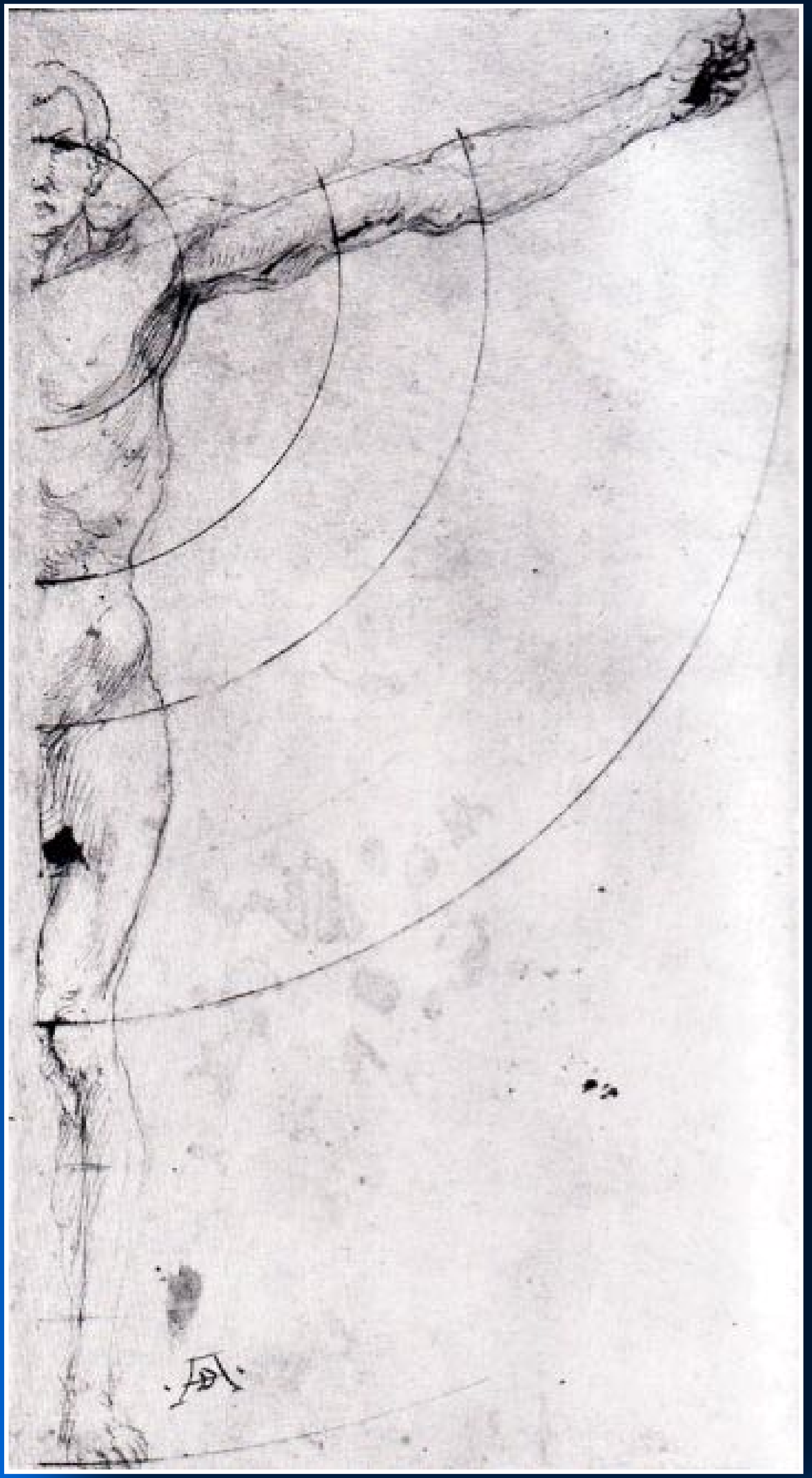


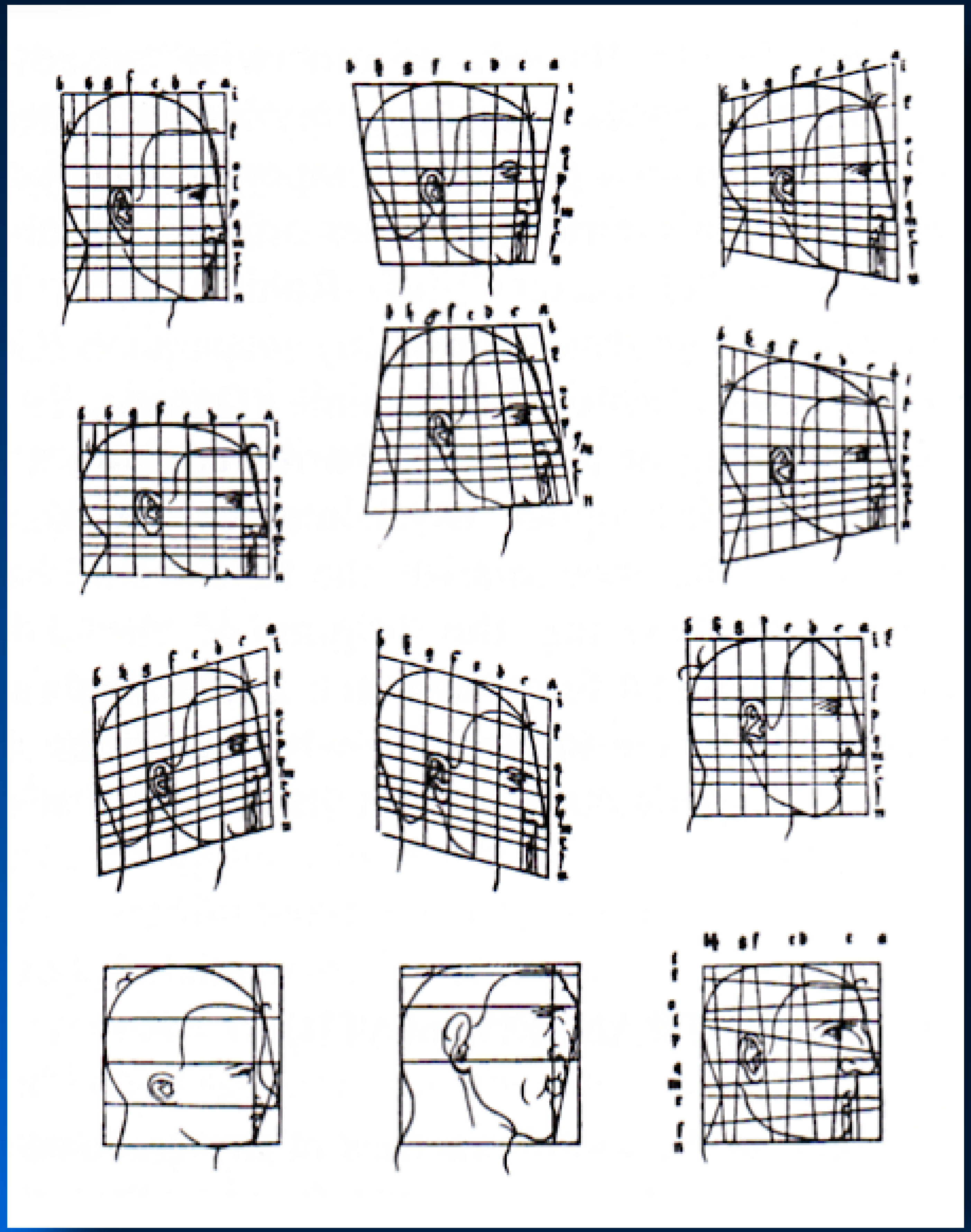
Thompsonian Deformation Grids

Description & Analysis of Morphology

Artistic & Architectural Origins

Albrecht Dürer
(1471 - 1528)

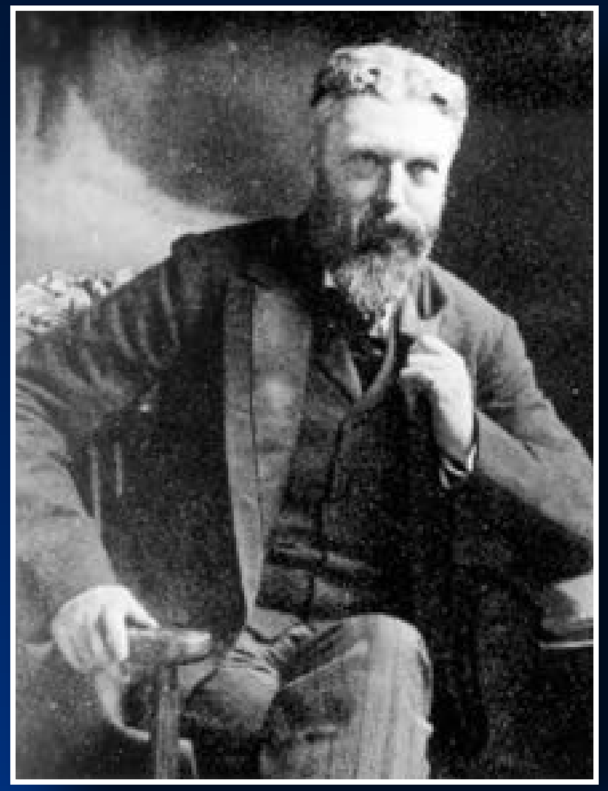


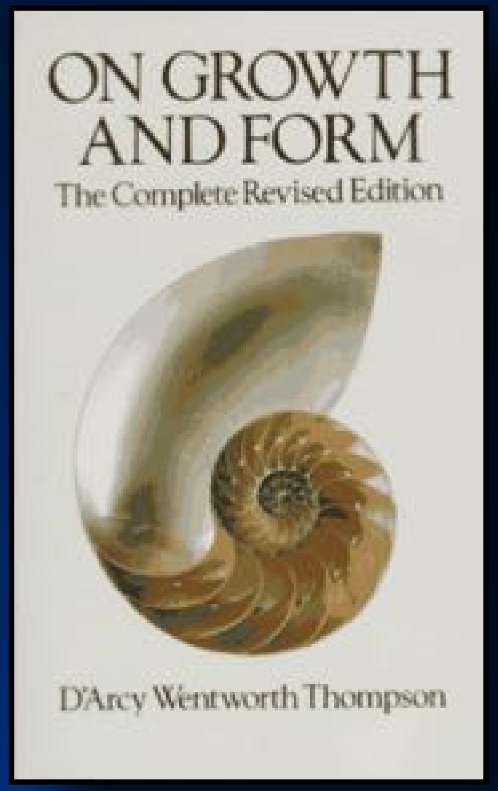


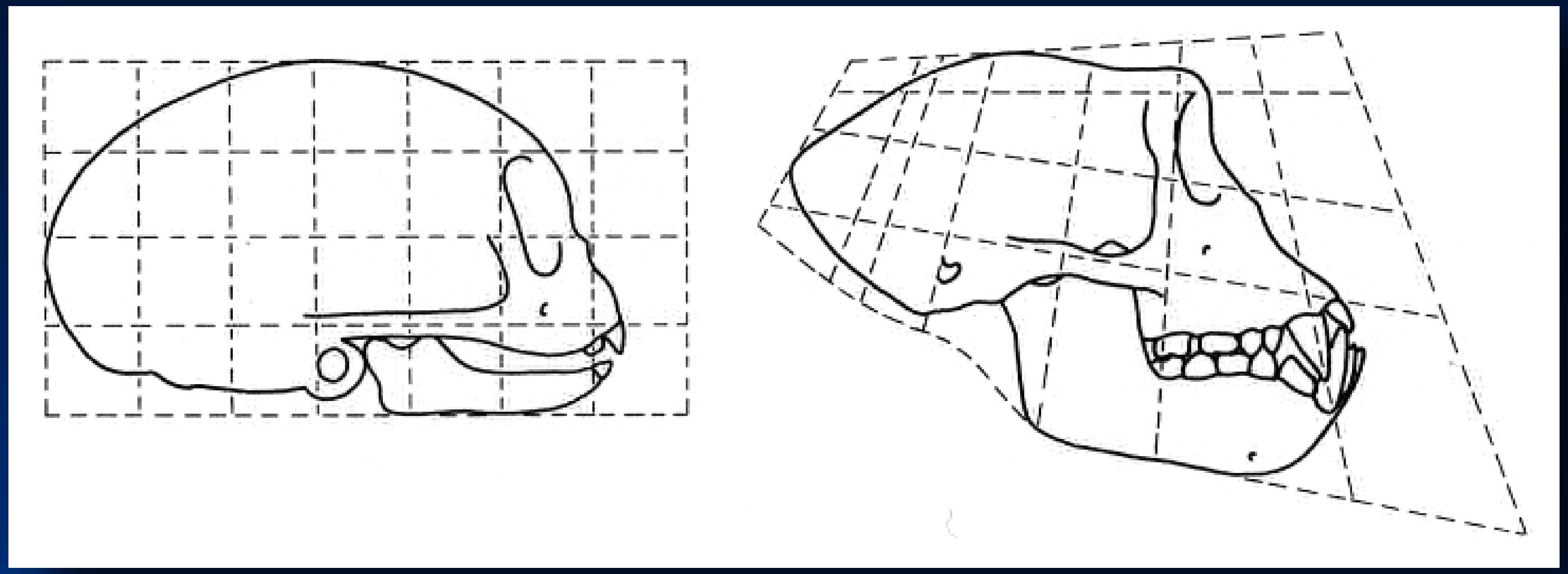
Vire Bücher von Menchlicher
Proportion (1524)

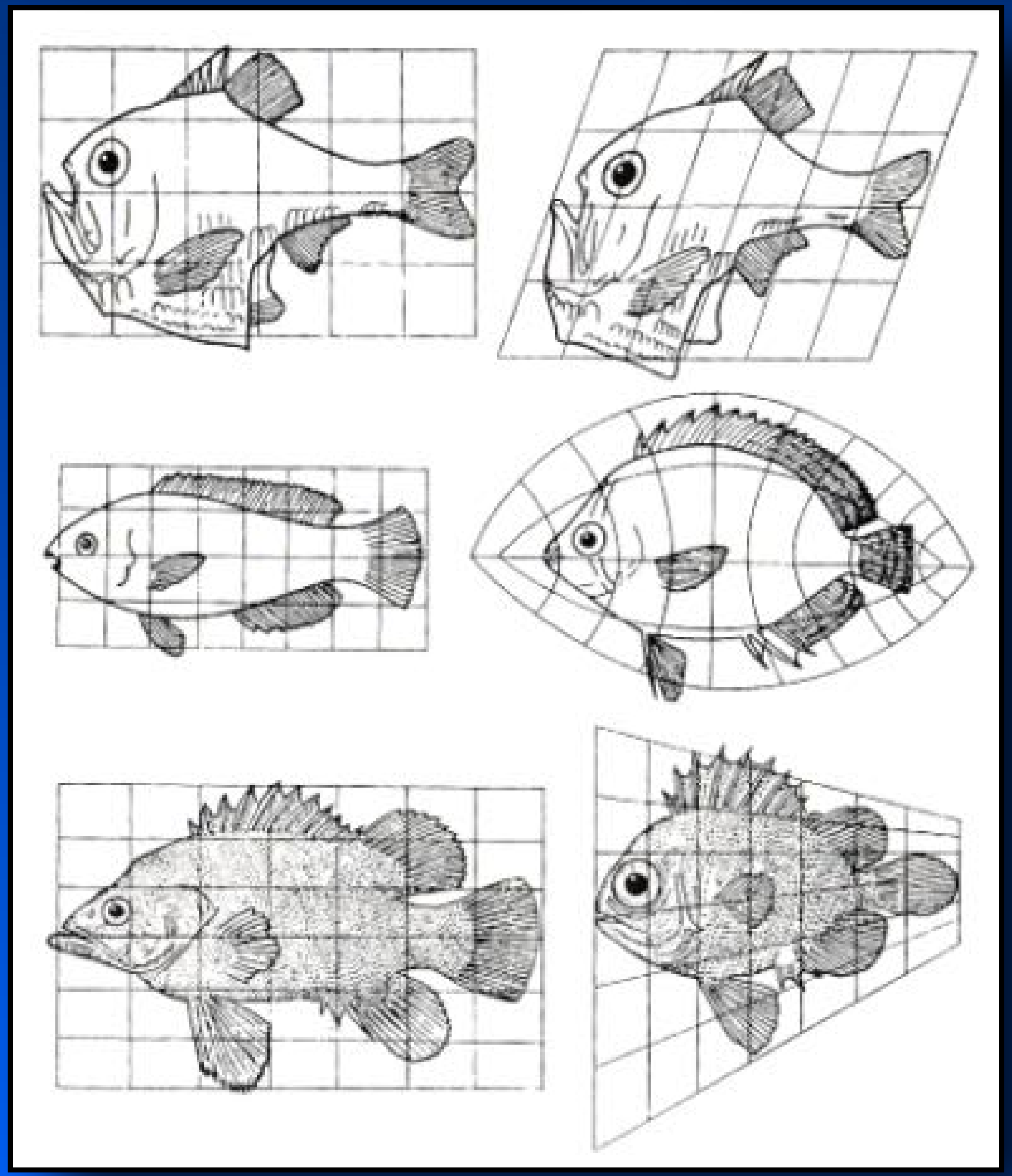
Deformation Morphometrics

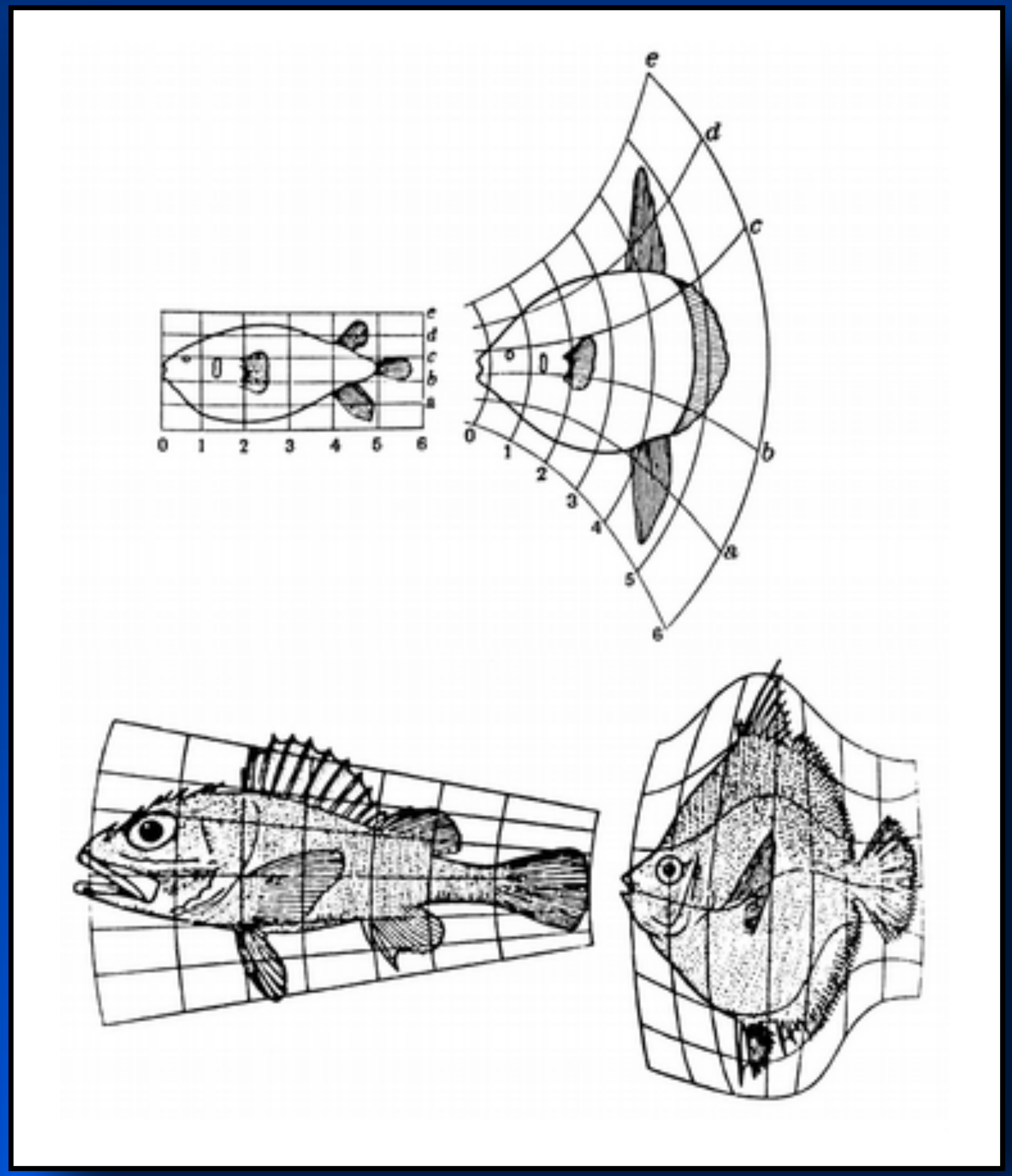
Representative Transformation Grids

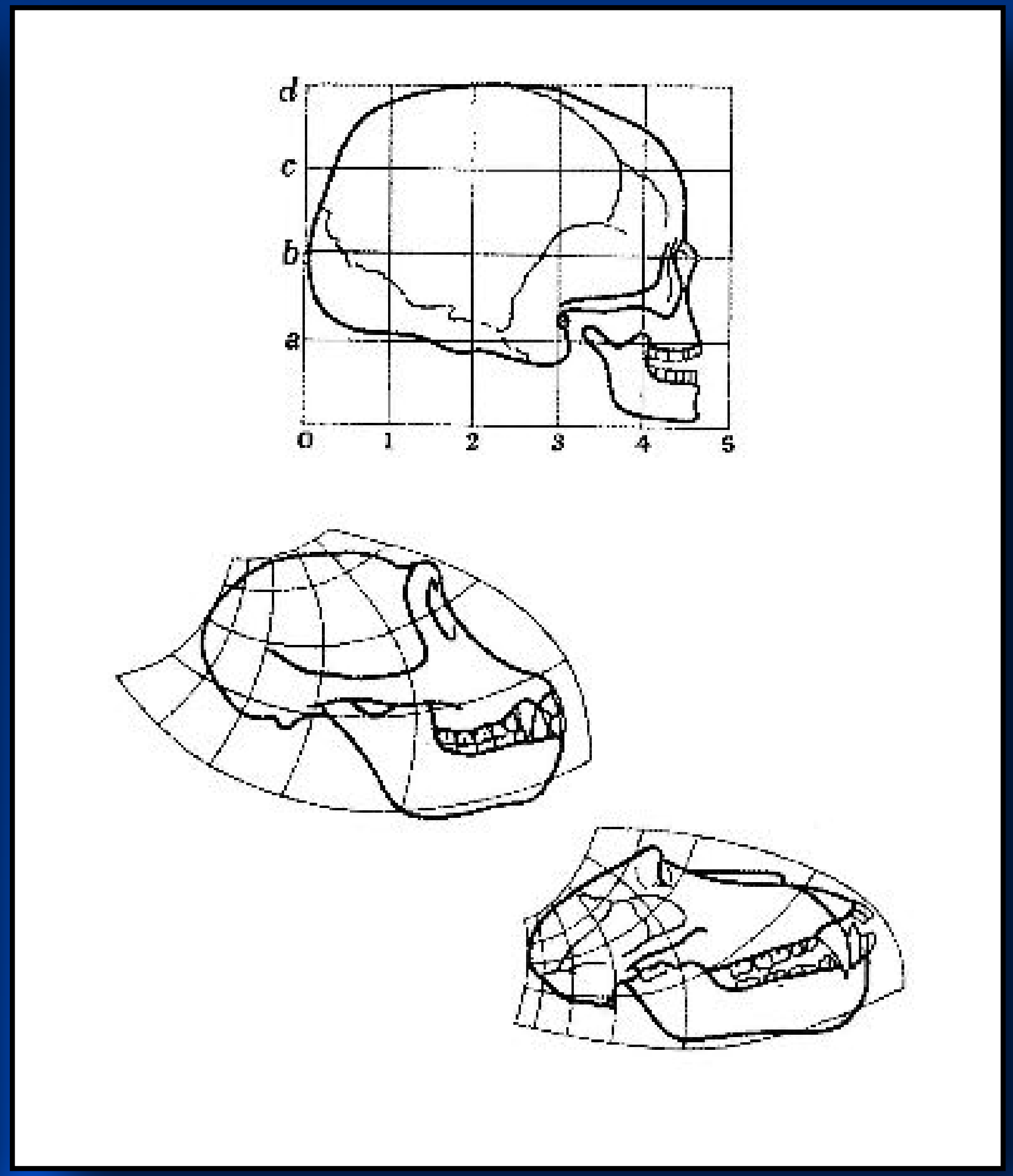




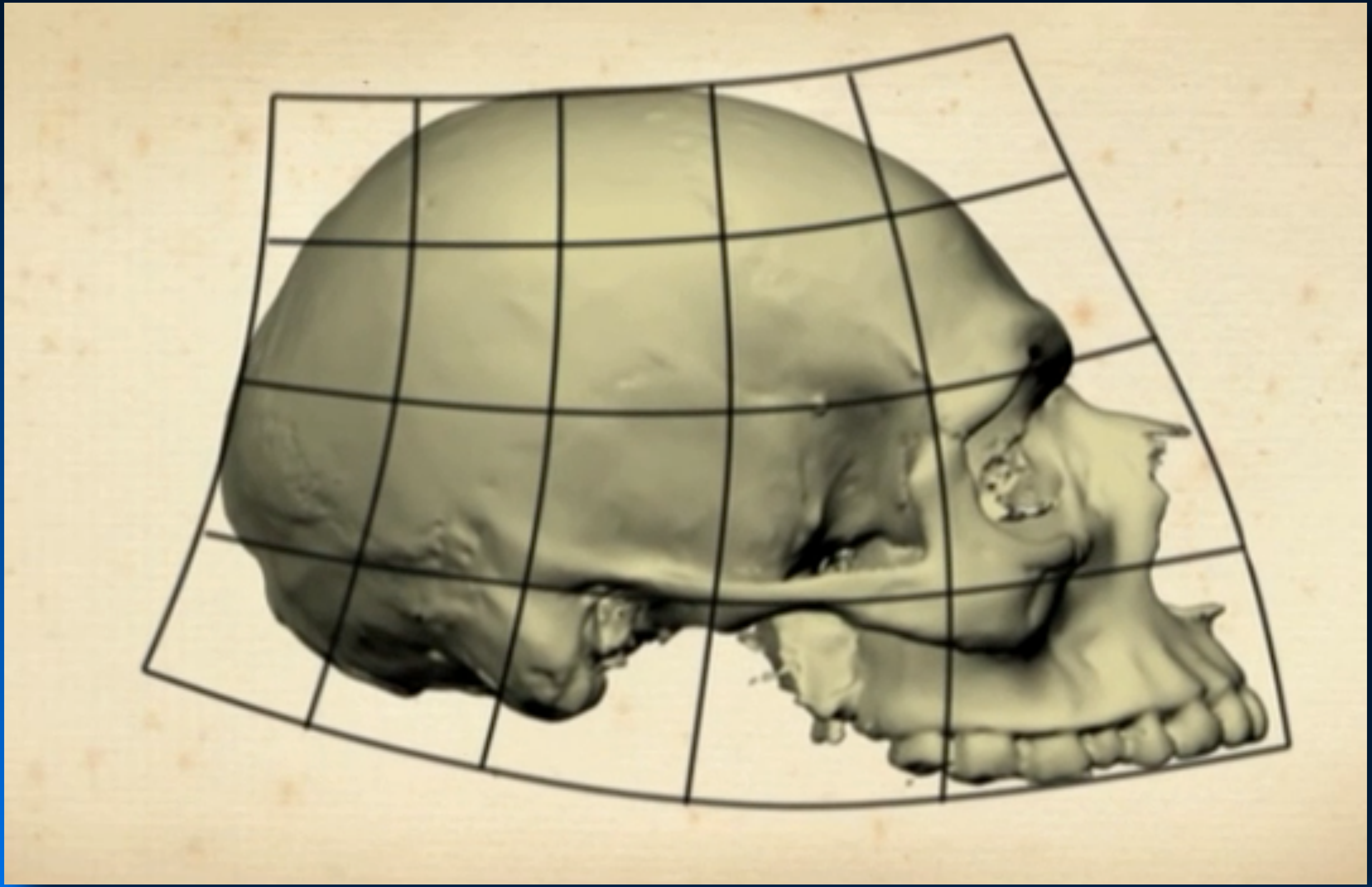


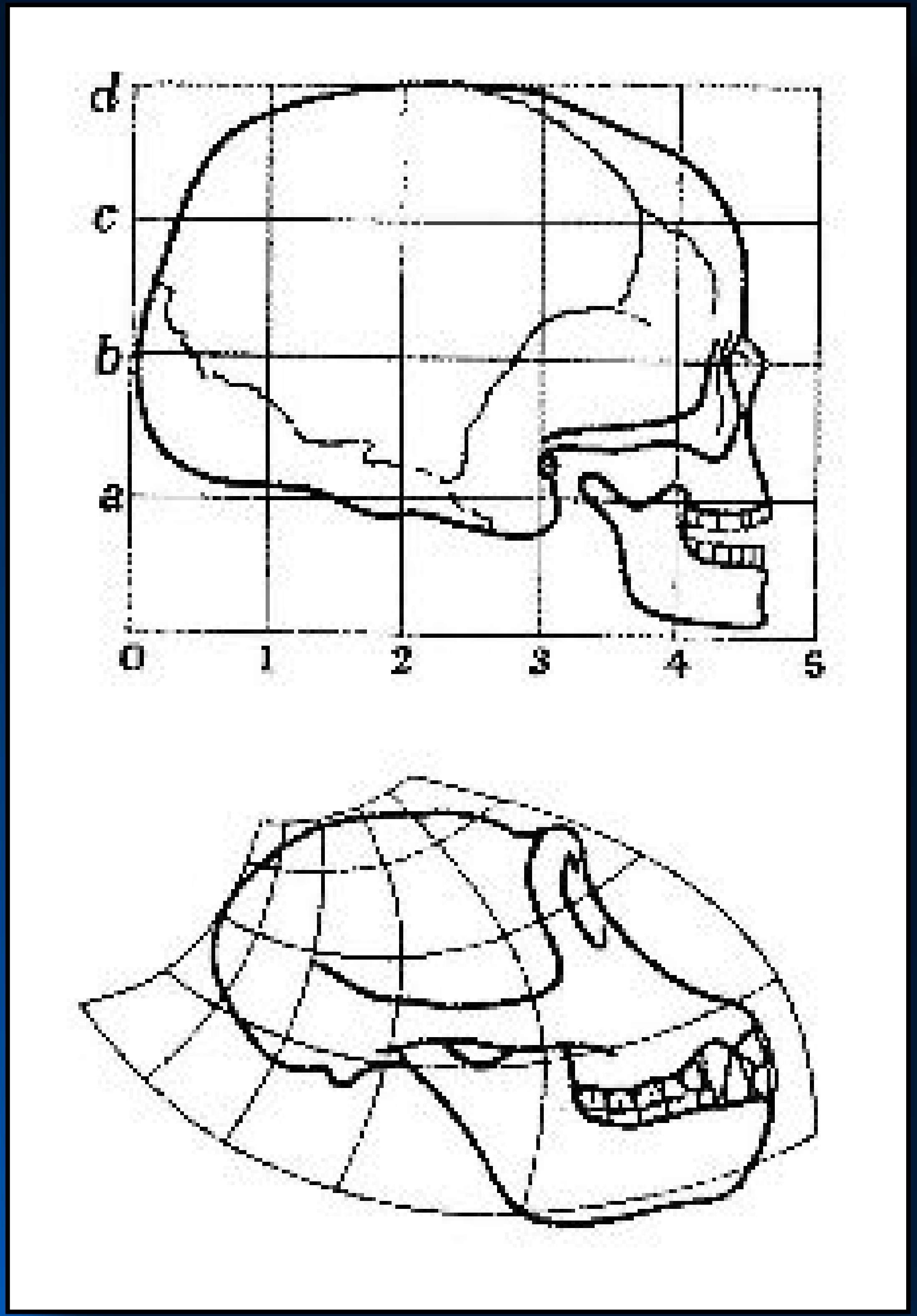




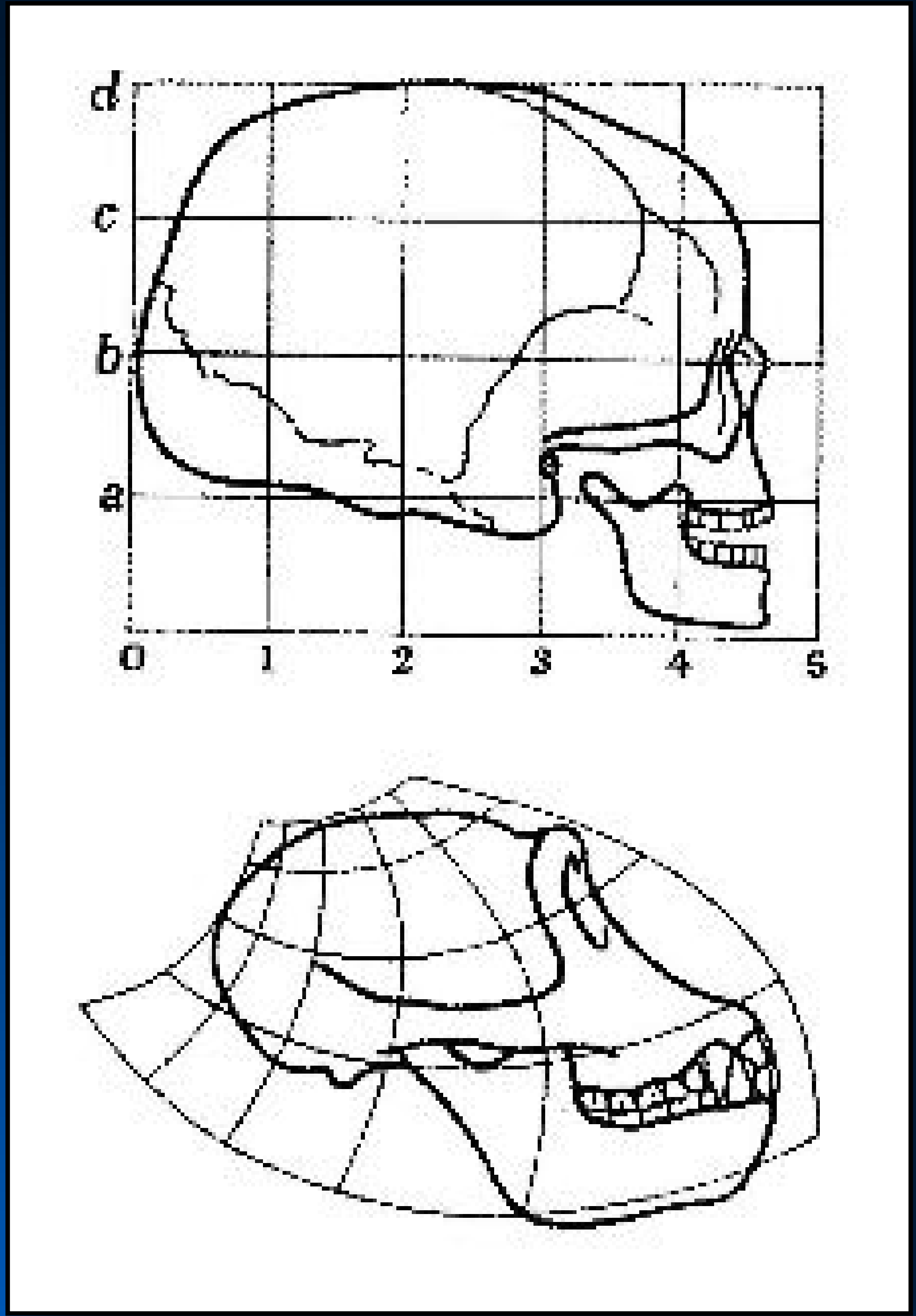


Deformation Morphometrics





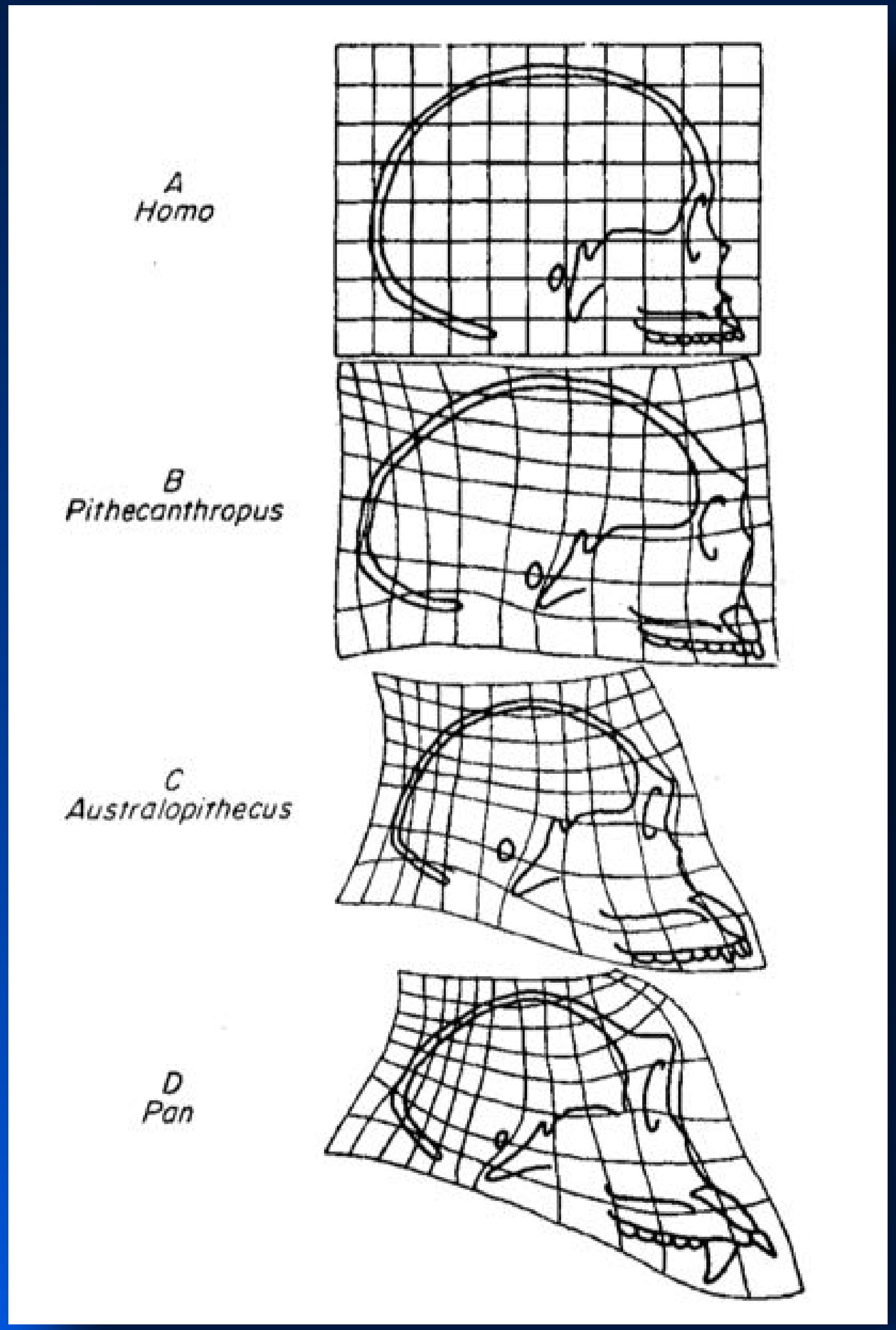
Deformation Morphometrics



Deformation Morphometrics

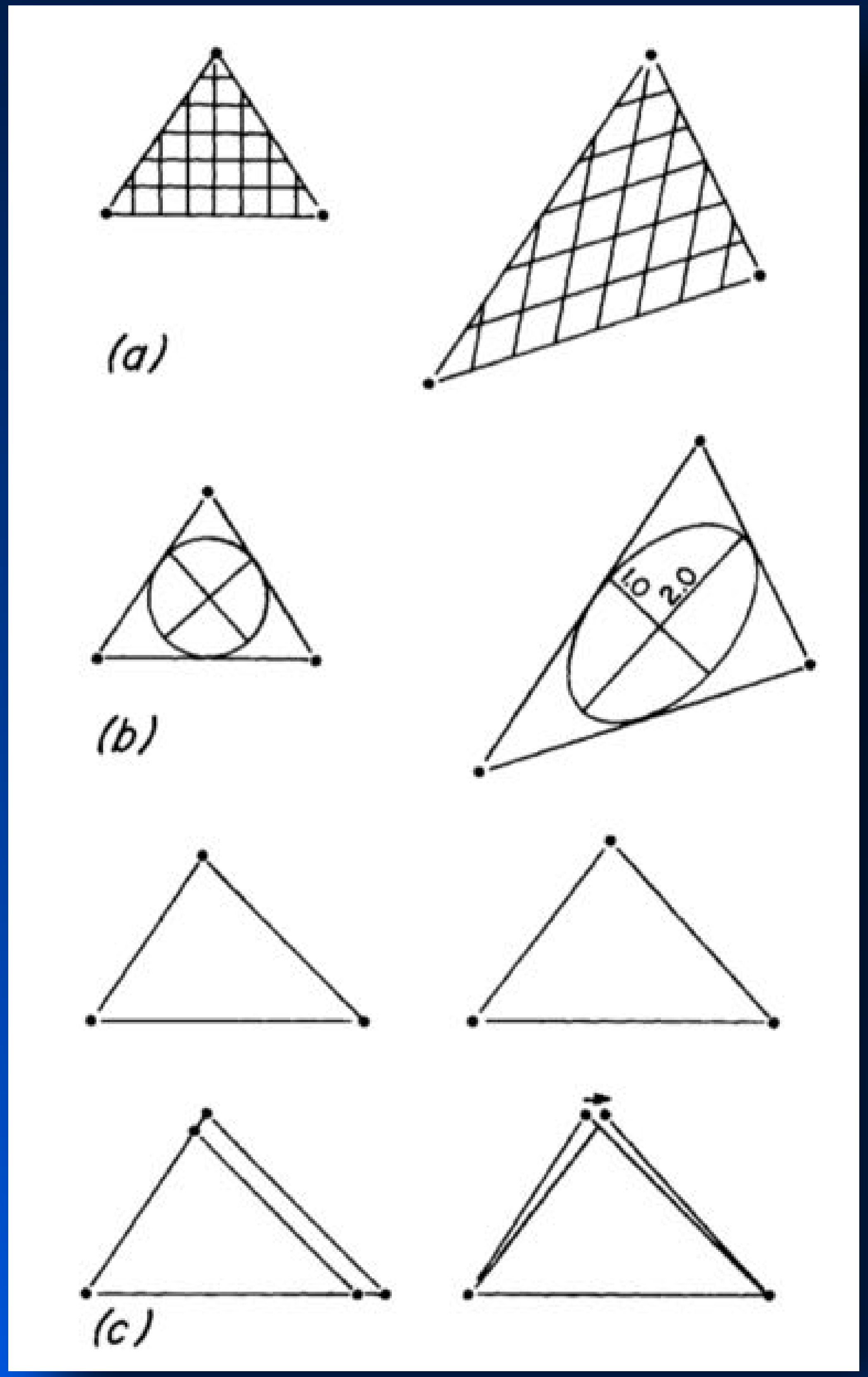
Early Attempts to Quantify Thompsonian Deformation Grids

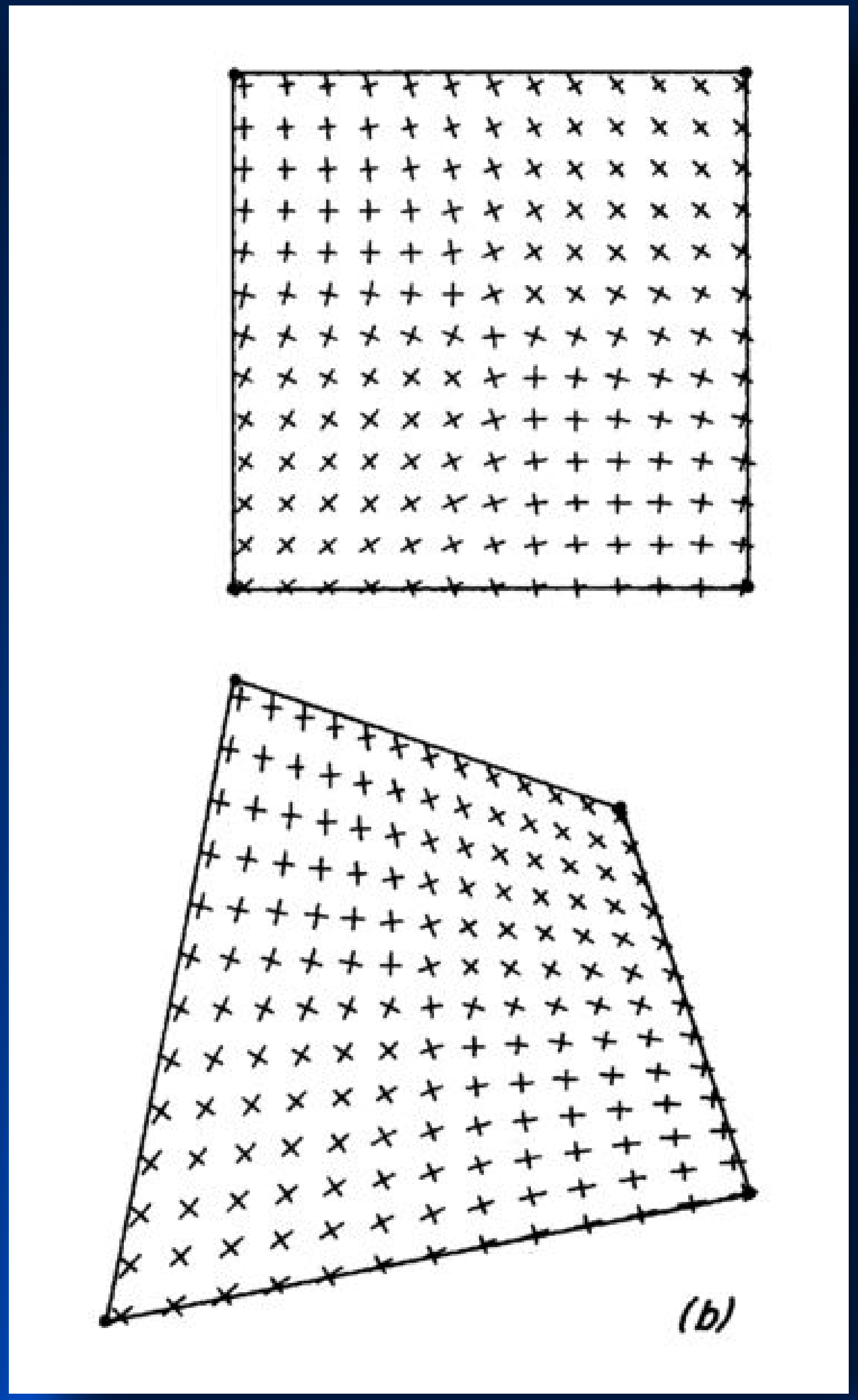
Trend Surface Fit

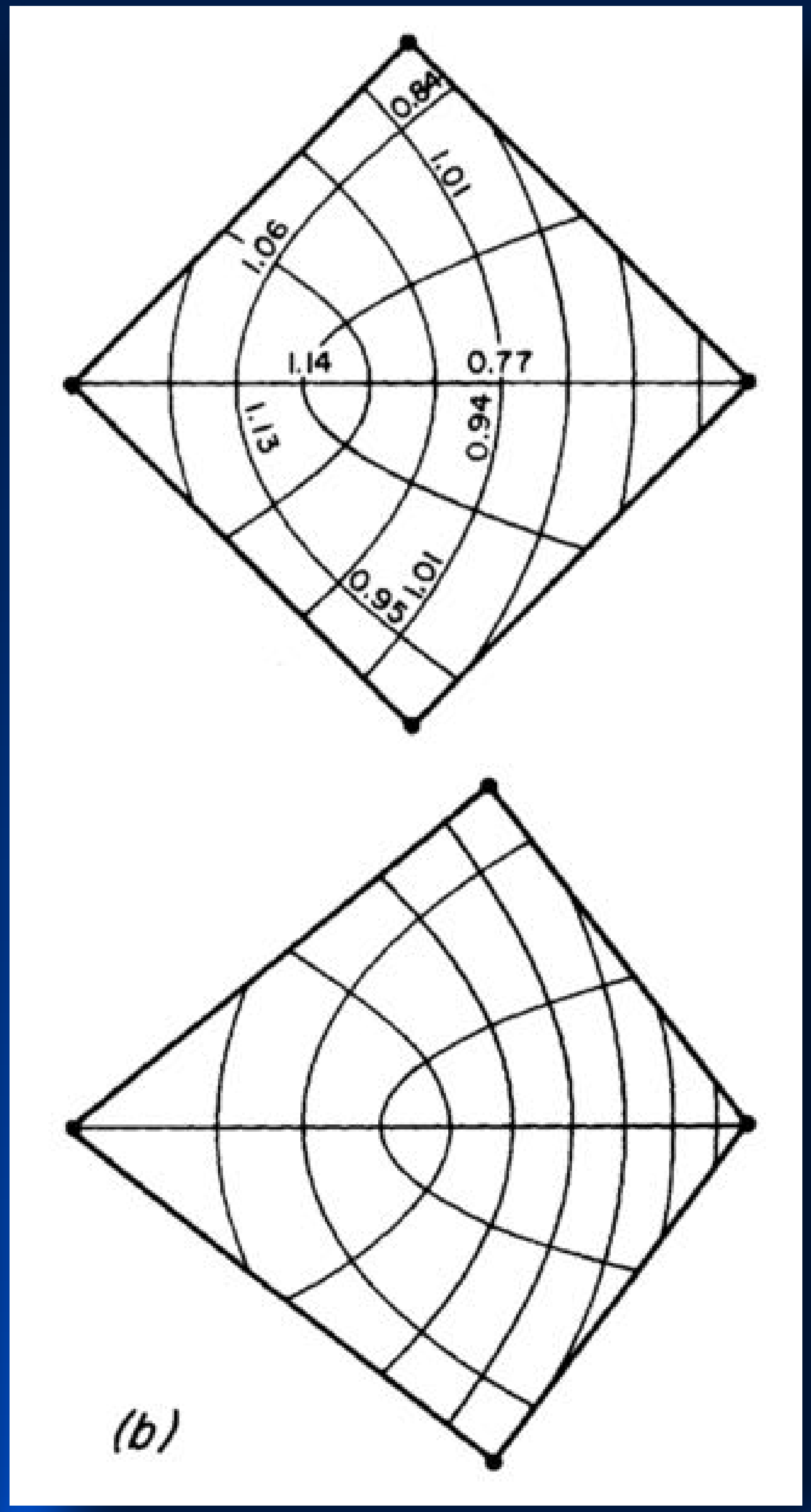


Sneath (1967)

Biorthogonal Grid Fit

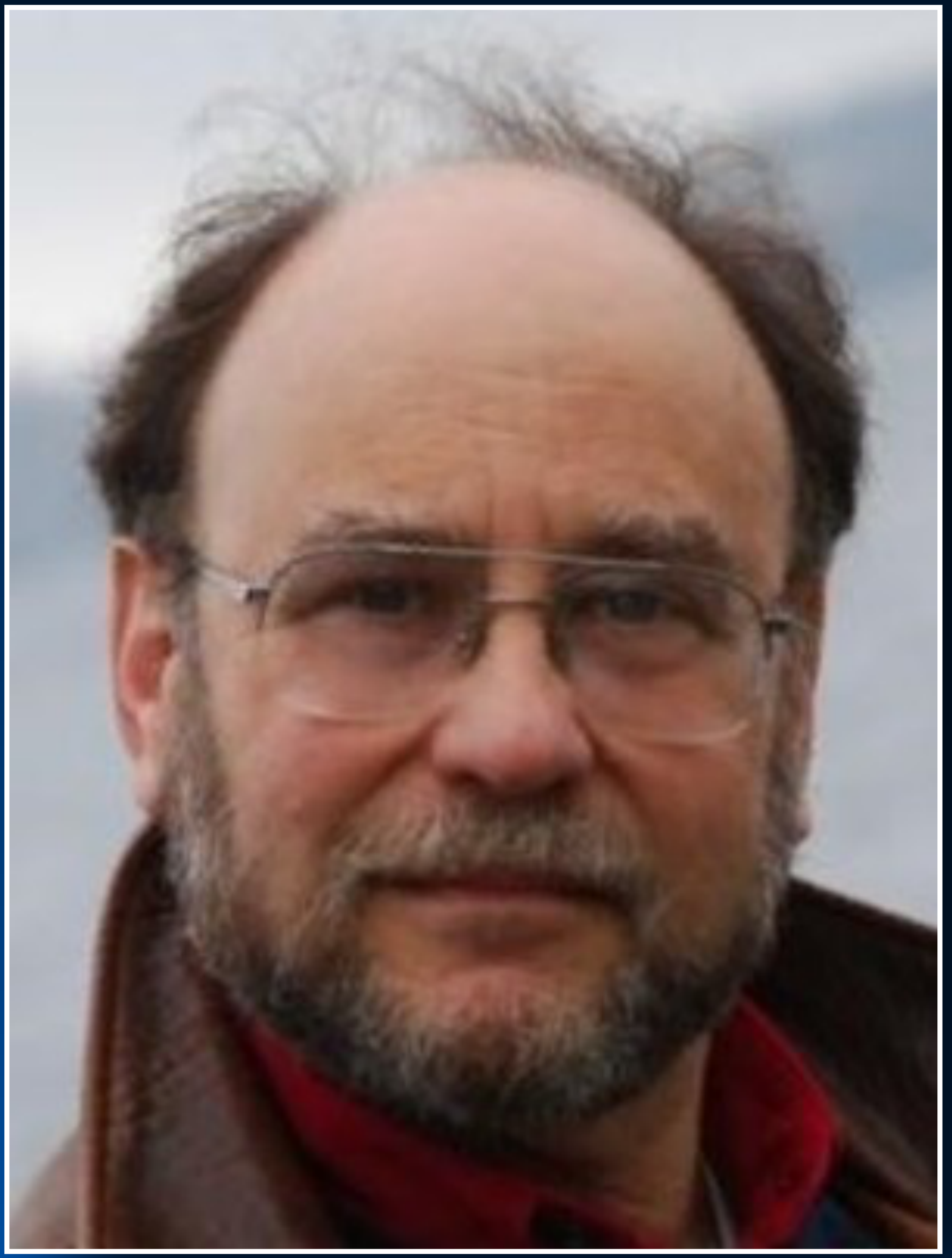




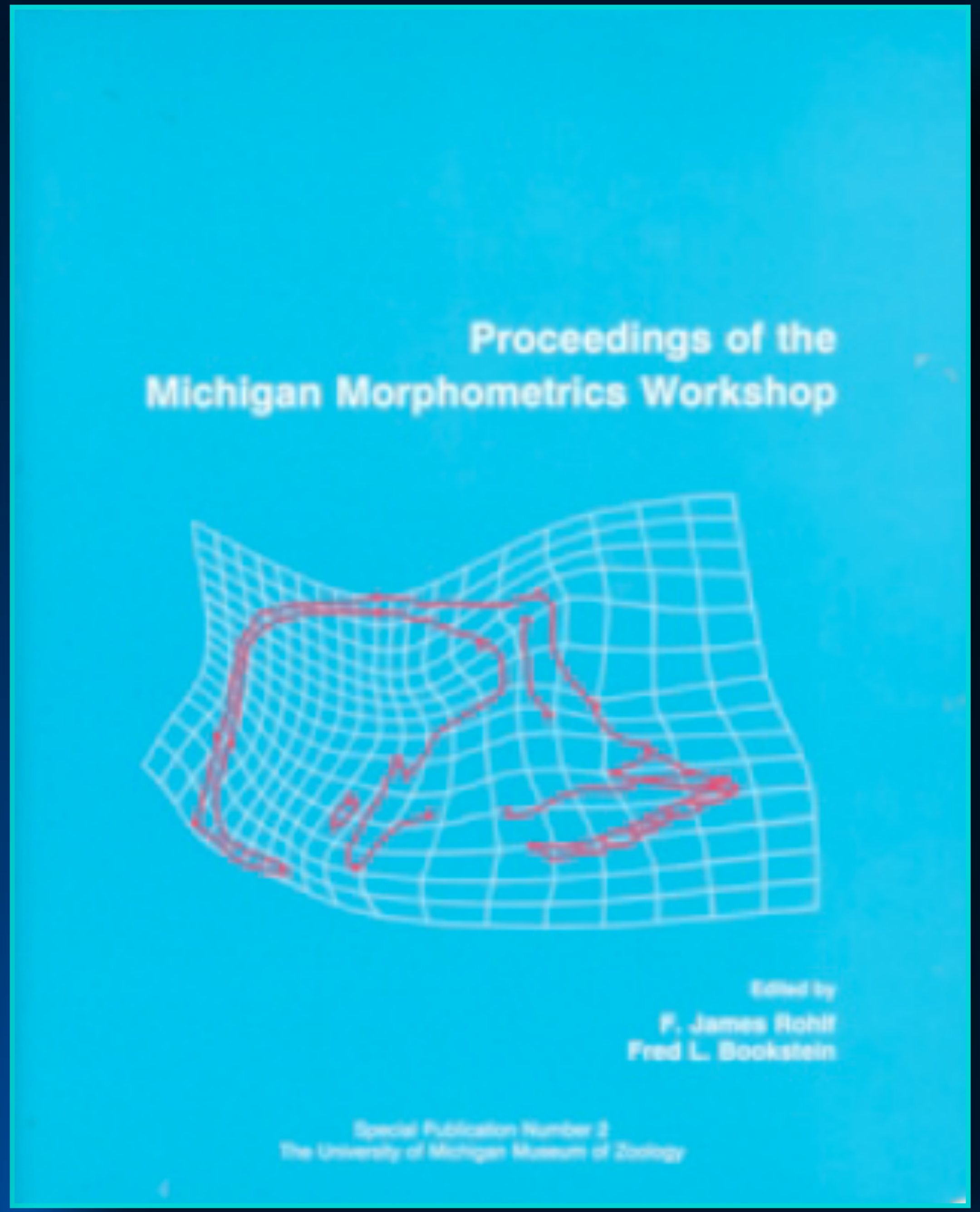


Bookstein (1978, 1985)

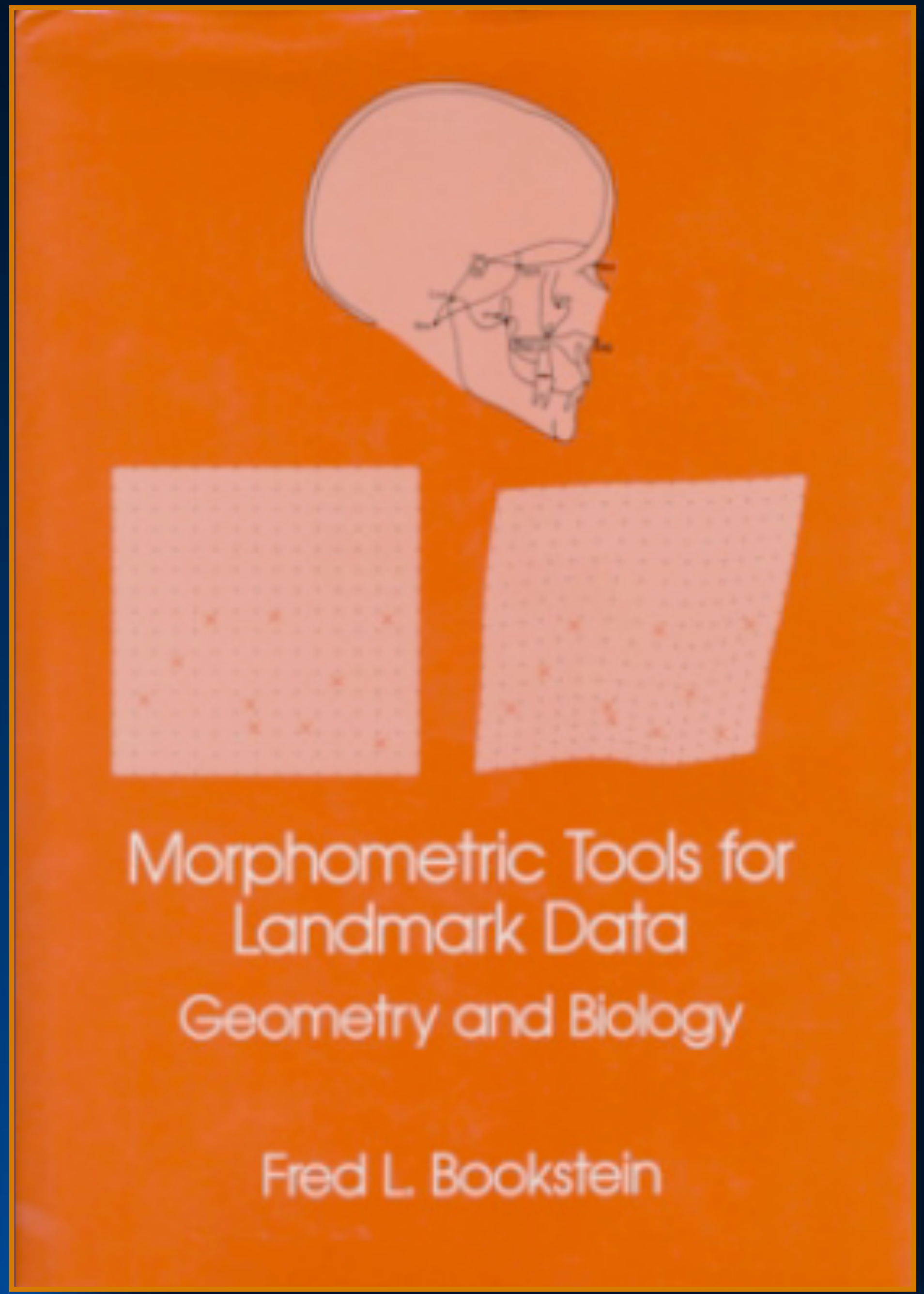
The Morphometric Synthesis



F. L. Bookstein



1990



1991

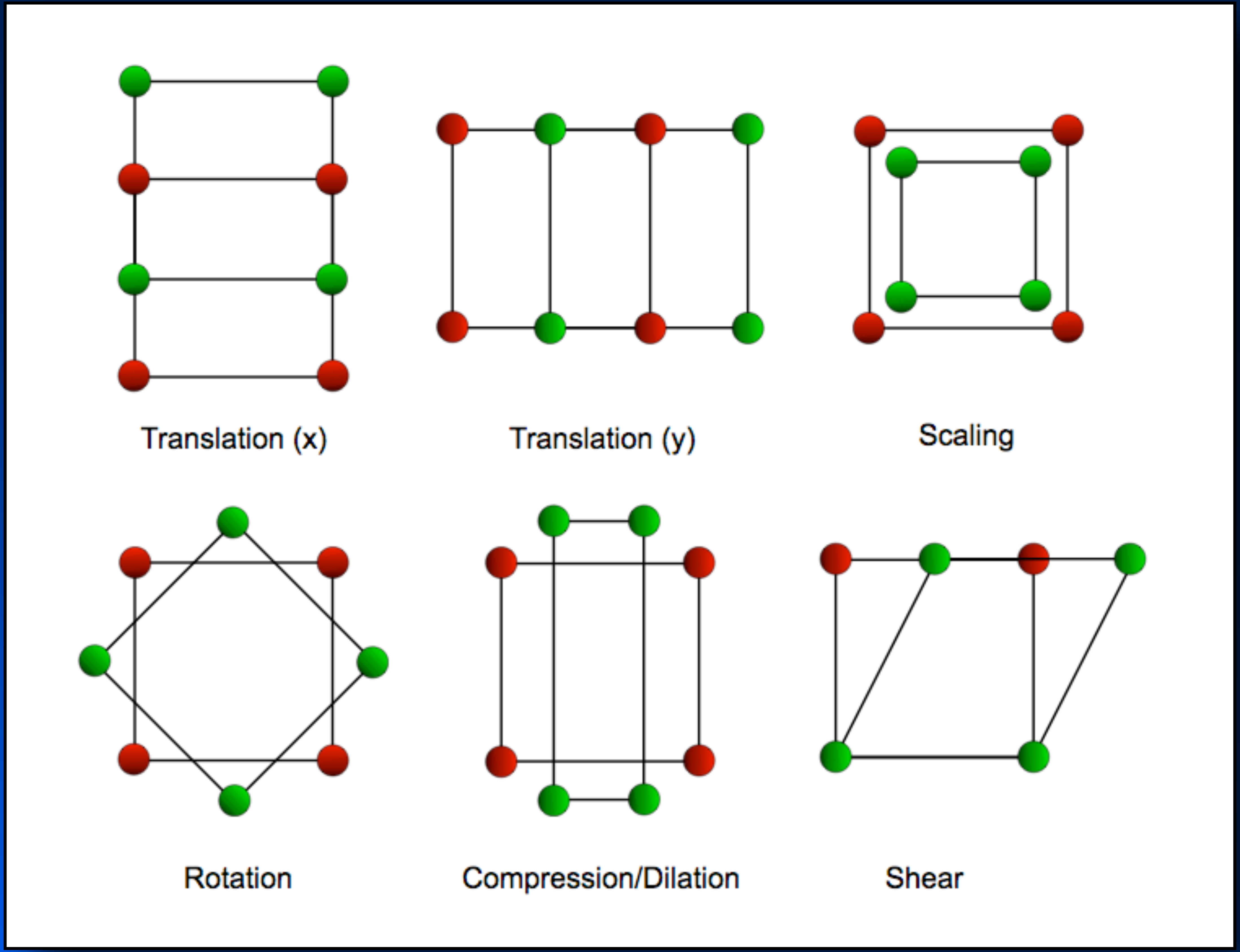
F. J. Rohlf

Through the mid-1980's to 2000 Fred Bookstein, Jim Rohlf, and others synthesized the multivariate and superposition approaches to morphometrics.

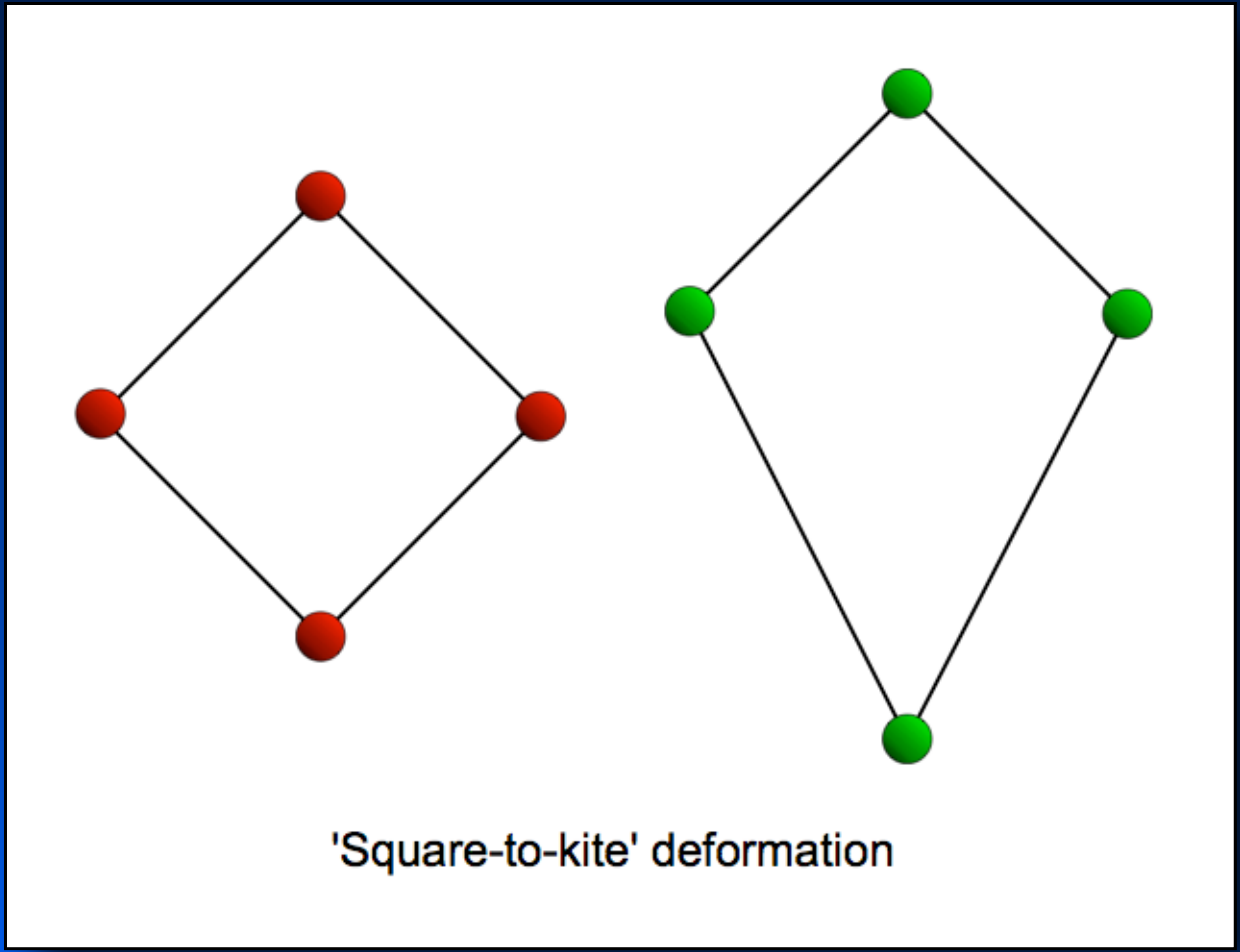
Geometric Morphometrics

Shape Change As a Deformation

Isotropic Transformations



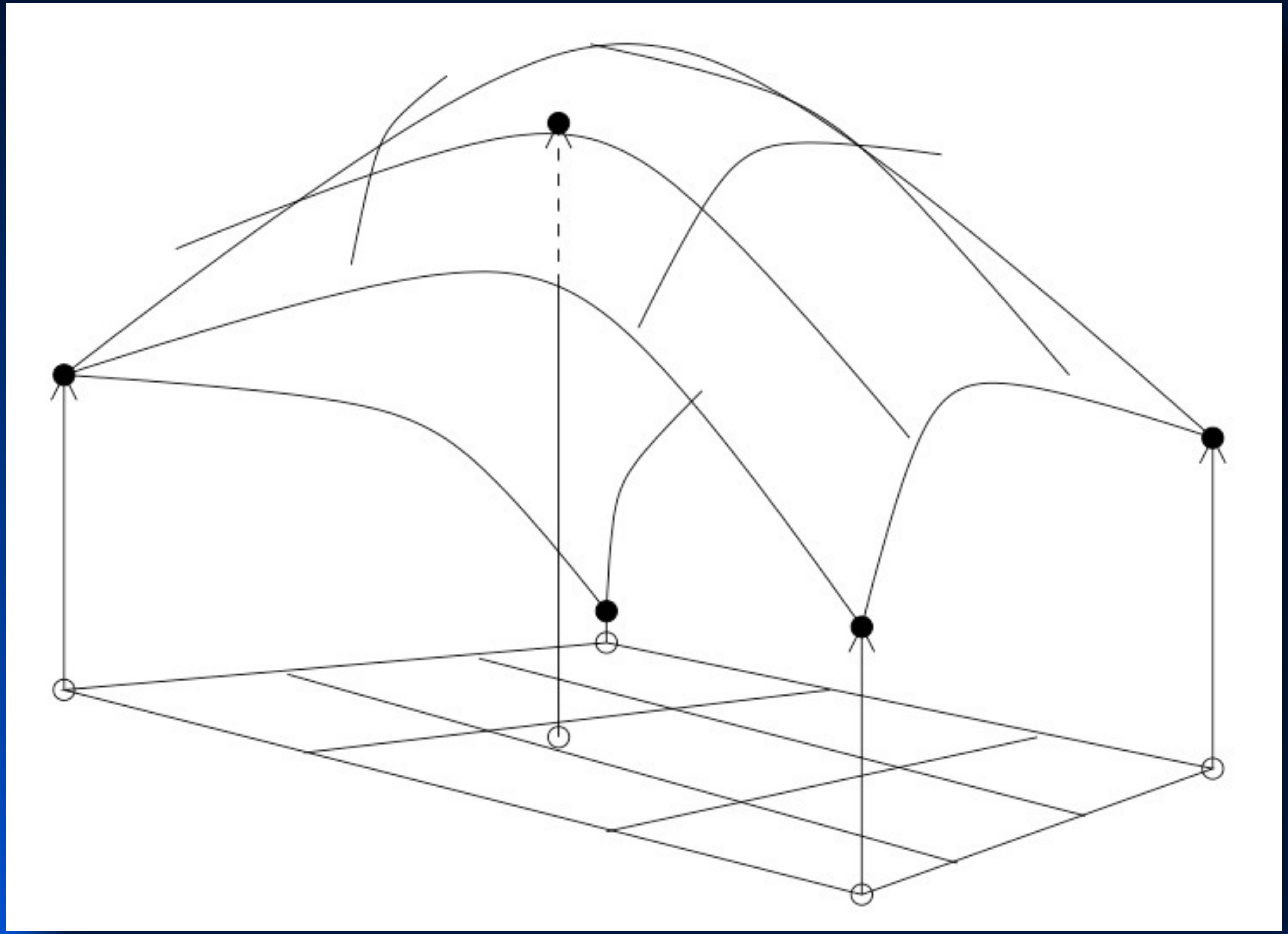
Anisotropic Transformation



Geometric Morphometrics

The Thin Plate Spline

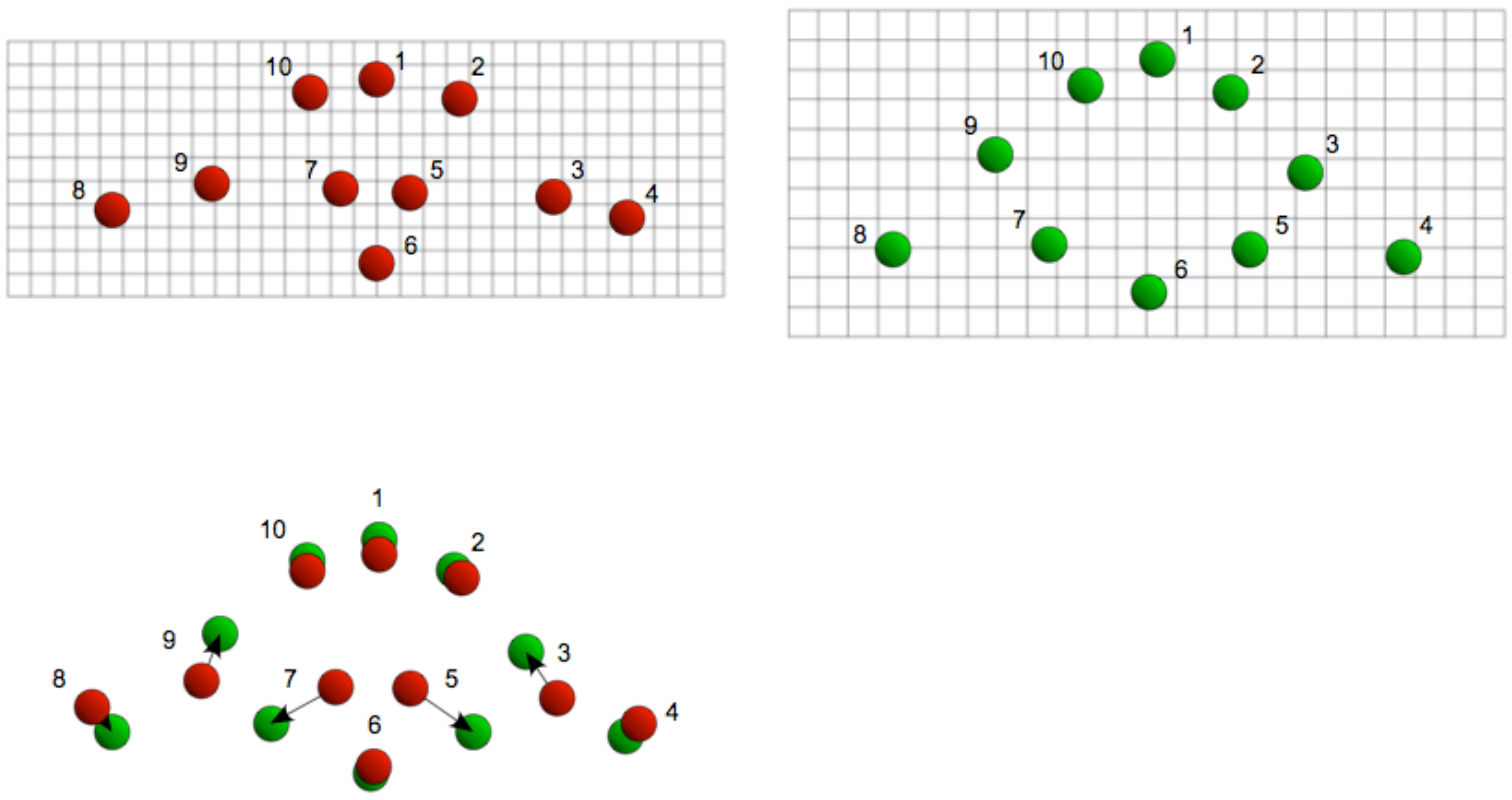
A visual metaphor used to express transformational shape change as a mathematical deformation. This mathematical device represents a series of point displacements in two dimensions (x,y) as if the displacement represents a third (z) axis. Once the landmark-specific displacements have been established, a 2D surface is interpolated over the (z) displaced points such that the degree of surgical bending across the entire interpolated surface is minimized. Any pair of forms that have been (or that can be) qualified by a corresponding series of landmark point locations can be compared in the form of a deformation surface using this technique.



Geometric Morphometrics

The Thin Plate Spline

Acaste



Acaste → Calymene

Calymene

Geometric Morphometrics

The Bending Energy Matrix

$$U\left(r_{ij}\right) = r_{ij}^2 \ln r_{ij}^2$$

= distance between land-marks *i* and *j*.

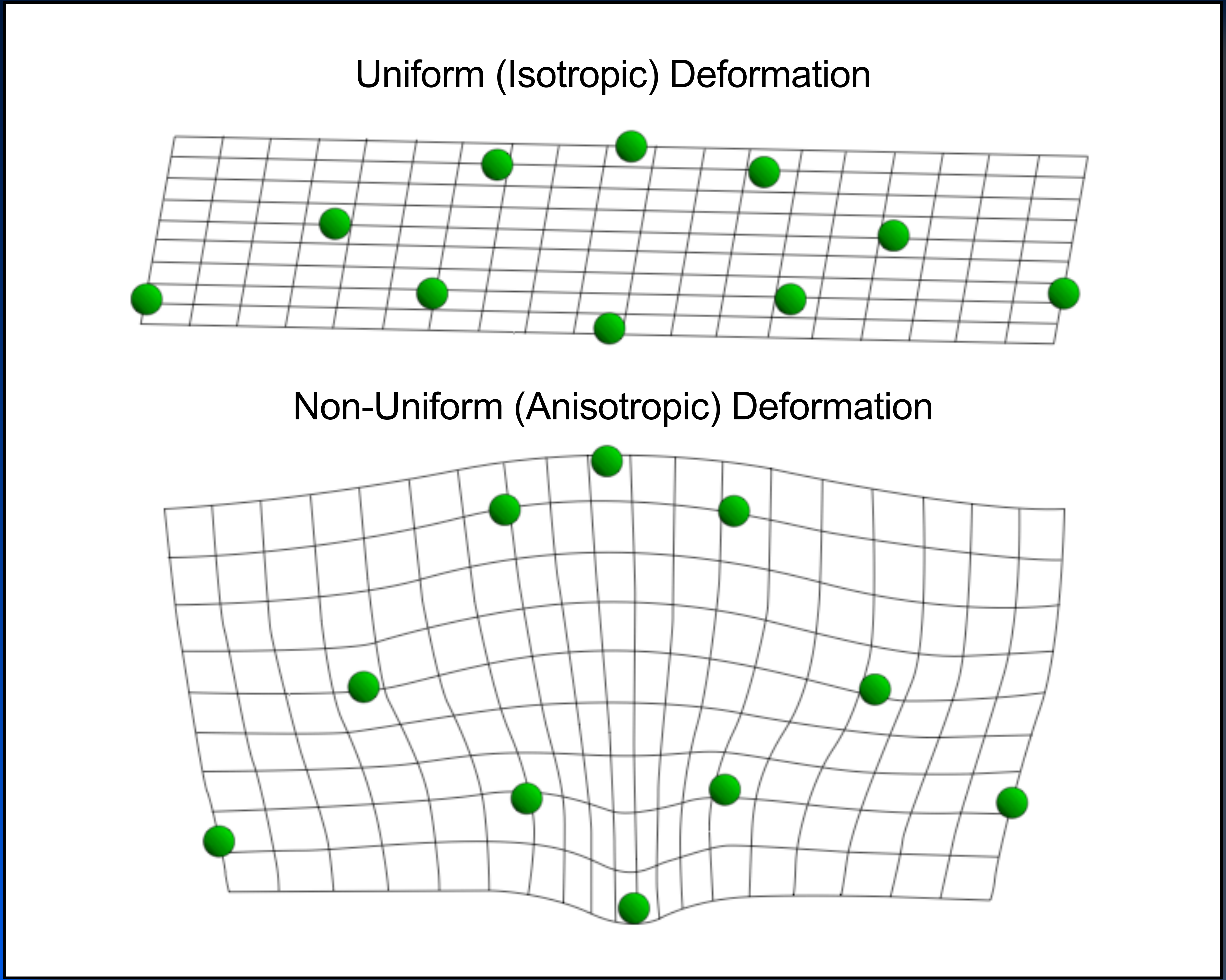
$$L = \begin{bmatrix} 0 & U_{1,2} & U_{1,3} & \cdots & U_{1,p} & 1 & x_1 & y_1 \\ U_{2,1} & 0 & U_{2,3} & \cdots & U_{2,p} & 1 & x_2 & y_2 \\ U_{3,1} & U_{3,2} & 0 & \cdots & U_{3,p} & 1 & x_3 & y_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ U_{p,1} & U_{p,2} & U_{p,3} & \cdots & 0 & 1 & x_p & y_p \\ 1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 \\ x_1 & x_2 & x_3 & \cdots & x_p & 0 & 0 & 0 \\ y_1 & y_2 & y_3 & \cdots & y_p & 0 & 0 & 0 \end{bmatrix}$$

In practice, patterns of form deformation across a sample are calculate by calculating the degree of displacement between each form and a reference form (usually the sample mean shape; x_n, y_n) across all landmarks scaled by the proximity between landmarks ($U(r)$). These values are then combined to form a square, symmetric landmark deformation matrix (L). The bending energy matrix is L^{-1}.

$$L = \begin{bmatrix} P & Q \\ Q^t & 0 \end{bmatrix}$$

Geometric Morphometrics

The Bending Energy Matrix



Part of the mathematical elegance of the thin-plate spline approach to the quantification of form change as a deformation is that a simple matrix-arithmetic procedure is available to calculate not only the total deformation, but also the isotropic and anisotropic components of deformation, and to display these as an intuitively pleasing set of Thompsonian-style deformation grids.

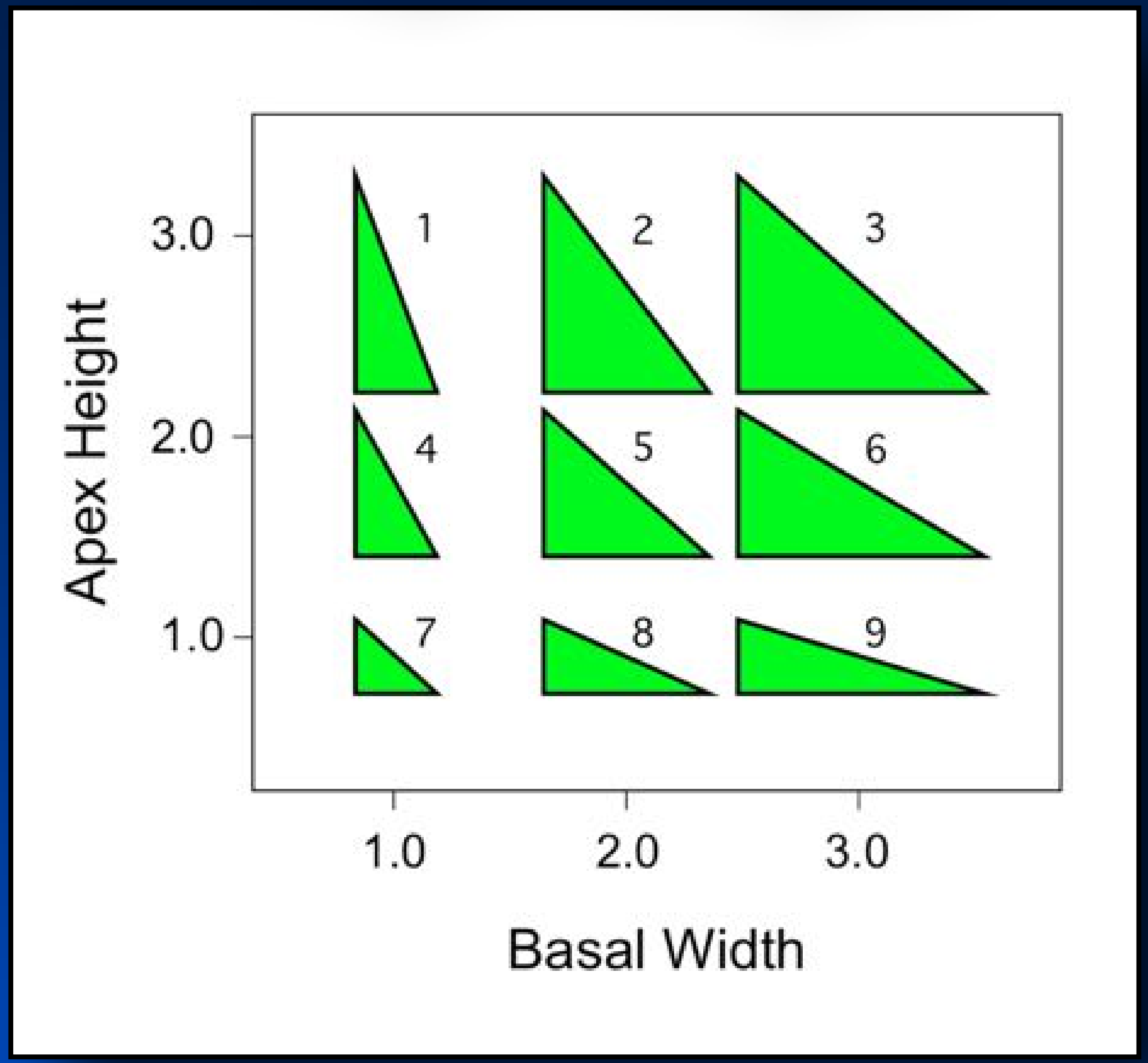
$$W_{uniform} = X_c L_q^{-1}$$

$$W_{non-uniform} = X_c L_p^{-1}$$

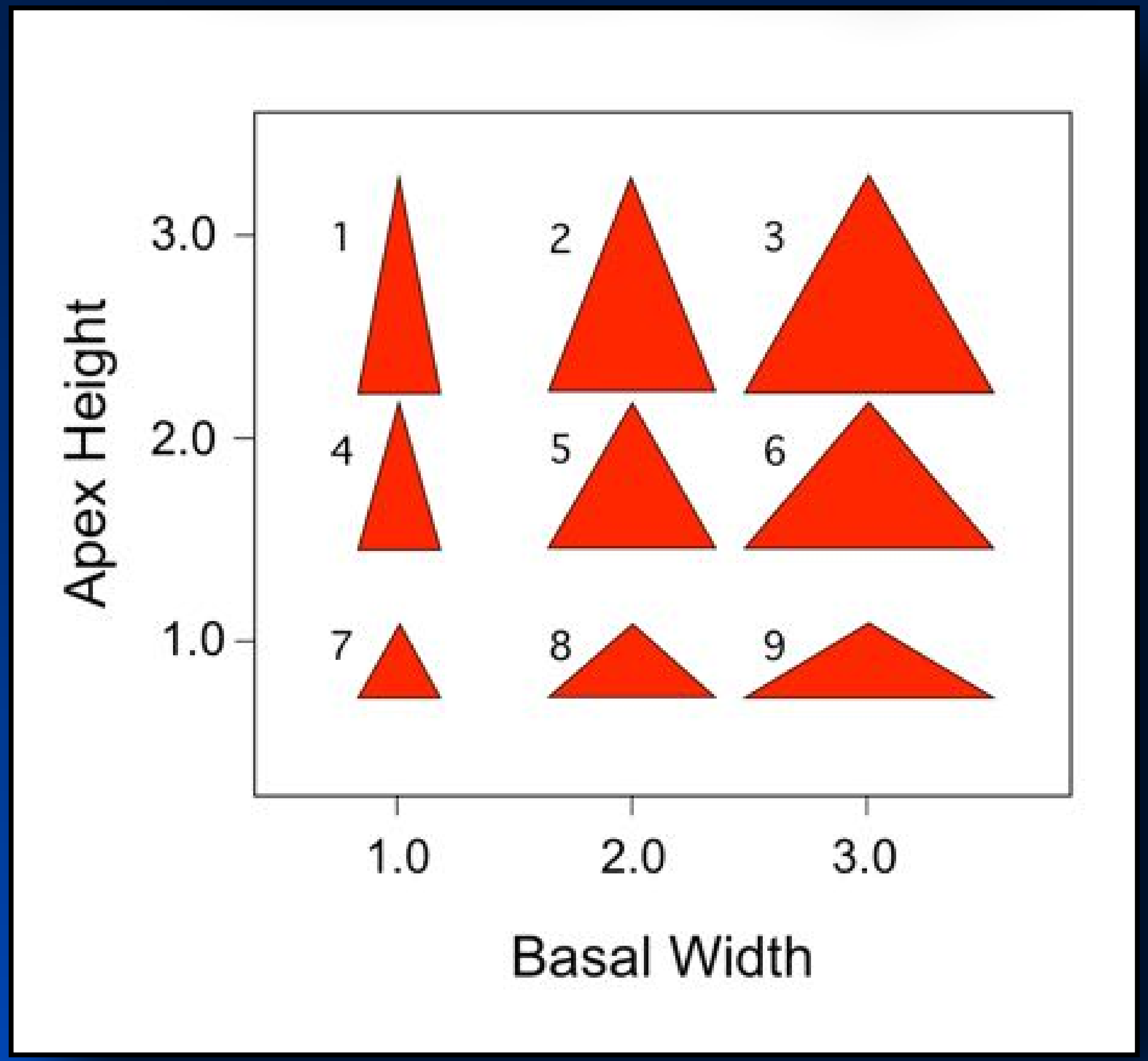
Geometric Morphometrics

Mathematical Shape Theory

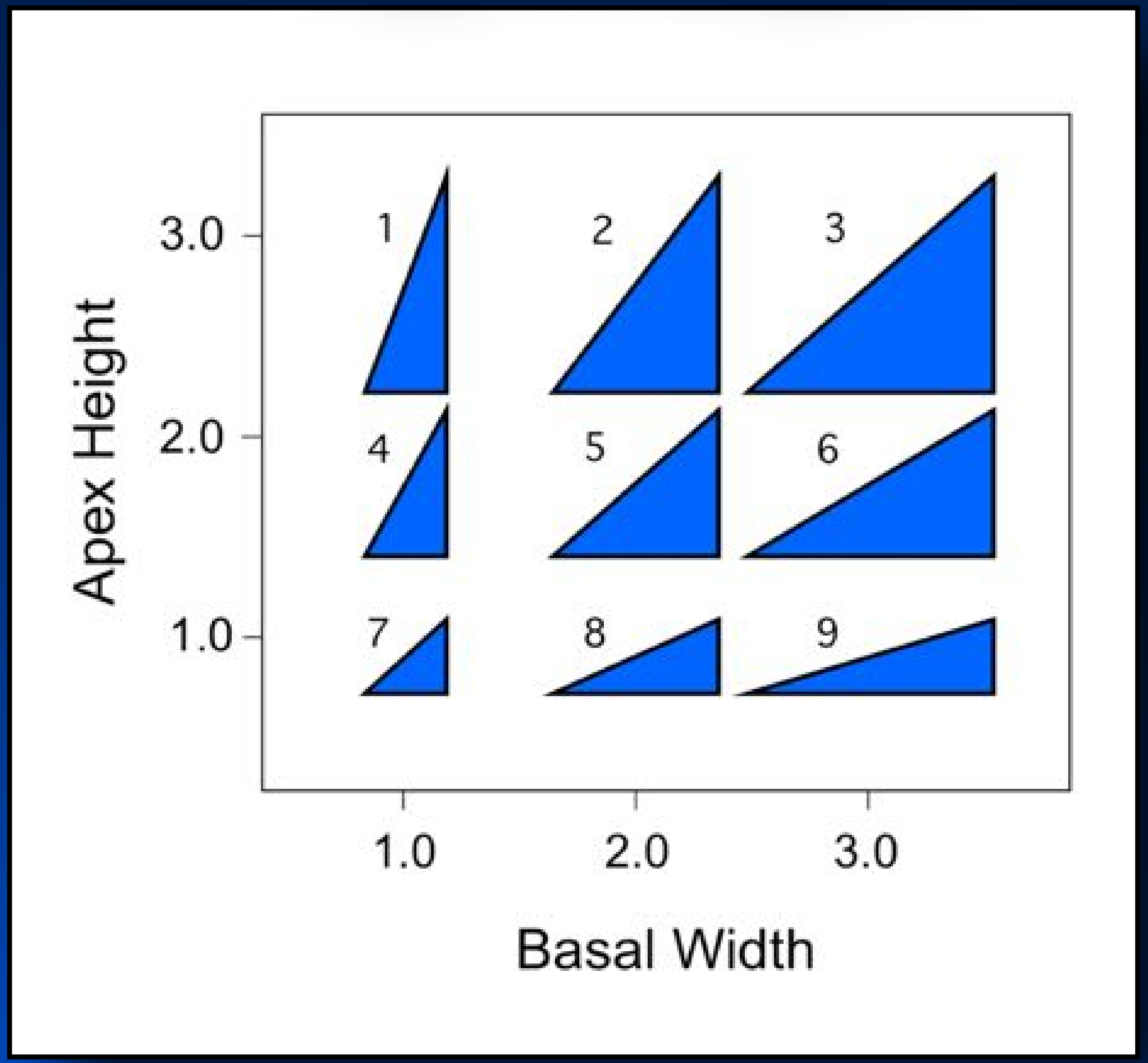
Right Triangles



Isosceles Triangles



Right Triangles

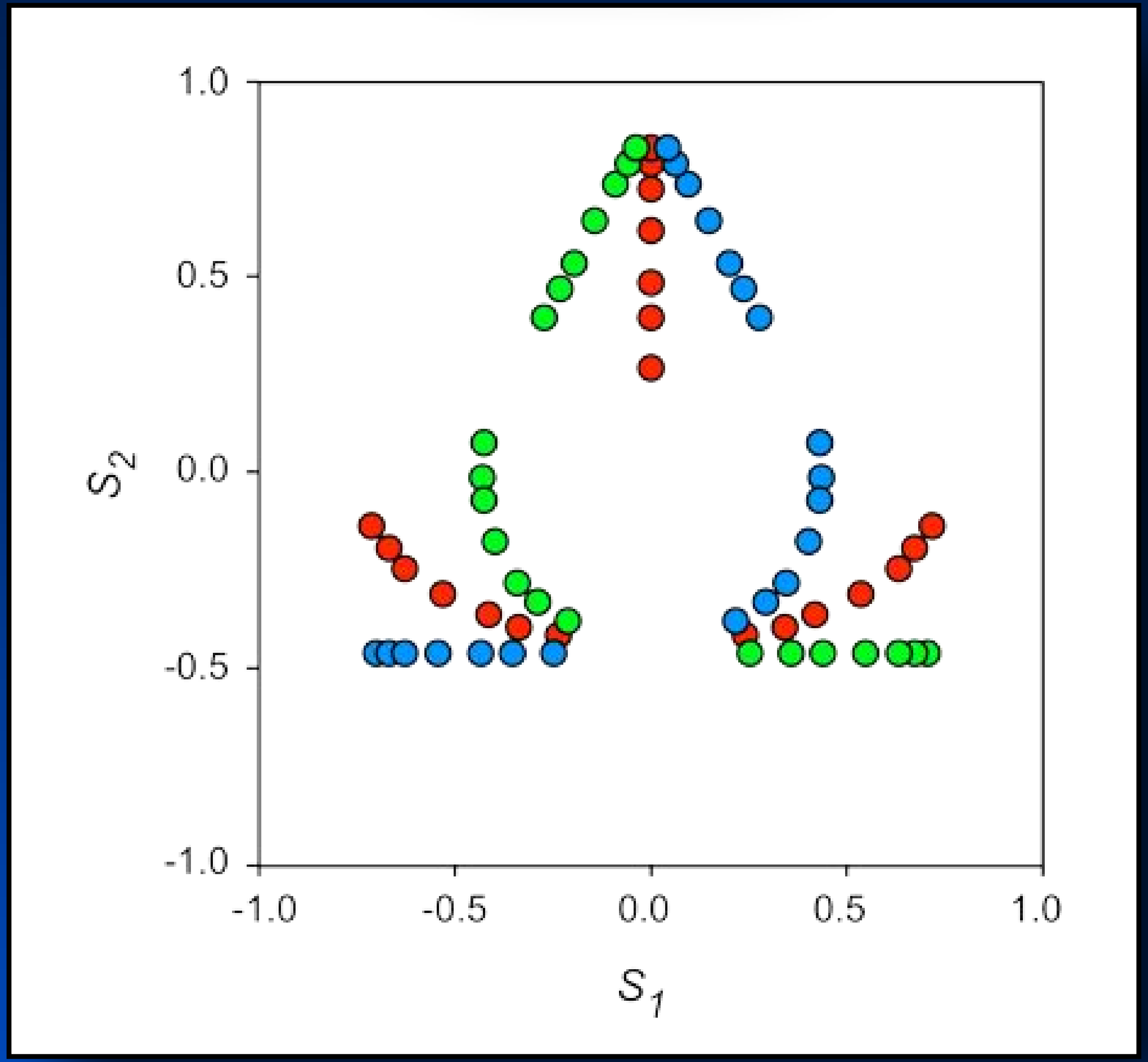


All three sets of triangles have different forms. However, when the gross (linear distance) dimensions are located in a Cartesian space, they plot to exactly the same positions. This suggests simple linear distance-based characterizations and/or comparisons of form (or shape) are insufficient to represent these differences correctly.

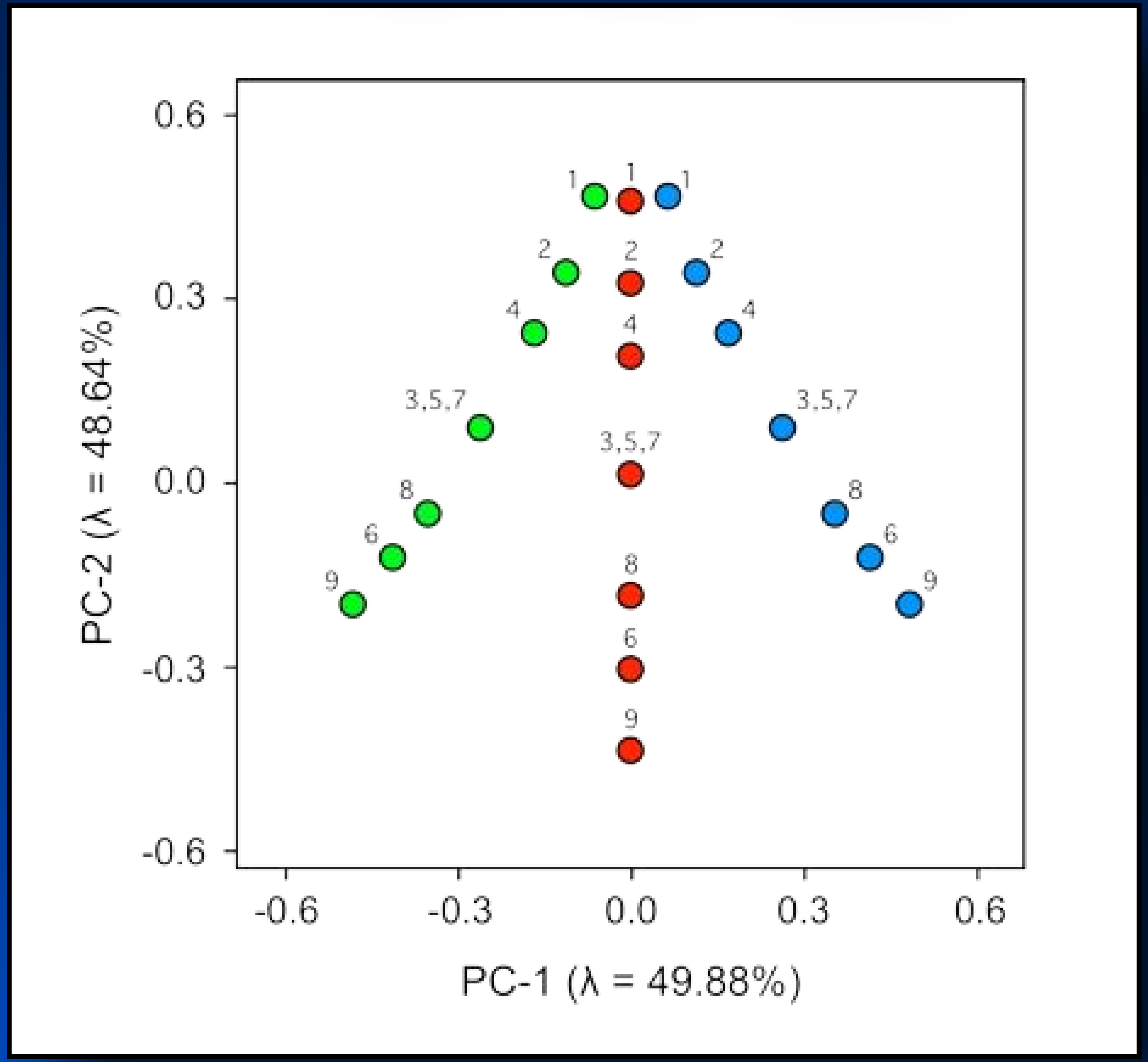
Geometric Morphometrics

Mathematical Shape Theory

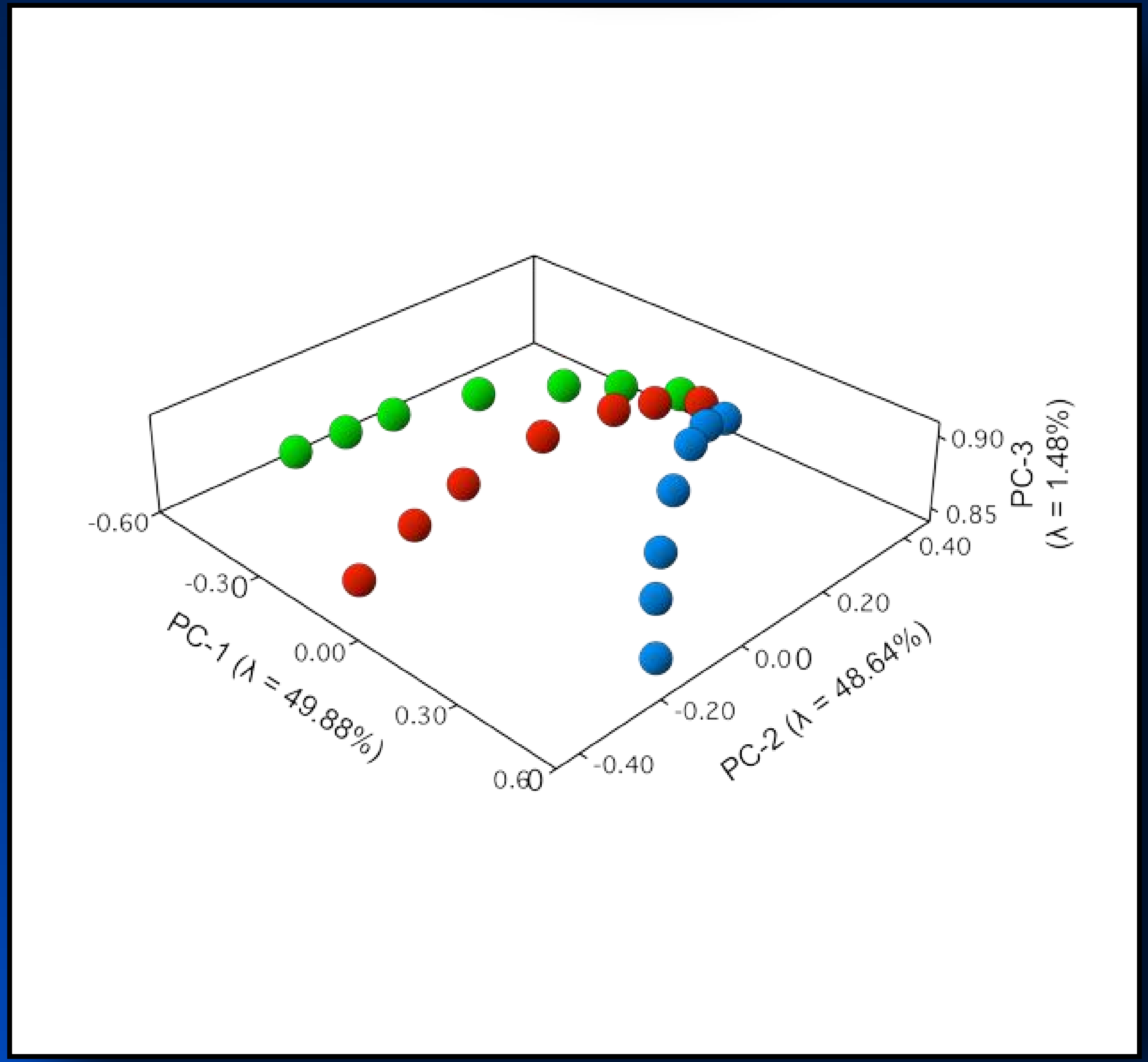
Procrustes Shape Coords.



PC-1 vs. PC-2



PC-1 vs. PC-2 vs. PC-3

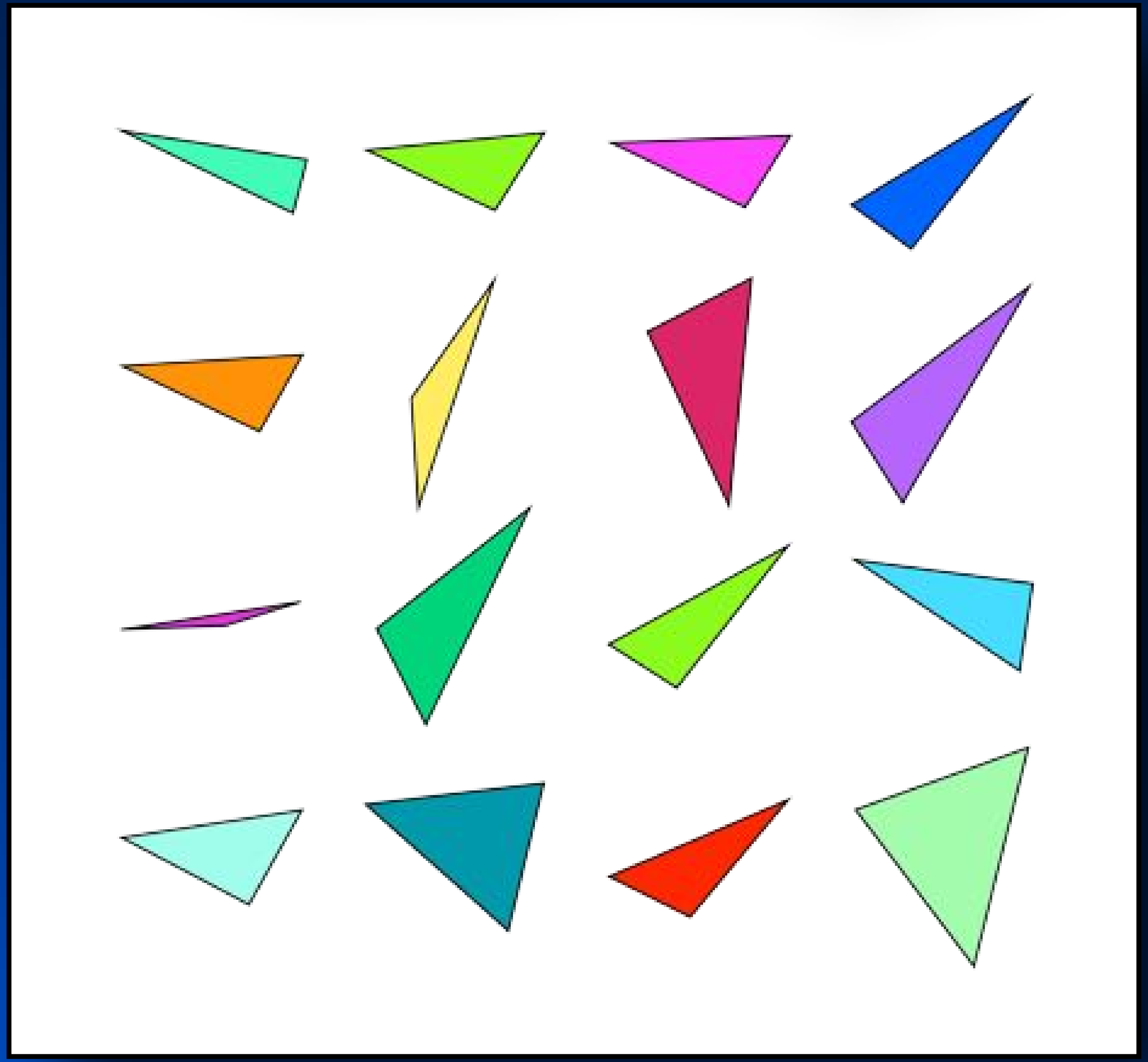


When the triangles are analyzed as Procrustes shape coordinates, most seem to exhibit unique positions when superposed. When these superposed coordinates are subjected to PCA the different shape sets are distinguished clearly and appear to lie on curved trajectories.

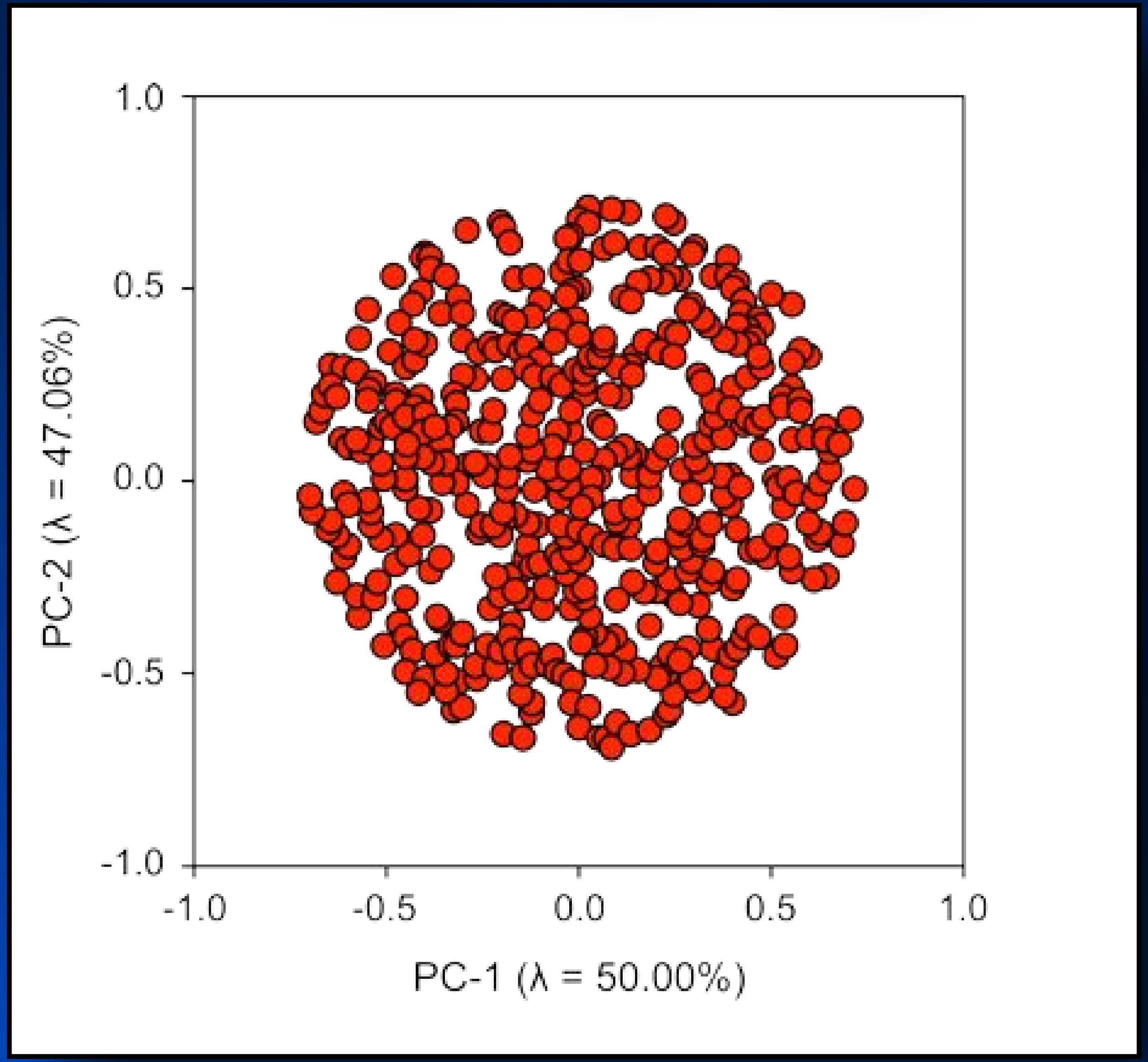
Geometric Morphometrics

Mathematical Shape Theory

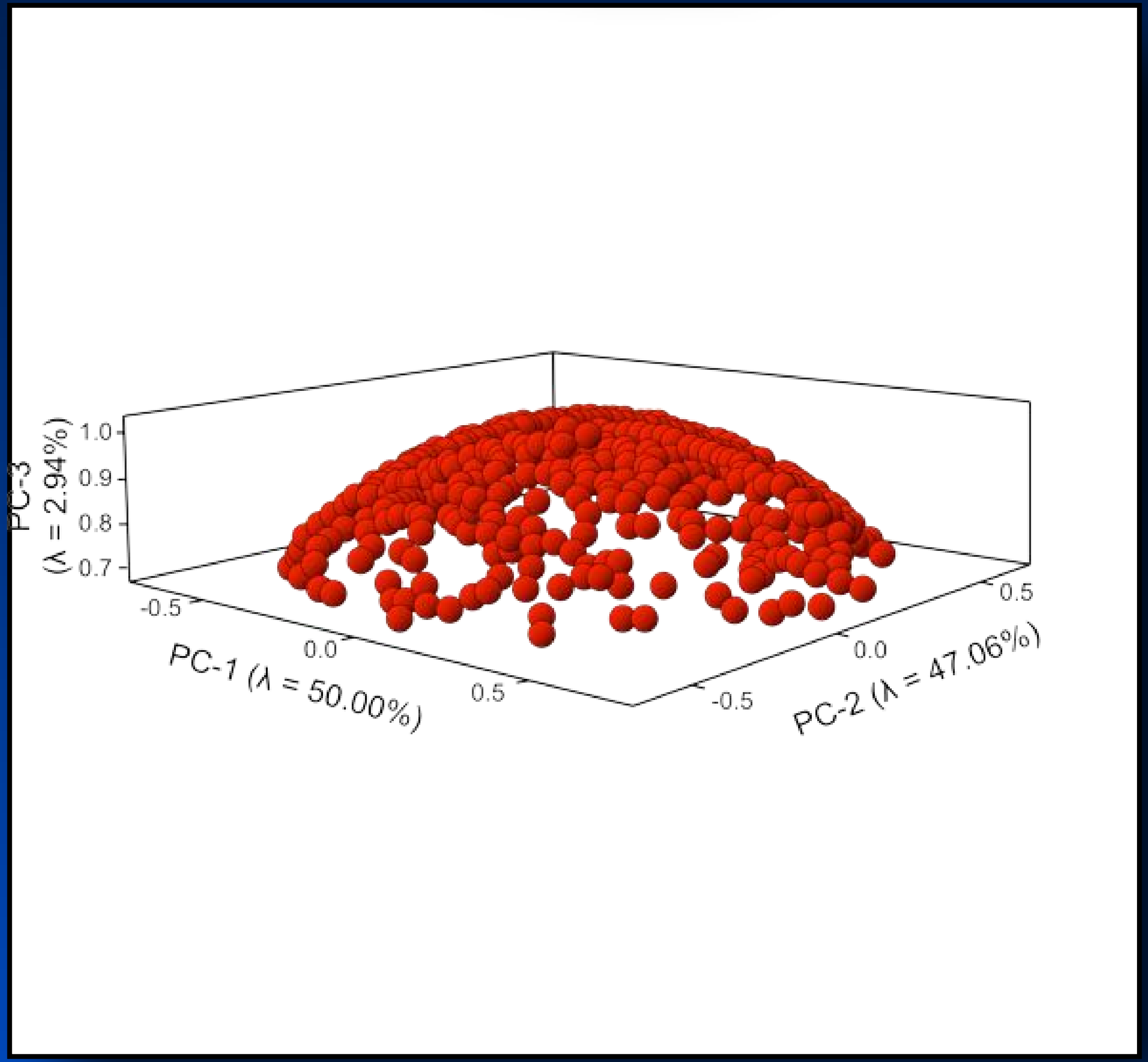
Random Triangles



PC-1 vs. PC-2



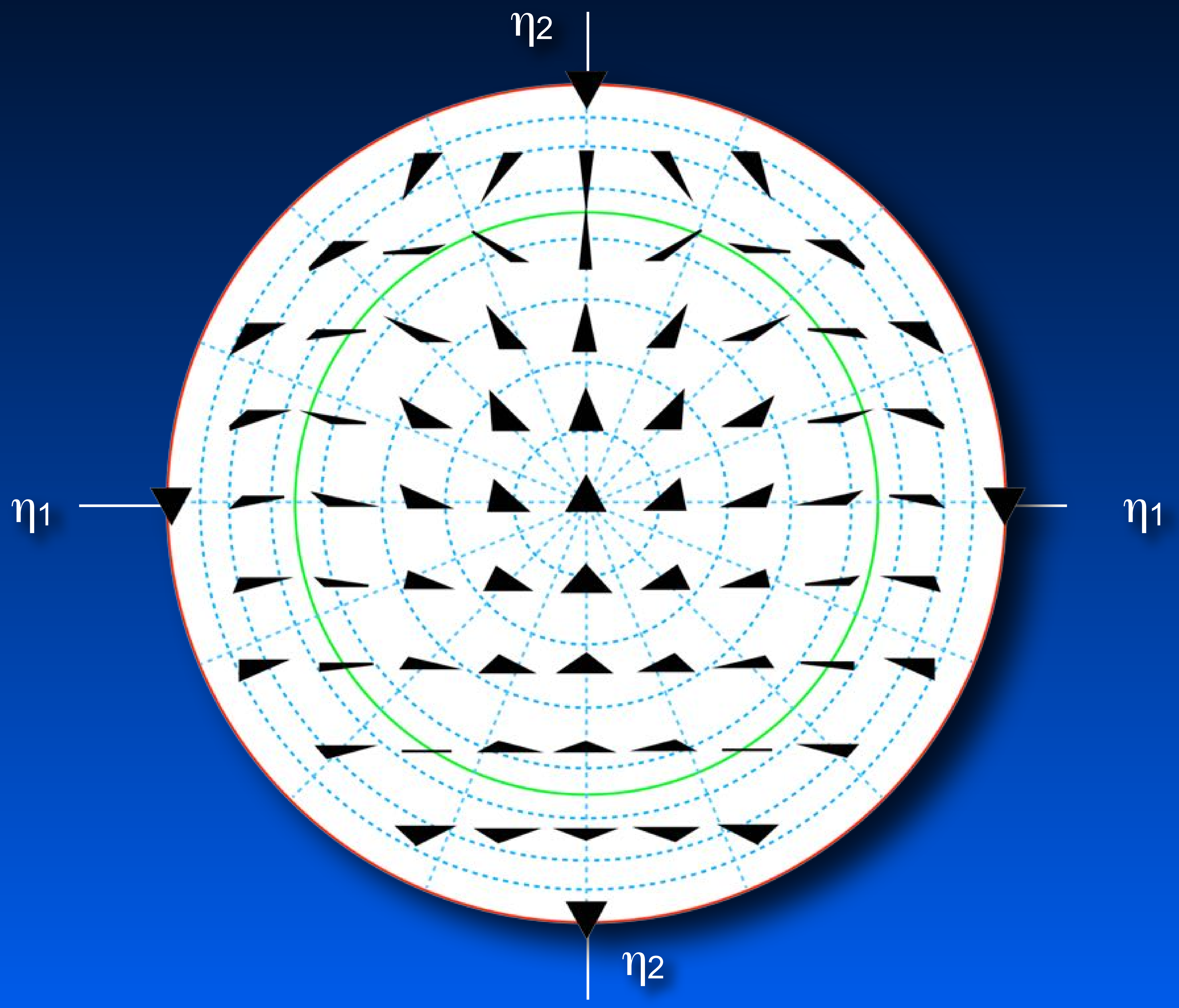
PC-1 vs. PC-2 vs. PC-3



When many triangles (n = 100) are analyzed in this way they form a circular distribution in the PC-1 vs. PC-2 space and a hemispherical distribution in the PC-1 vs. PC-2 vs. PC-3 space; almost as if they were lying on the surface of a sphere.

Geometric Morphometrics

The Triangle Shape Manifold

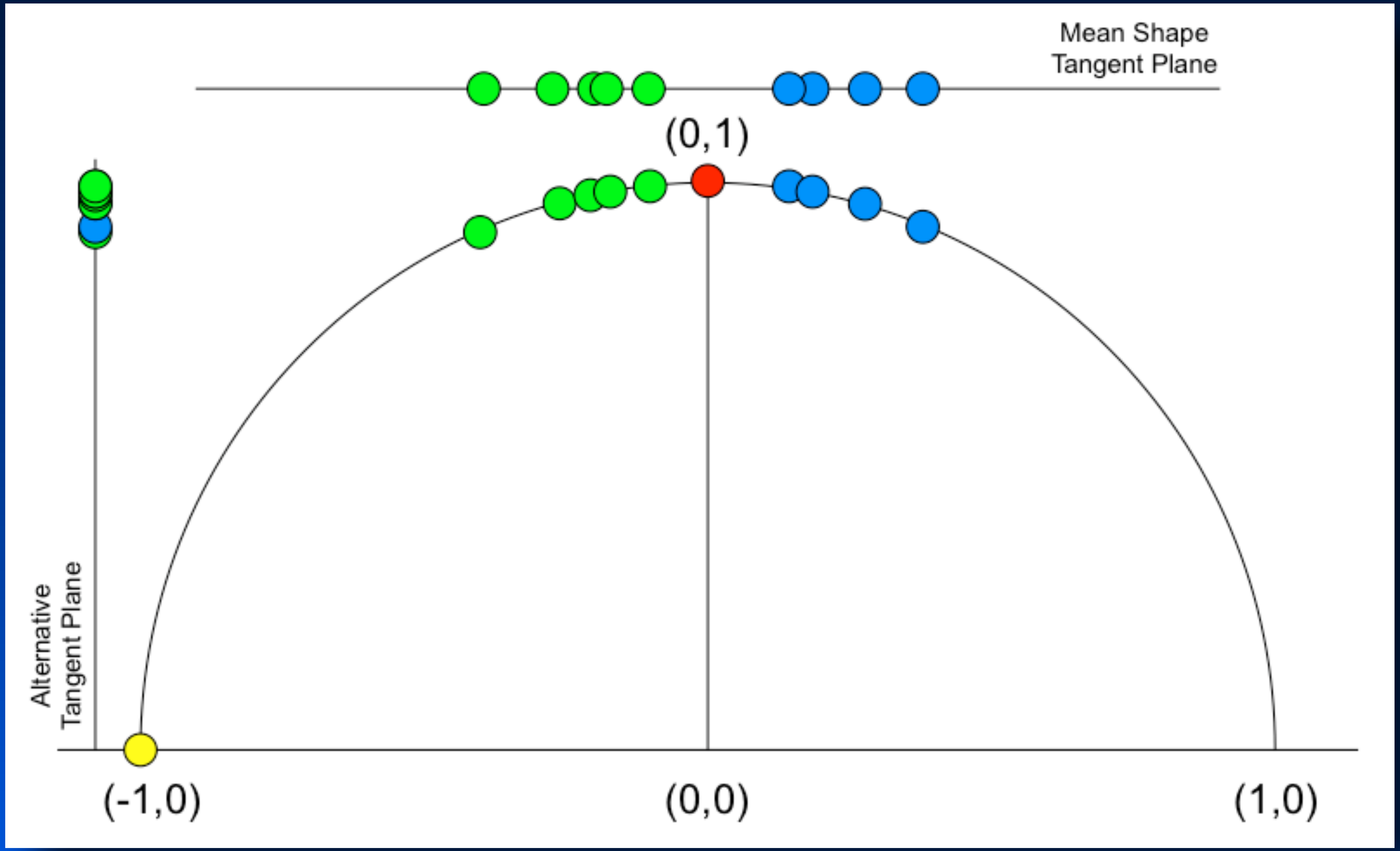


- All possible configurations of landmarks of constant root centroid size and common centroid location are geometrically constrained to lie on the surface of a hyperdimensional manifold ($d = n - 3$).
- If these landmark configurations are rotated to a common orientation (= Procrustes alignment) this shape manifold collapses to a hyperdimensional shape half-space.
- Since radius of this manifold is determined by the size RCS value landmark configurations of different sizes (= form configurations) will be located along radii projecting from the hypermanifold's centroid through the configuration's position when size standardized.

Diagram from Rohlf (2000)

Geometric Morphometrics

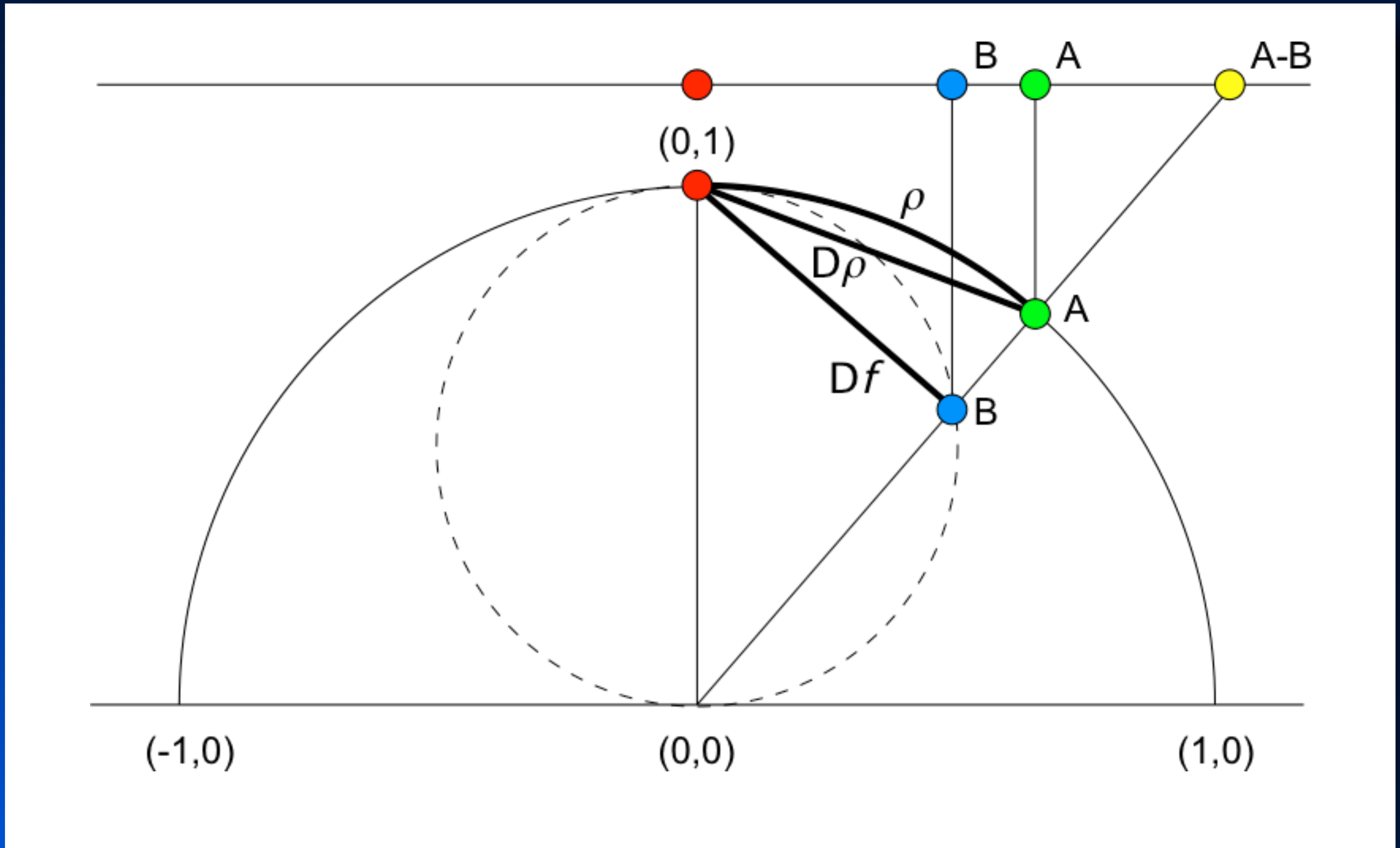
Projection Onto a Linear, Tangent (Ordination) Plane



The reference shape designated during Procrustes alignment establishes the position of a linear (hyper)plane space tangent to the shape half-space onto which landmark configurations may be projected (e.g., via PCA) to obtain a visual summary of form/shape relations. Note the choice of any alternative (e.g, yellow configuration) to the sample mean shape (e.g, red configuration) would be suboptimal for the purpose of visualizing shape/form relations.

Geometric Morphometrics

Alternative Linear Tangent Plane Projections



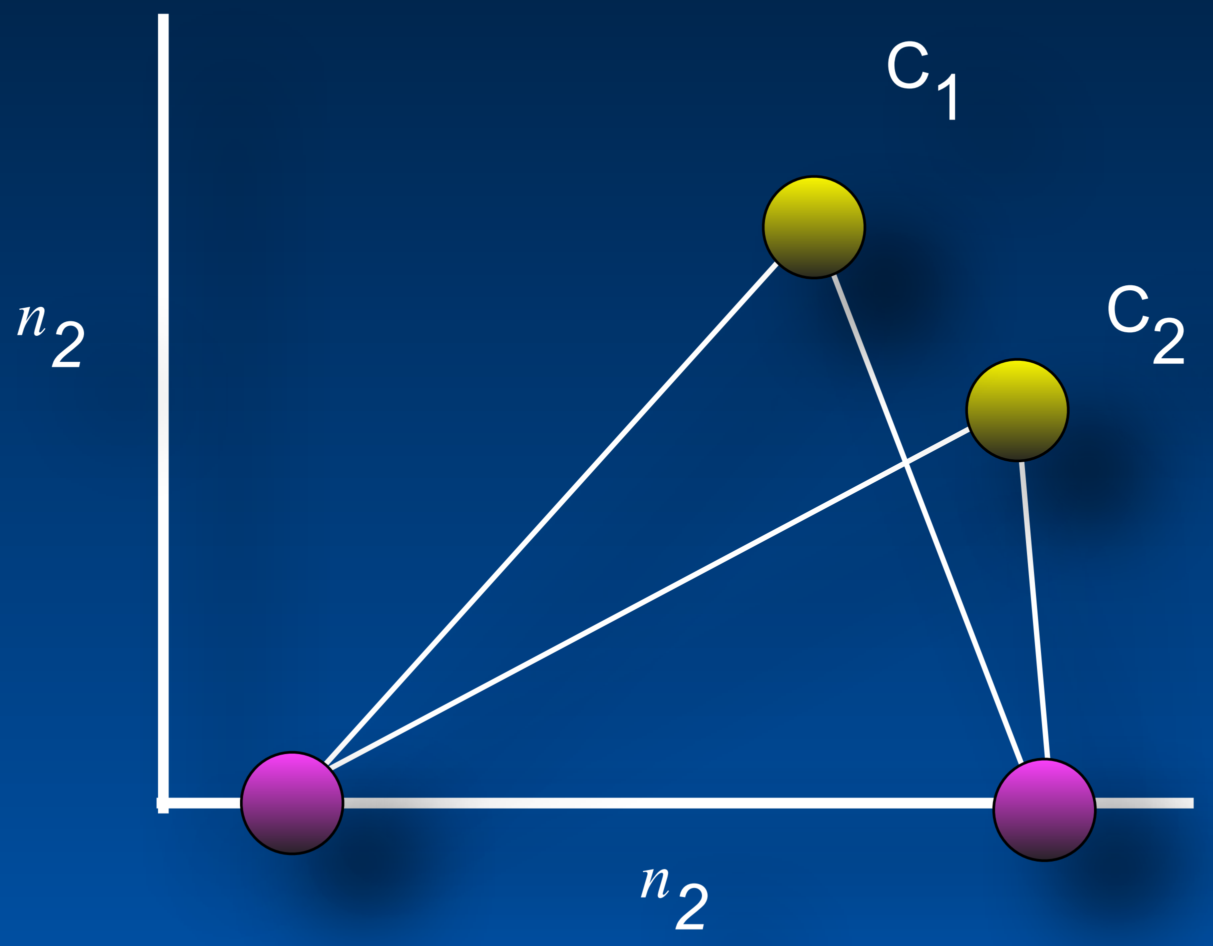
ρ = Procrustes distance. Dρ = Partial Procrustes distance. Df = Full Procrustes distance. Dashed circle = Procrustes Form Manifold.

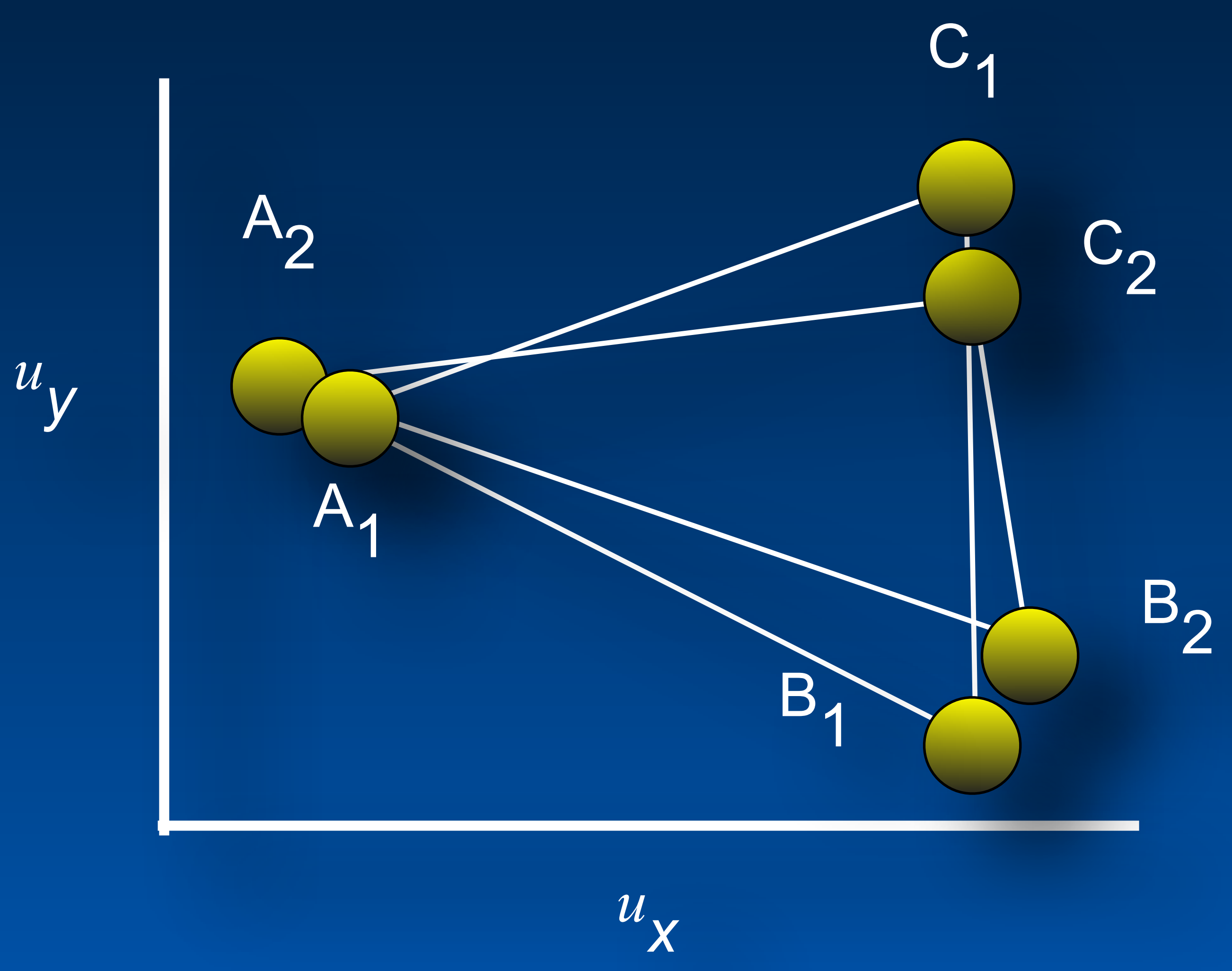
- As with maps of the earth, a wide range of projection options are available (e.g., orthographic, stereographic).
- So long as the range of landmark shape configurations is modest, projection distortion should be minimal.
- Linear-plane projection satisfied the need for shape/form coordinate data to be visualized which greatly aids interpretation.
- Since each coordinate position on the surface of the shape/form half-space represents a unique shape/form configuration, the GM approach supports linear shape modelling.

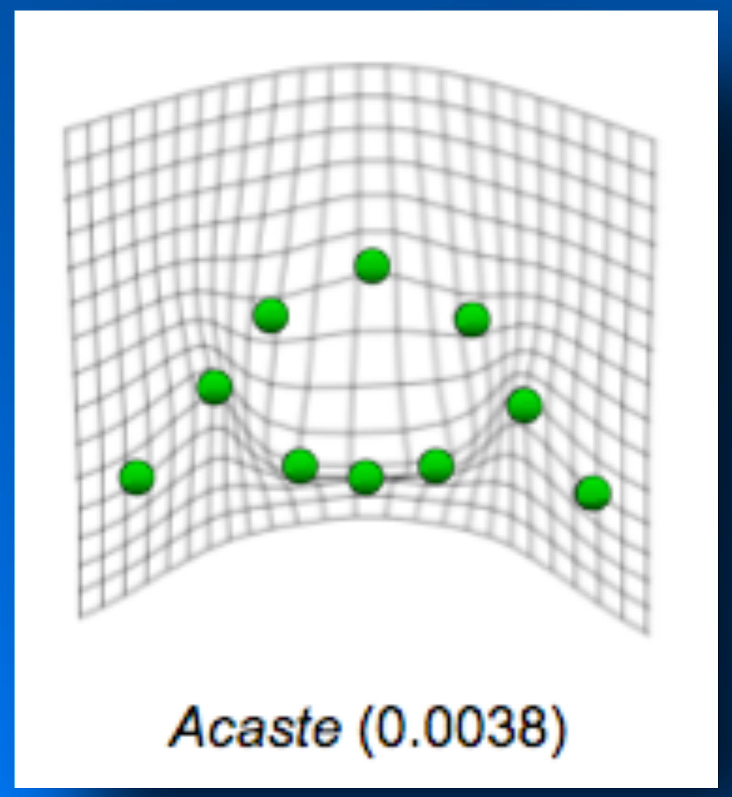
Form & Shape Analysis I

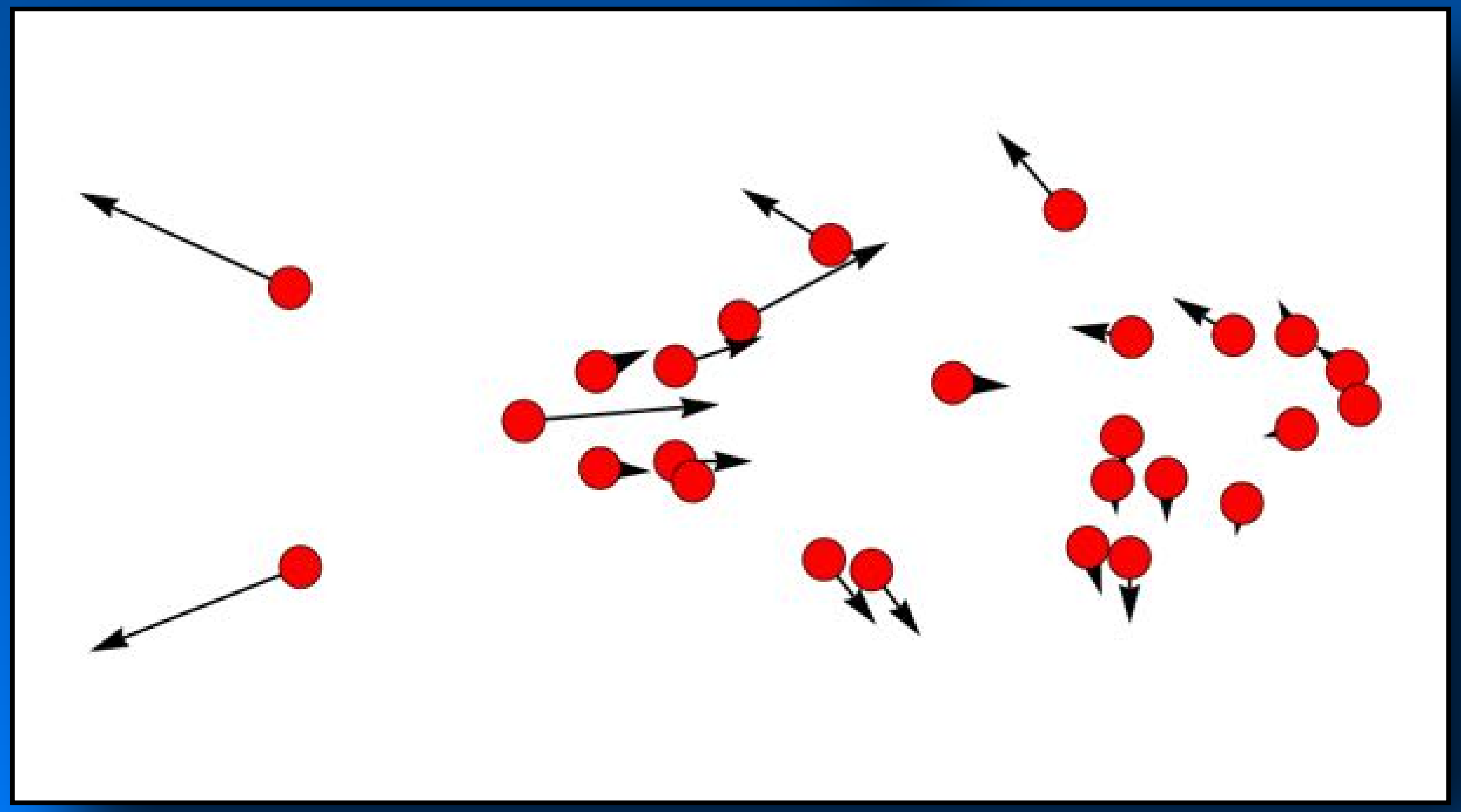
Multivariate Morphometrics, Thin-Plate Spines & Geometric Morphometrics

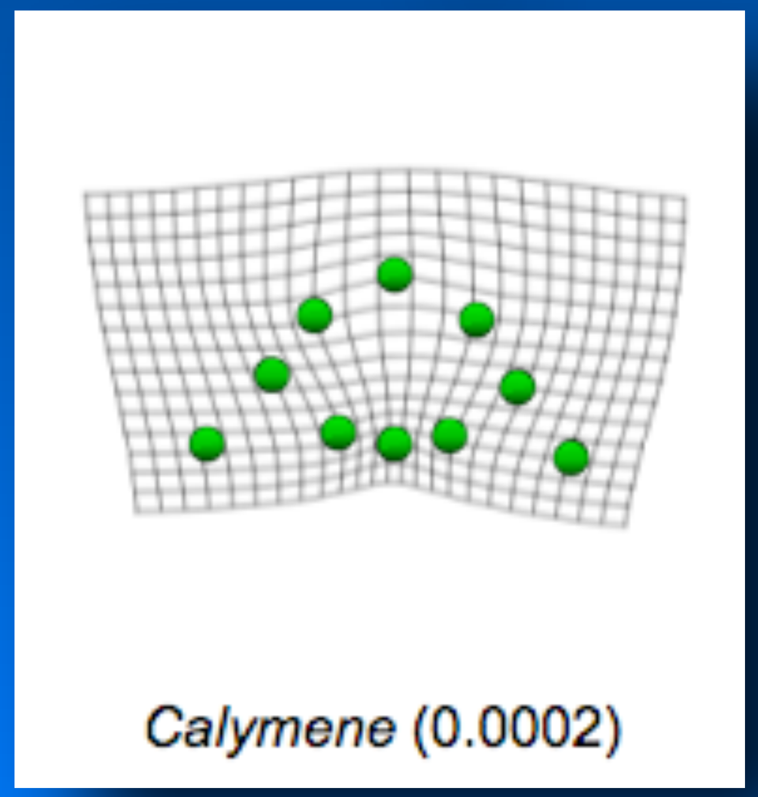
Prof. Norman MacLeod
School of Earth Sciences & Engineering, Nanjing University











Geometric Morphometrics: An Example Analysis

Acaste

Balizoma

Calymene

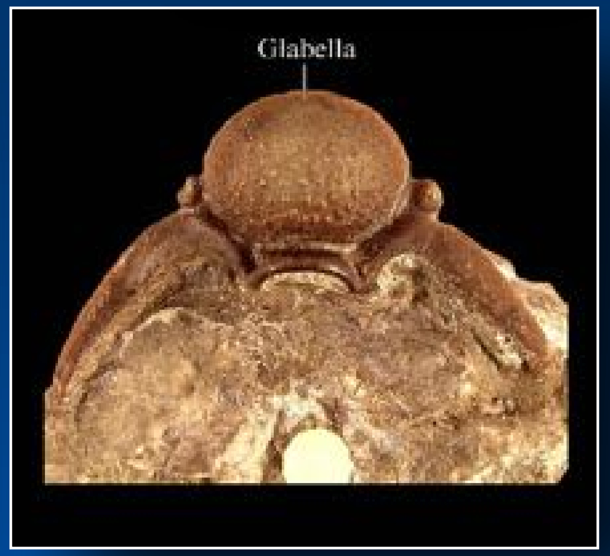
Caraurus

Cheirurus

Cybantyx

Cybeloides

Dalamanities



Delphion

Narroia

Ormathops

Phacopidina

Phacops

Placoparia

Pricylopyg

Ptychoparia

Rhenops

Sphaerexochus

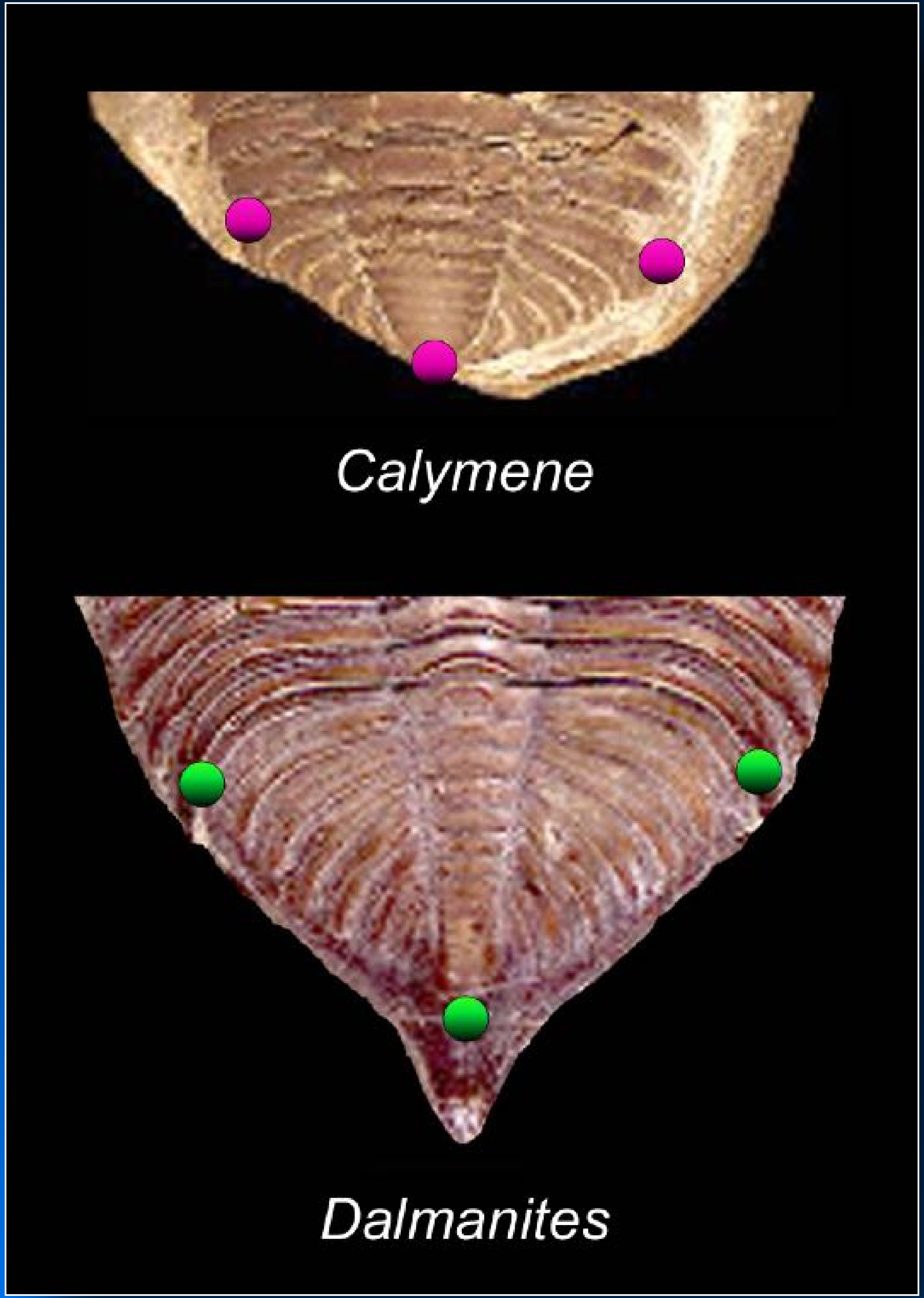
Toxochasmops

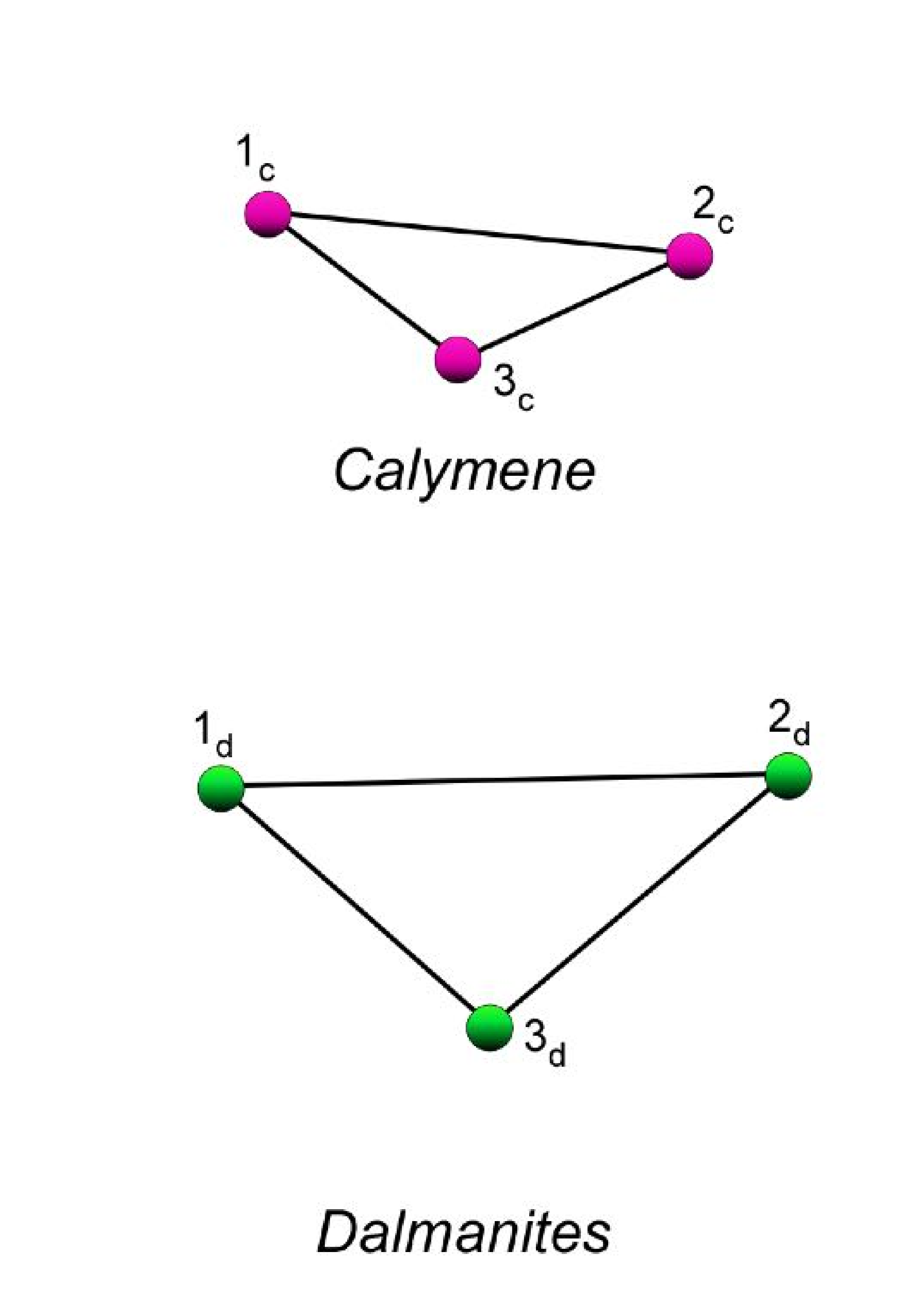
Trimerus

Zacanthoides

Geometric Morphometrics: An Example Analysis

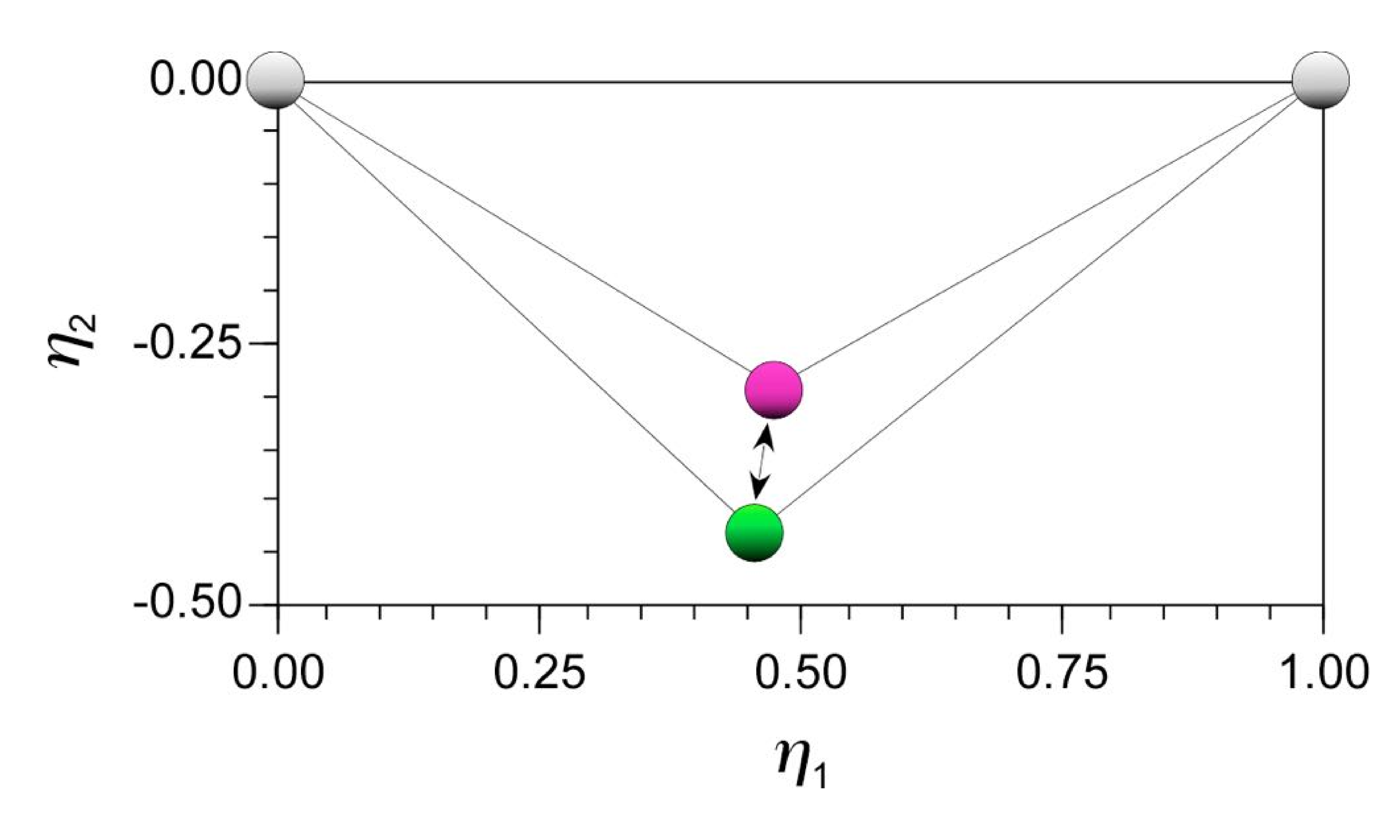
Bookstein Shape Coordinates

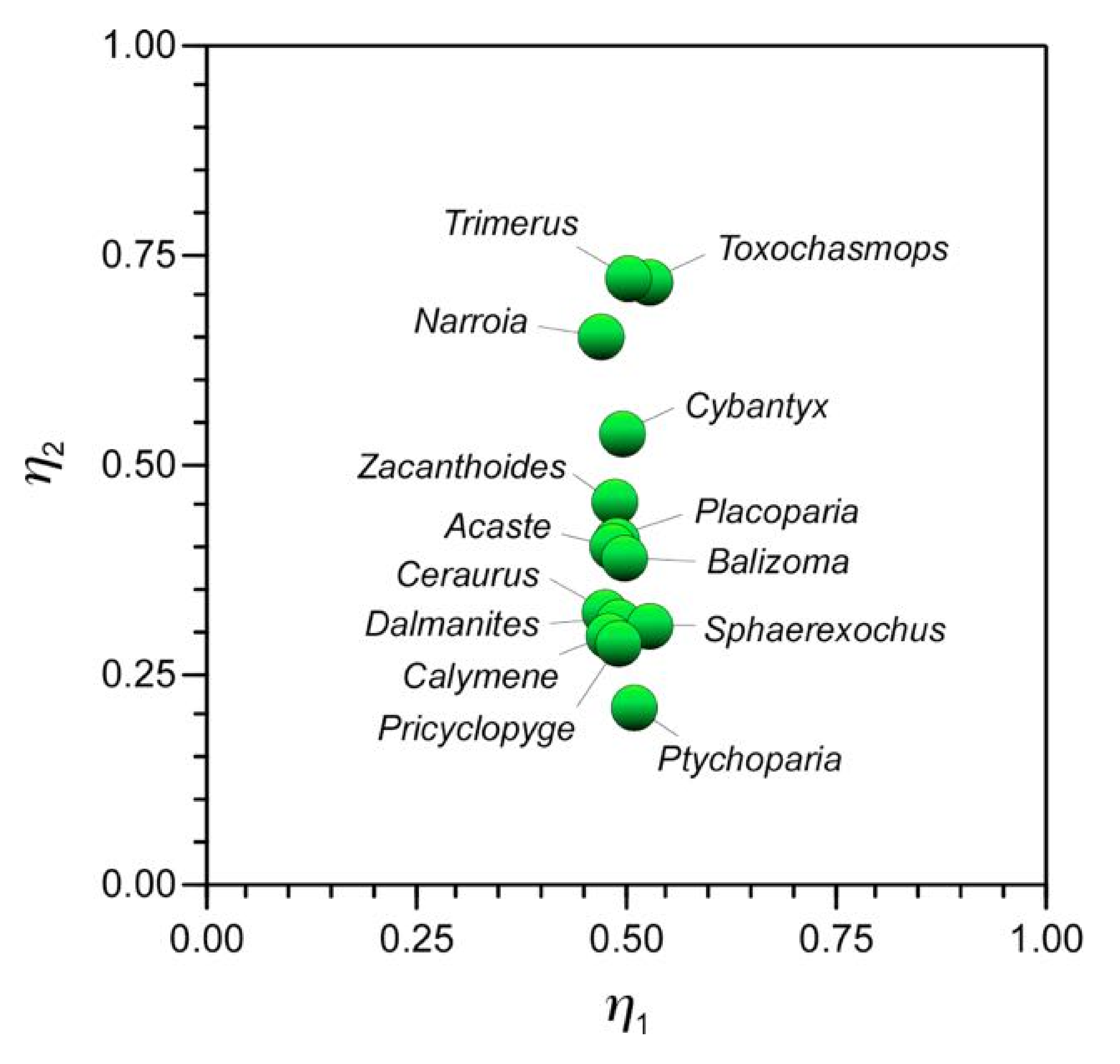




Geometric Morphometrics: An Example Analysis

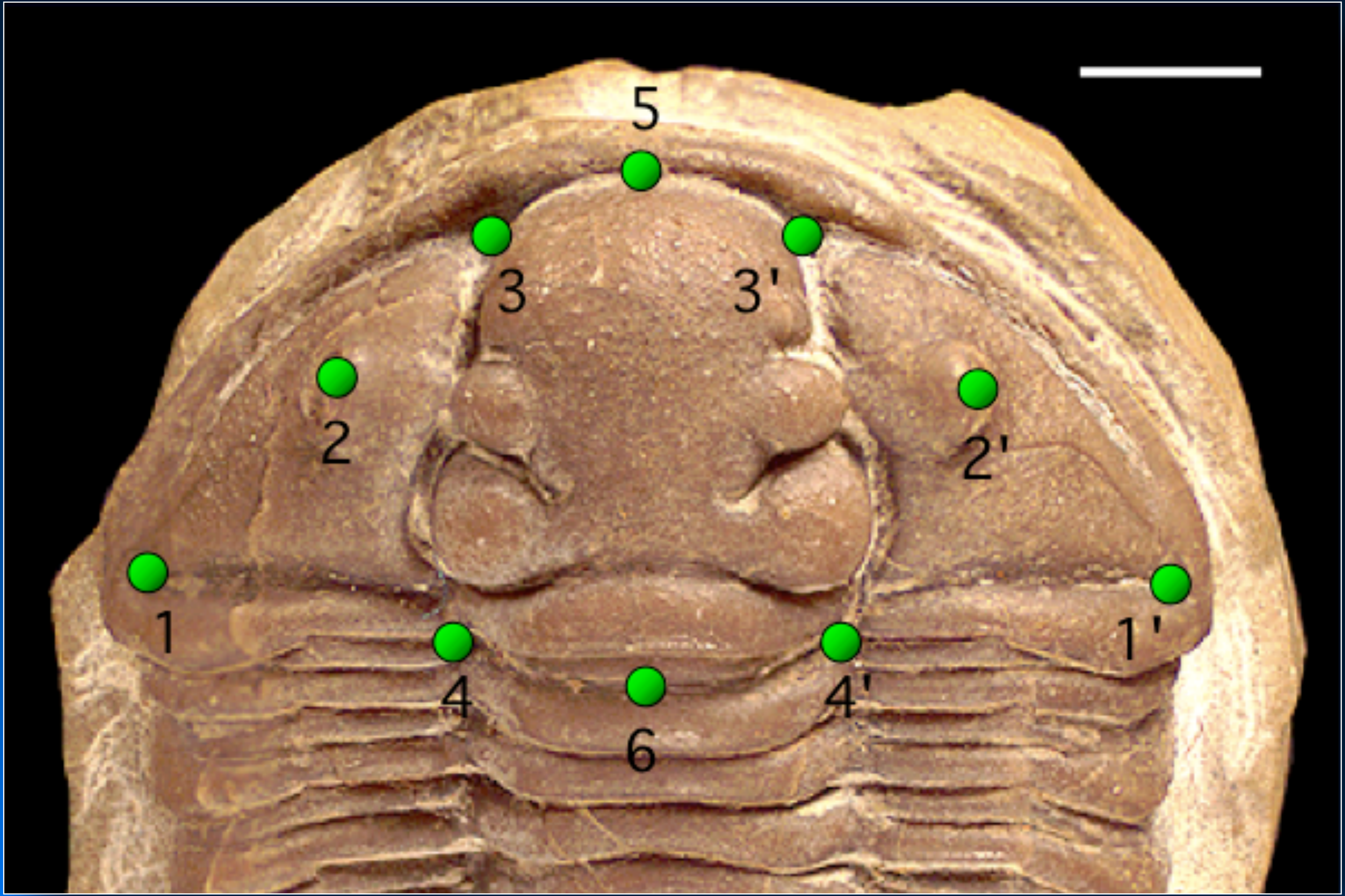
Bookstein Shape Coordinates





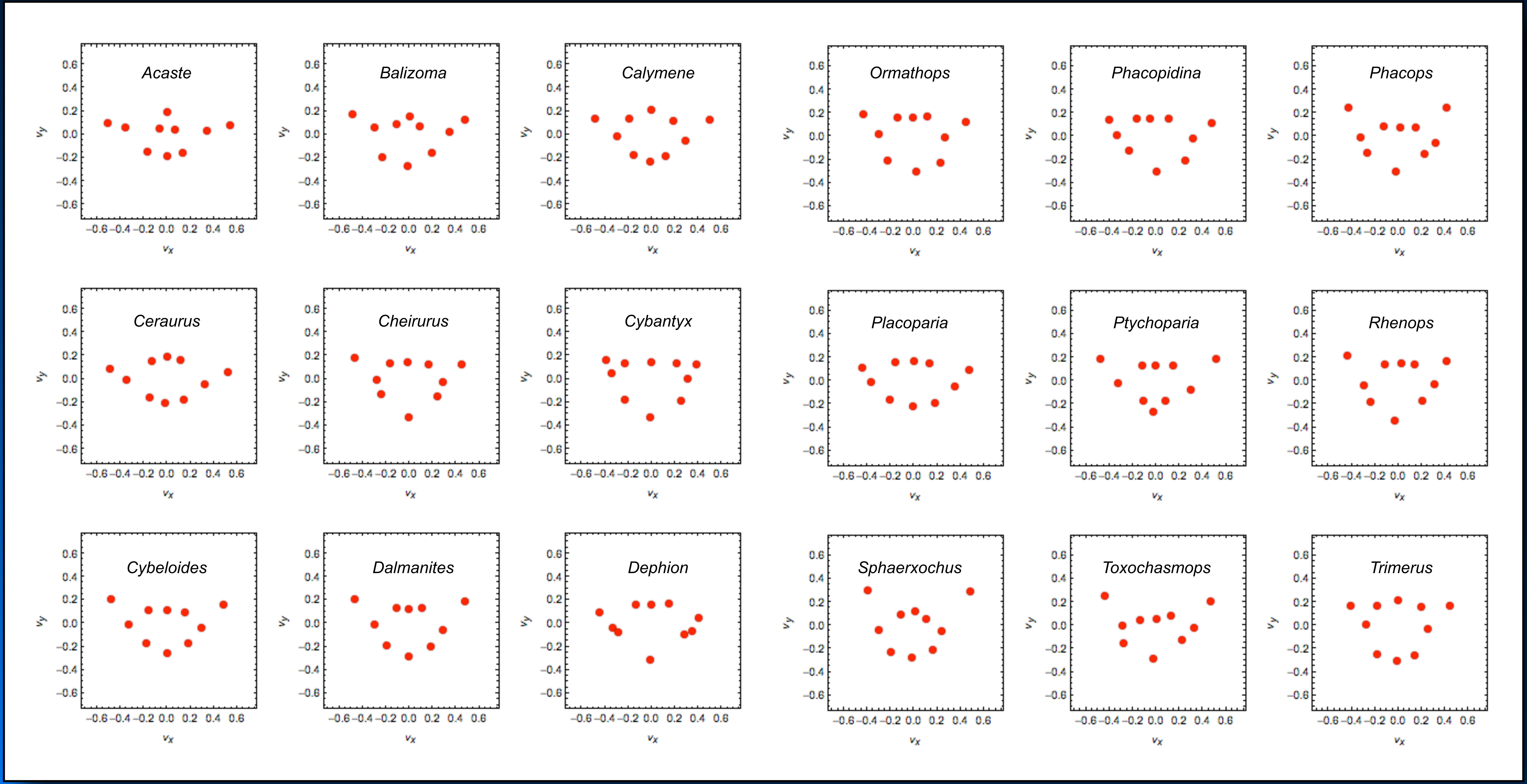
Geometric Morphometrics: An Example Analysis

Procrustes Superposition



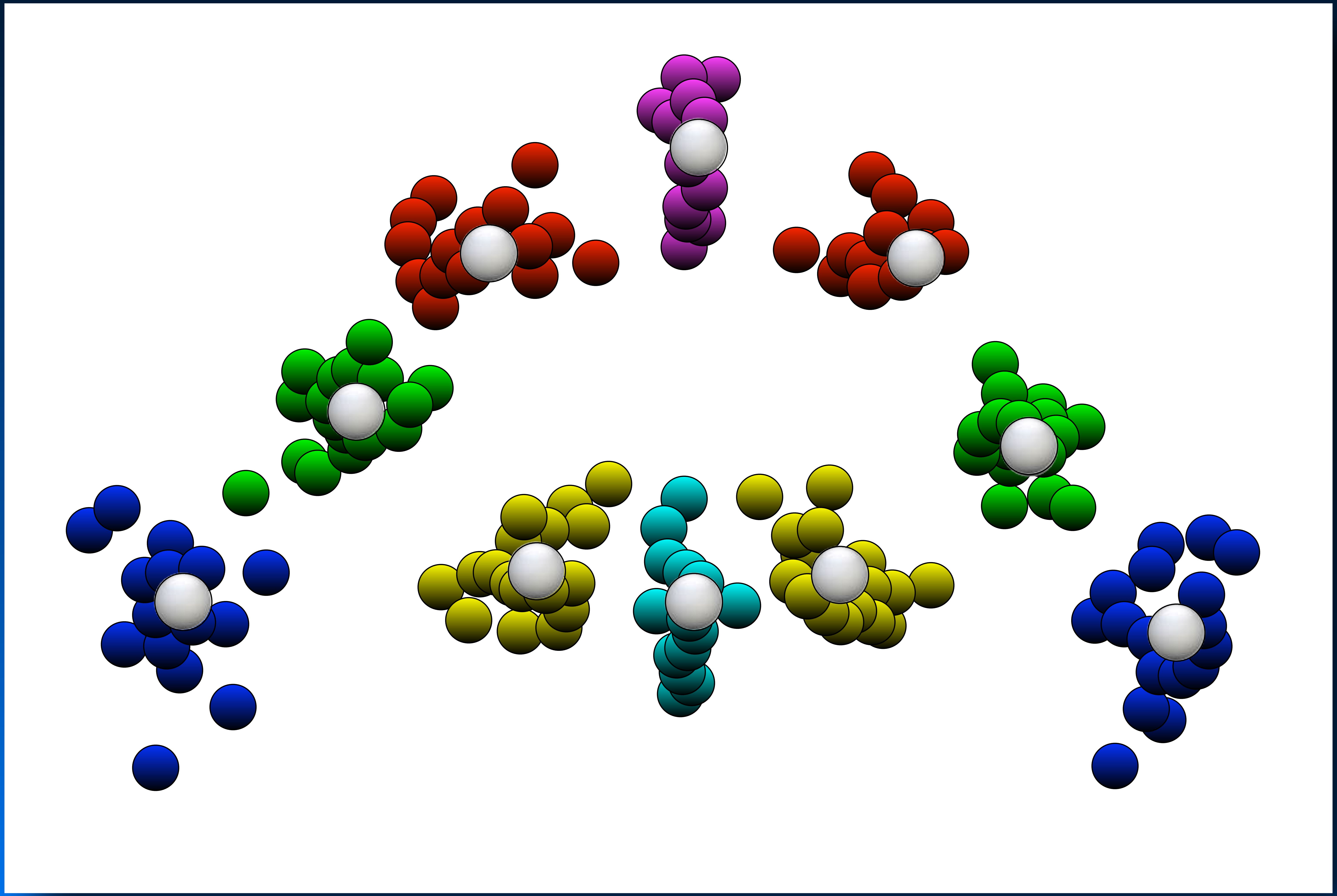
Geometric Morphometrics: An Example Analysis

Procrustes Superposition



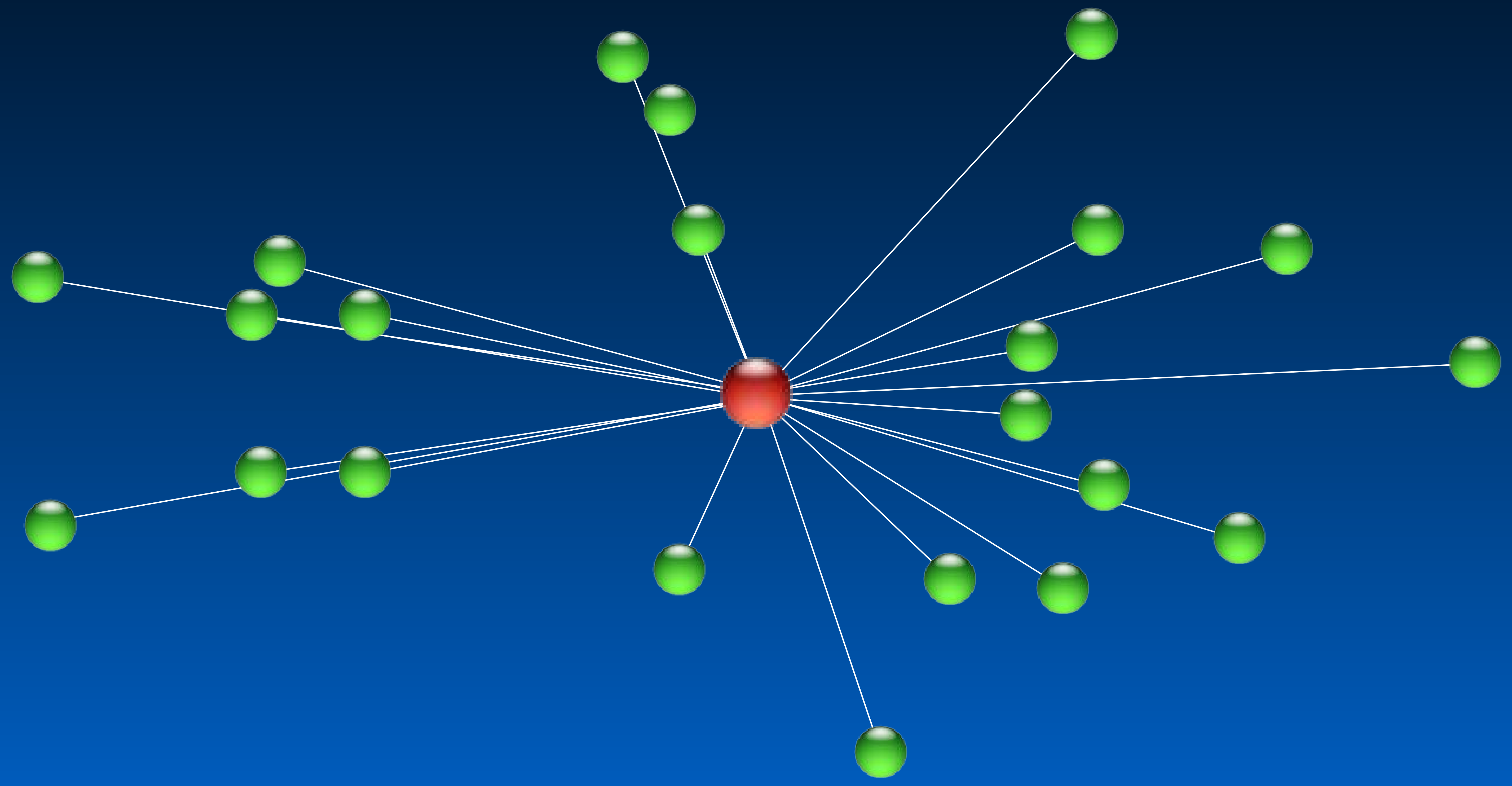
Geometric Morphometrics: An Example Analysis

Procrustes Superposition



Geometric Morphometrics: An Example Analysis

Root Centroid Size



$$RCS = \sqrt{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2 + \left(y_i - \bar{y}\right)^2}$$

Geometric Morphometrics: An Example Analysis

Bending Energy

Interlandmark Distance Equation

$$U\left(r_{ij}\right) = r_{ij}^2 \ln r_{ij}^2$$

r_{ij} = distance between land-marks *i* and *j*.

This quantity is calculated among the landmarks of the reference configuration.

Geometric Morphometrics: An Example Analysis

Bending Energy

Landmark Deformation Matrix: General Format

$$L = \begin{bmatrix} P & Q \\ Q^t & 0 \end{bmatrix}$$

Each term is a partition that expresses a different type of deformation.

Geometric Morphometrics: An Example Analysis

Bending Energy

Landmark Deformation Matrix: General Format

$$P = \begin{bmatrix} 0 & U_{1,2} & U_{1,3} & \cdots & U_{1,p} \\ U_{2,1} & 0 & U_{2,3} & \cdots & U_{2,p} \\ U_{3,1} & U_{3,2} & 0 & \cdots & U_{3,p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ U_{p,1} & U_{p,2} & U_{p,3} & \cdots & 0 \end{bmatrix}$$

Summary of reciprocal of the distances (bending energies) between reference -configuration landmarks.

Geometric Morphometrics: An Example Analysis

Bending Energy

Landmark Deformation Matrix: General Format

$$Q = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & \vdots & \vdots \\ 1 & x_p & y_p \end{bmatrix}$$

Coordinates of the reference landmark configuration.

Geometric Morphometrics: An Example Analysis

Bending Energy

Landmark Deformation Matrix: General Format

$$0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Pad to complete the matrix and express the absence of position, rotation, and scaling information

Geometric Morphometrics: An Example Analysis

Bending Energy

Landmark Deformation Matrix

$$L = \left[\begin{array}{ccccc|ccc} 0 & U_{1,2} & U_{1,3} & \cdots & U_{1,p} & 1 & x_1 & y_1 \\ U_{2,1} & 0 & U_{2,3} & \cdots & U_{2,p} & 1 & x_2 & y_2 \\ U_{3,1} & U_{3,2} & 0 & \cdots & U_{3,p} & 1 & x_3 & y_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ U_{p,1} & U_{p,2} & U_{p,3} & \cdots & 0 & 1 & x_p & y_p \\ \hline 1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 \\ x_1 & x_2 & x_3 & \cdots & x_p & 0 & 0 & 0 \\ y_1 & y_2 & y_3 & \cdots & y_p & 0 & 0 & 0 \end{array}\right]$$

Geometric Morphometrics: An Example Analysis

The Thin Plate Spline

$$L_p^{-1}$$

$$W_{non-uniform} = X_c L_p^{-1}$$

Quantifies the amount and the spatial distribution of ‘energy’ required to achieve any anisotropic deformation of the reference landmark configuration where X_c represents that deformation.

Geometric Morphometrics: An Example Analysis

The Uniform Component of the Bending Energy Matrix

$$L_q^{-1}$$

$$W_{uniform} = X_c L_q^{-1}$$

Quantifies the amount and the spatial distribution of 'energy' required to achieve any isotropic deformation of the reference landmark configuration where X_c represents that deformation.

Geometric Morphometrics: An Example Analysis

The Total Deformation Matrix

$$L^{-1}$$

$$W = L^{-1} X_{+}$$

Quantifies the amount and the spatial distribution of total 'energy' required to achieve any deformation of the reference landmark configuration where X_{+} represents that deformation.

Geometric Morphometrics: An Example Analysis

The Total Deformation Matrix

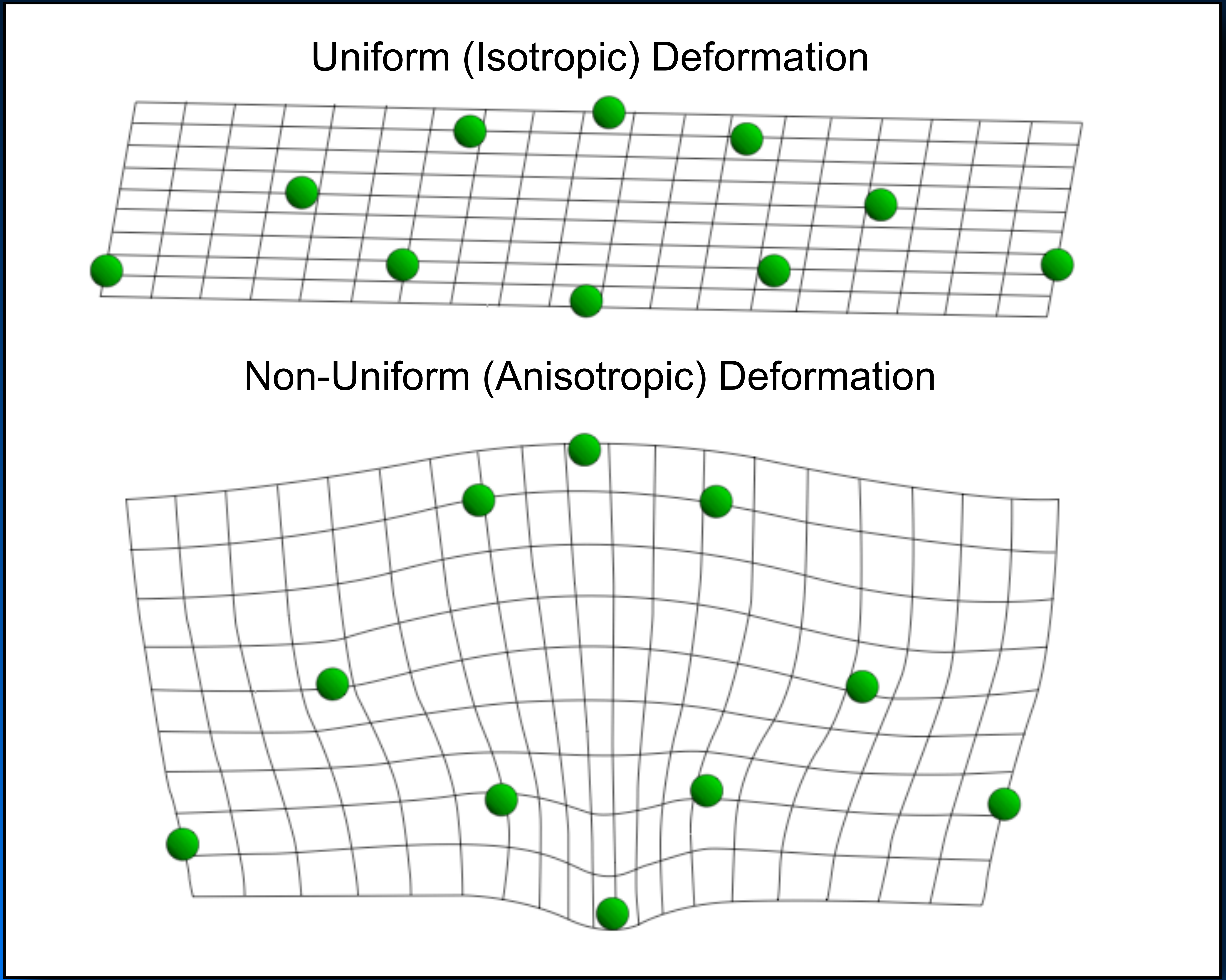
$$z_x(x,y) = W_{p+1,1} + W_{p+2,1}x + W_{p+3,1}y + \sum_{i=1}^{p} W_{i,1} U\left(\sqrt{\left(r_{i,1} - x_{i,1}\right)^2 + \left(r_{i,2} - x_{i,2}\right)^2}\right)$$

$$z_y(x,y) = W_{p+1,2} + W_{p+2,2}x + W_{p+3,2}y + \sum_{i=1}^{p} W_{i,2} U\left(\sqrt{\left(r_{i,1} - x_{i,1}\right)^2 + \left(r_{i,2} - x_{i,2}\right)^2}\right)$$

These equations are used to translate the weight matrix (*W*) into a set of coordinate positions for the vertices of the TPS grid. The grid itself is formed by joining these vertices together with straight line segments.

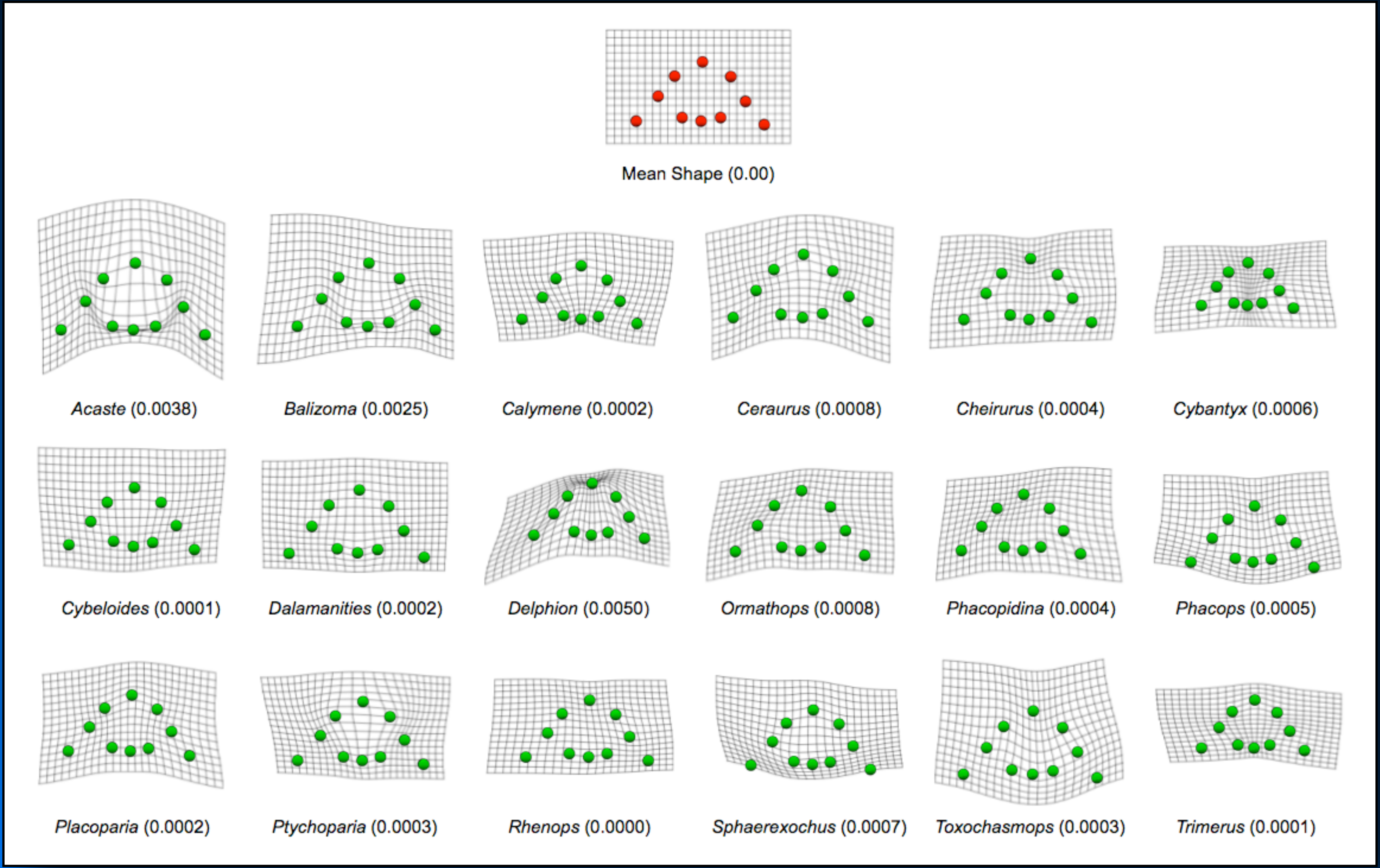
Geometric Morphometrics: An Example Analysis

The Thin Plate Spline



Geometric Morphometrics: An Example Analysis

The Thin Plate Spline



Geometric Morphometrics: An Example Analysis

Principal Warps

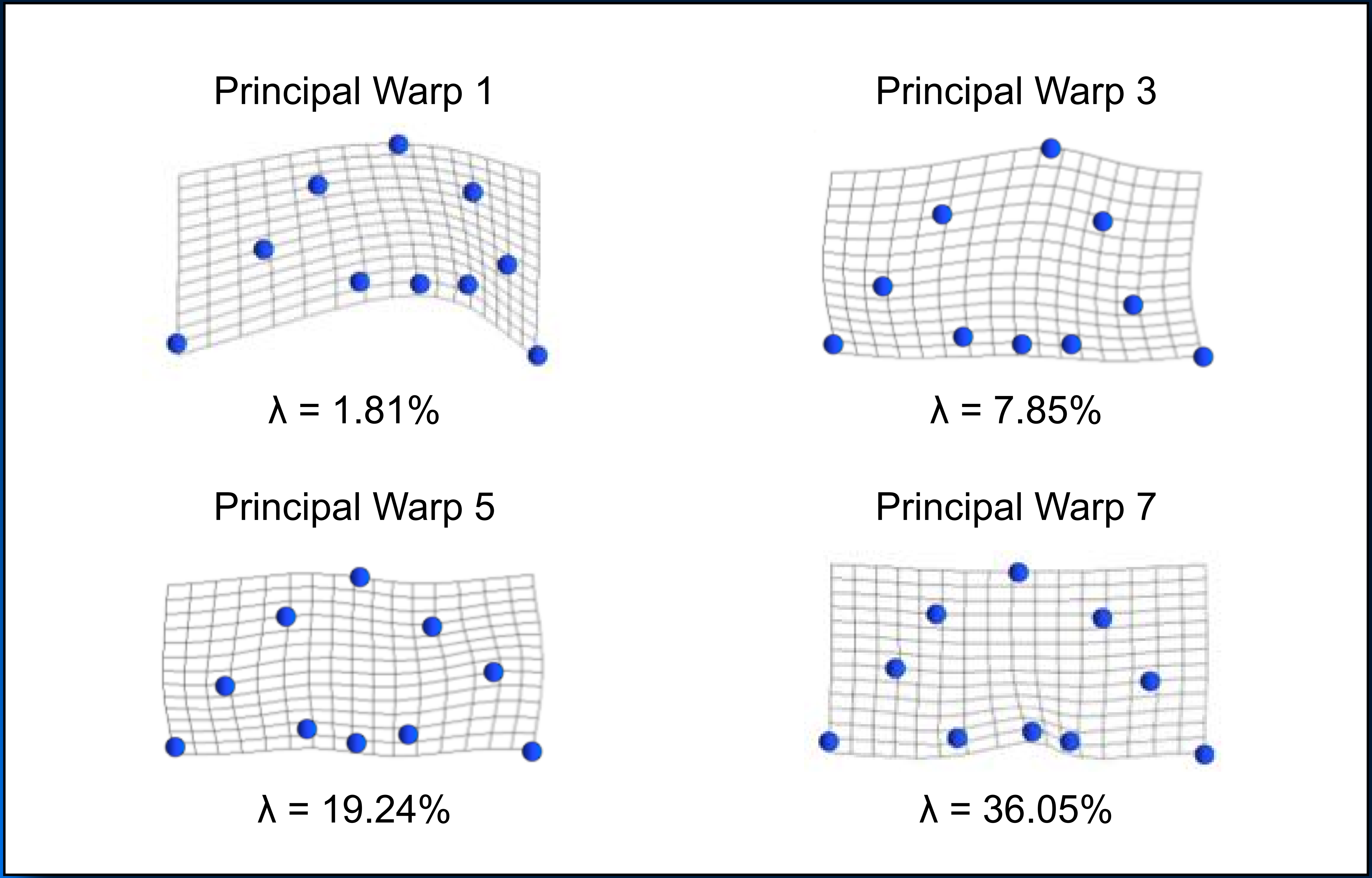
$$L_p^{-1} = \left(Z \Lambda Z' \right)$$

Λ = set of eigenvalues of the bending energy matrix.

Z = set of eigenvectors of the bending energy matrix (= principal warps).

Geometric Morphometrics: An Example Analysis

Principal Warps



Geometric Morphometrics: An Example Analysis

Principal Warps

L Matrix: Reference -Trilobite Mean Shape

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	U	Rx	Ry
L1	0	-0.148	-0.306	-0.228	-0.367	-0.322	-0.335	-0.367	-0.3	-0.15	1	-0.446	0.17
L2	-0.148	0	-0.127	-0.141	-0.301	-0.255	-0.358	-0.333	-0.367	-0.291	1	-0.313	-0.006
L3	-0.306	-0.127	0	-0.215	-0.152	-0.277	-0.292	-0.323	-0.357	-0.328	1	-0.206	-0.173
L4	-0.228	-0.141	-0.215	0	-0.307	-0.078	-0.321	-0.198	-0.335	-0.368	1	-0.138	0.117
L5	-0.367	-0.301	-0.152	-0.307	0	-0.31	-0.151	-0.309	-0.293	-0.366	1	-0.008	-0.284
L6	-0.322	-0.255	-0.277	-0.078	-0.31	0	-0.277	-0.079	-0.262	-0.332	1	0.001	0.142
L7	-0.335	-0.358	-0.292	-0.321	-0.151	-0.277	0	-0.218	-0.115	-0.308	1	0.193	-0.182
L8	-0.367	-0.333	-0.323	-0.198	-0.309	-0.079	-0.218	0	-0.153	-0.24	1	0.14	0.114
L9	-0.3	-0.367	-0.357	-0.335	-0.293	-0.262	-0.115	-0.153	0	-0.164	1	0.308	-0.039
L10	-0.15	-0.291	-0.328	-0.368	-0.366	-0.332	-0.308	-0.24	-0.164	0	1	0.467	0.141
U	1	1	1	1	1	1	1	1	1	1	0	0	0
Rx	-0.446	-0.313	-0.206	-0.138	-0.008	0.001	0.193	0.14	0.308	0.467	0	0	0
Ry	0.17	-0.006	-0.173	0.117	-0.284	0.142	-0.182	0.114	-0.039	0.141	0	0	0

Geometric Morphometrics: An Example Analysis

Principal Warps

L_{inv} Matrix: Reference -Trilobite Mean Shape

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	U	Rx	Ry
L1	2.13	-3.24	1.23	-0.61	0.06	-0.02	0.33	-0.07	0.22	-0.03	0.16	-1.31	1.02
L2	-3.24	10.62	-6.48	-4.26	1.43	2.25	-0.24	-0.00	-0.27	0.21	0.06	0.32	-1.09
L3	1.23	-6.48	8.68	0.47	-5.12	-0.10	1.48	-0.23	-0.24	0.32	0.07	-0.56	-0.22
L4	-0.61	-4.26	0.47	13.49	0.31	-12.25	-0.26	3.17	0.01	-0.07	0.05	0.79	0.73
L5	0.06	1.43	-5.12	0.31	6.66	0.05	-5.31	0.37	1.50	0.05	0.20	-0.02	-1.26
L6	-0.02	2.25	-0.10	-12.25	0.05	20.06	-0.20	-11.95	2.04	0.11	0.13	0.03	0.37
L7	0.33	-0.24	1.48	-0.26	-5.31	-0.20	9.60	0.49	-7.10	1.21	0.05	0.52	-0.16
L8	-0.07	-0.00	-0.23	3.17	0.37	-11.95	0.49	12.70	-3.71	-0.76	0.07	-0.73	0.81
L9	0.22	-0.27	-0.24	0.01	1.50	2.04	-7.10	-3.71	10.46	-2.91	0.07	-0.38	-1.08
L10	-0.03	0.21	0.32	-0.07	0.05	0.11	1.21	-0.76	-2.91	1.88	0.15	1.34	0.87
U	0.16	0.06	0.07	0.05	0.20	0.13	0.05	0.07	0.07	0.15	0.24	-0.00	-0.02
Rx	-1.31	0.32	-0.56	0.79	-0.02	0.03	0.52	-0.73	-0.38	1.34	-0.00	-0.14	-0.05
Ry	1.02	-1.09	-0.22	0.73	-1.26	0.37	-0.16	0.81	-1.08	0.87	-0.02	-0.05	-2.10

Geometric Morphometrics: An Example Analysis

Warp Taxonomy

Principal Warps - Eigenvectors of the bending energy matrix ($L^{-1}{}_{p}$).

Partial Warps - Projections of the x and y landmark coordinate values into the principal warp space that has been scaled by the α-coefficient.

Relative Warps - Eigenvectors of the covariance matrix found using of the entire partial warps score matrix + the uniform component of shape deformation.

Geometric Morphometrics: An Example Analysis

Principal / Partial Warps

- Uses an eigen-decomposition of the bending energy matrix to define a set of progressive more spatially localized shape descriptors (the principal warps).
- Shape descriptors are inherently non-linear and completely tied to selection of the reference shape. As such they are relatively unstable under change on the reference shape.
- These shape descriptors can be used as axes of an ordination space (the partial warps space) to portray shape similarities and differences between sets of landmark configurations.
- This shape space can be (crudely) ‘tuned’ to emphasize different spatial scales of deformation using the α-coefficient.
- Scores of shapes projected into the space of the principal/partial warps can be used in subsequent analyses (e.g., PCA, CVA).

Geometric Morphometrics: An Example Analysis

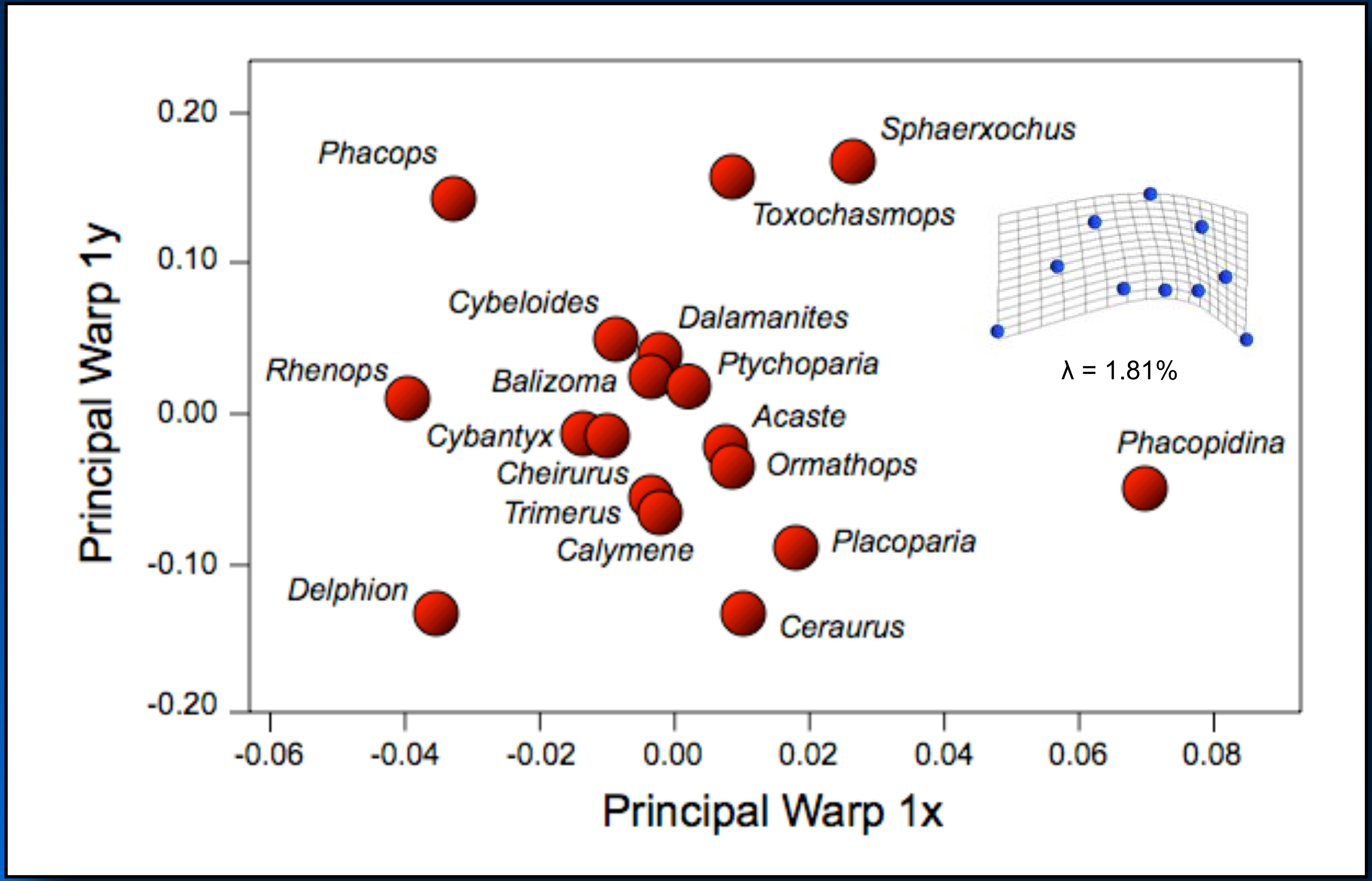
Partial Warps Score Matrix: Mean Shape as Reference, α = 0.0

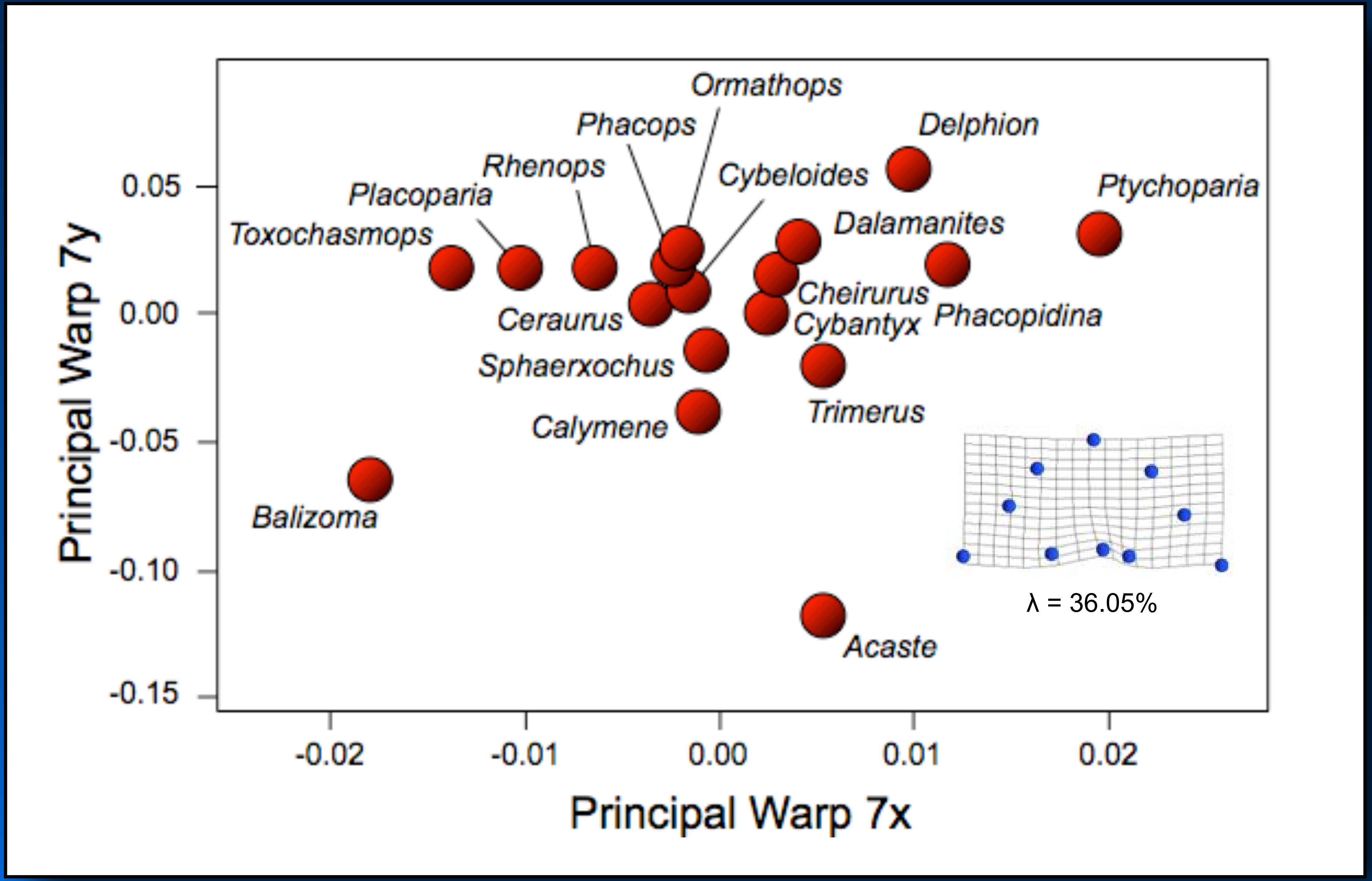
Genus	1x	1y	2x	2y	3x	3y	4x	4y	5x	5y	6x	6y	7x	7y	Unix	Uniy
Acaste	0.005	-0.118	-0.020	-0.060	0.097	-0.006	-0.073	-0.008	-0.008	0.036	0.093	0.004	0.008	-0.023	0.151	0.022
Balizoma	-0.018	-0.064	-0.030	-0.038	0.013	-0.021	-0.062	-0.018	0.038	0.051	0.006	-0.031	-0.003	0.024	0.041	-0.000
Calymene	-0.001	-0.038	-0.012	-0.010	-0.011	0.000	0.085	-0.011	0.006	-0.045	0.088	0.019	-0.002	-0.066	0.016	0.009
Ceraurus	-0.003	0.004	0.005	-0.037	0.068	-0.007	-0.000	0.010	0.001	-0.017	0.082	0.009	0.010	-0.131	0.077	0.009
Cheirurus	0.003	0.015	0.001	0.055	-0.074	0.020	0.007	0.009	0.018	0.047	-0.033	-0.013	-0.010	-0.013	0.006	-0.029
Cybantyx	0.002	0.002	0.029	-0.019	-0.041	-0.014	0.106	0.004	-0.009	0.078	-0.121	-0.007	-0.014	-0.012	-0.031	-0.028
Cybeloides	-0.002	0.009	0.013	-0.004	0.003	0.006	0.028	-0.010	-0.007	-0.022	0.040	-0.011	-0.008	0.047	0.034	-0.018
Dalamanites	0.004	0.027	0.002	-0.007	0.007	-0.012	-0.044	-0.007	0.003	-0.031	0.042	0.005	-0.002	0.035	-0.042	-0.011
Delphion	0.009	0.057	-0.003	0.103	-0.029	0.015	-0.024	0.020	0.022	0.054	-0.130	0.001	-0.035	-0.133	0.077	-0.015
Ormathops	-0.002	0.022	0.016	-0.049	-0.044	-0.003	-0.029	0.016	-0.025	0.021	-0.023	-0.013	0.008	-0.035	-0.063	-0.041
Phacopidina	0.012	0.020	0.020	-0.005	-0.007	0.029	-0.016	0.025	-0.020	0.041	-0.064	0.035	0.070	-0.049	0.007	-0.028
Phacops	-0.002	0.019	-0.007	0.062	-0.011	-0.003	-0.019	-0.004	0.008	-0.024	-0.076	0.015	-0.033	0.143	-0.019	0.017
Placoparia	-0.010	0.017	-0.005	-0.035	0.047	-0.001	0.025	-0.002	-0.010	-0.028	-0.022	0.018	0.017	-0.090	0.058	0.004
Ptychoparia	0.019	0.030	-0.007	0.021	0.074	-0.020	0.041	-0.008	-0.003	-0.041	0.140	0.011	0.002	0.021	-0.015	0.023
Rhenops	-0.007	0.018	-0.008	0.038	-0.012	0.022	-0.024	0.002	0.020	-0.001	-0.039	-0.024	-0.039	0.010	-0.076	0.013
Sphaerxochus	-0.001	-0.015	0.023	0.003	-0.006	0.015	-0.037	-0.030	-0.038	-0.103	0.038	-0.005	0.026	0.169	-0.115	0.034

Geometric Morphometrics: An Example Analysis

Partial Warps

Linv Matrix: Reference -Trilobite Mean Shape, α = 0.0





Geometric Morphometrics: An Example Analysis

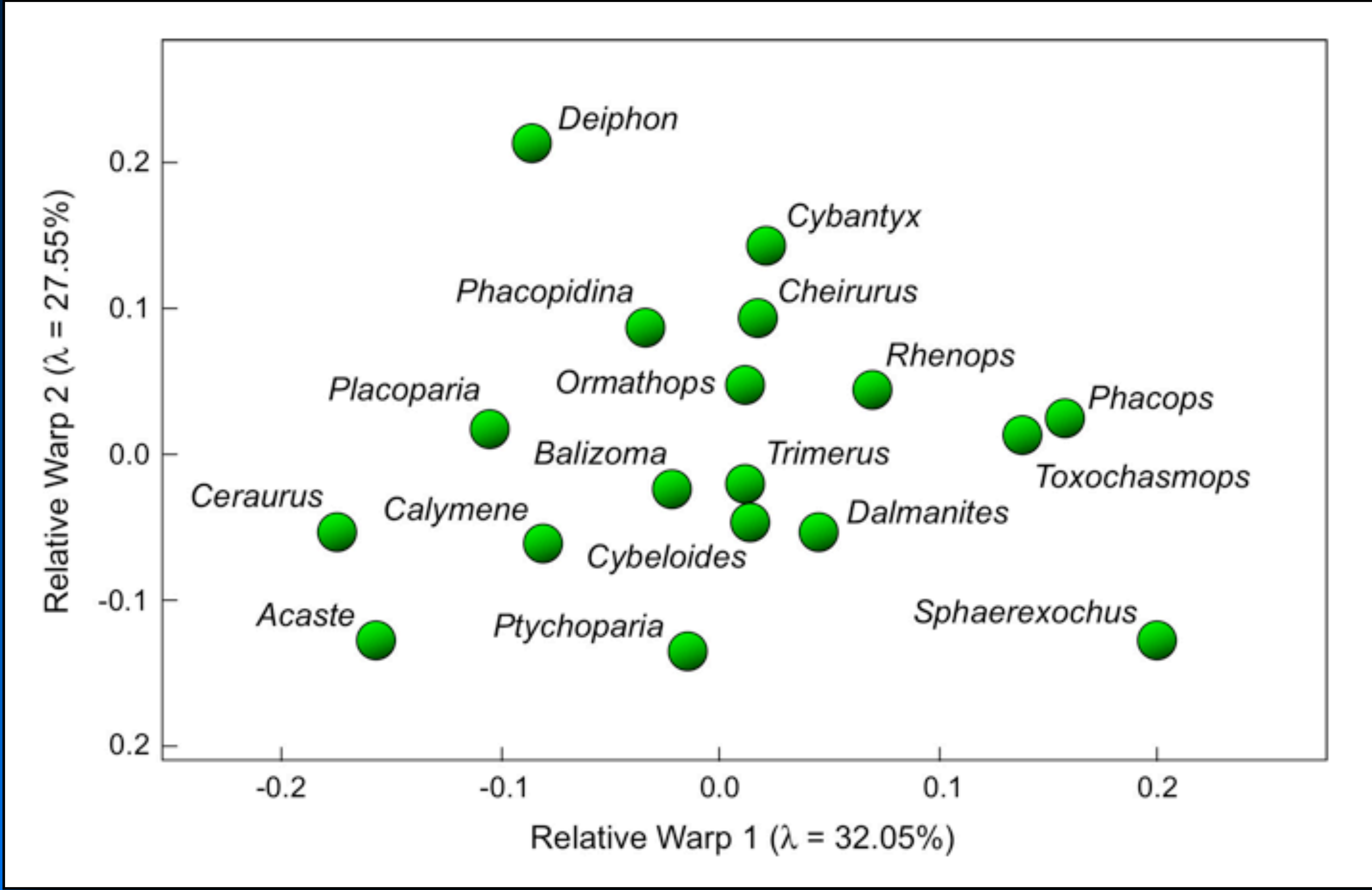
Eigenvalues of Partial Warps Score (Covariance) Matrix

Table 1. Eigenvalues for the trilobite principal warp weight data.

Relative Warp	Eigenvalue	Variance (%)	Cum. Variance (%)	Relative Warp	Eigenvalue	Variance (%)	Cum. Variance (%)
1	0.010165	32.054	32.054	9	0.000379	1.195	98.421
2	0.008737	27.551	59.605	10	0.000226	0.711	99.133
3	0.005782	18.233	77.838	11	0.000119	0.375	99.508
4	0.002277	7.180	85.018	12	0.000083	0.262	99.769
5	0.001594	5.027	90.045	13	0.000048	0.151	99.920
6	0.001098	3.463	93.508	14	0.000019	0.059	99.979
7	0.000663	2.089	95.598	15	0.000005	0.016	99.995
8	0.000517	1.629	97.226	16	0.000002	0.005	100.000

Geometric Morphometrics: An Example Analysis

Eigenscores of Landmark Configurations Projected onto Partial Warps Score Eigenvectors



Geometric Morphometrics: An Example Analysis

Warp Taxonomy

Relative Warps - Eigenvectors of the covariance matrix formed by use of the entire partial warps score matrix + the uniform component of shape deformation.

But the principal warps are just a re-description of the shape coordinates in terms of the reference shape’s bending energy matrix.

Procrustes PCA - Eigenvectors of the covariance matrix of the Procrustes superposed shape coordinates.

Geometric Morphometrics: An Example Analysis

Eigenvalues of Procrustes Shape Coordinate (Covariance) Matrix

Table 3. Eigenvalues for the *Procrustes* superposed data.

Principal Component	Eigenvalue	Variance (%)	Cum. Variance (%)	Principal Component	Eigenvalue	Variance (%)	Cum. Variance (%)
1	0.010167	31.972	31.972	10	0.000226	0.711	98.997
2	0.008739	27.483	59.455	11	0.000127	0.401	99.398
3	0.005788	18.201	77.655	12	0.000110	0.345	99.742
4	0.002277	7.159	84.815	13	0.000056	0.177	99.919
5	0.001595	5.014	89.829	14	0.000019	0.059	99.978
6	0.001099	3.455	93.284	15	0.000005	0.016	99.994
7	0.000677	2.130	95.414	16	0.000002	0.005	99.999
8	0.000522	1.641	97.055	17	0.000000	0.001	100.000
9	0.000392	1.231	98.286	18	0.000000	0.000	100.000

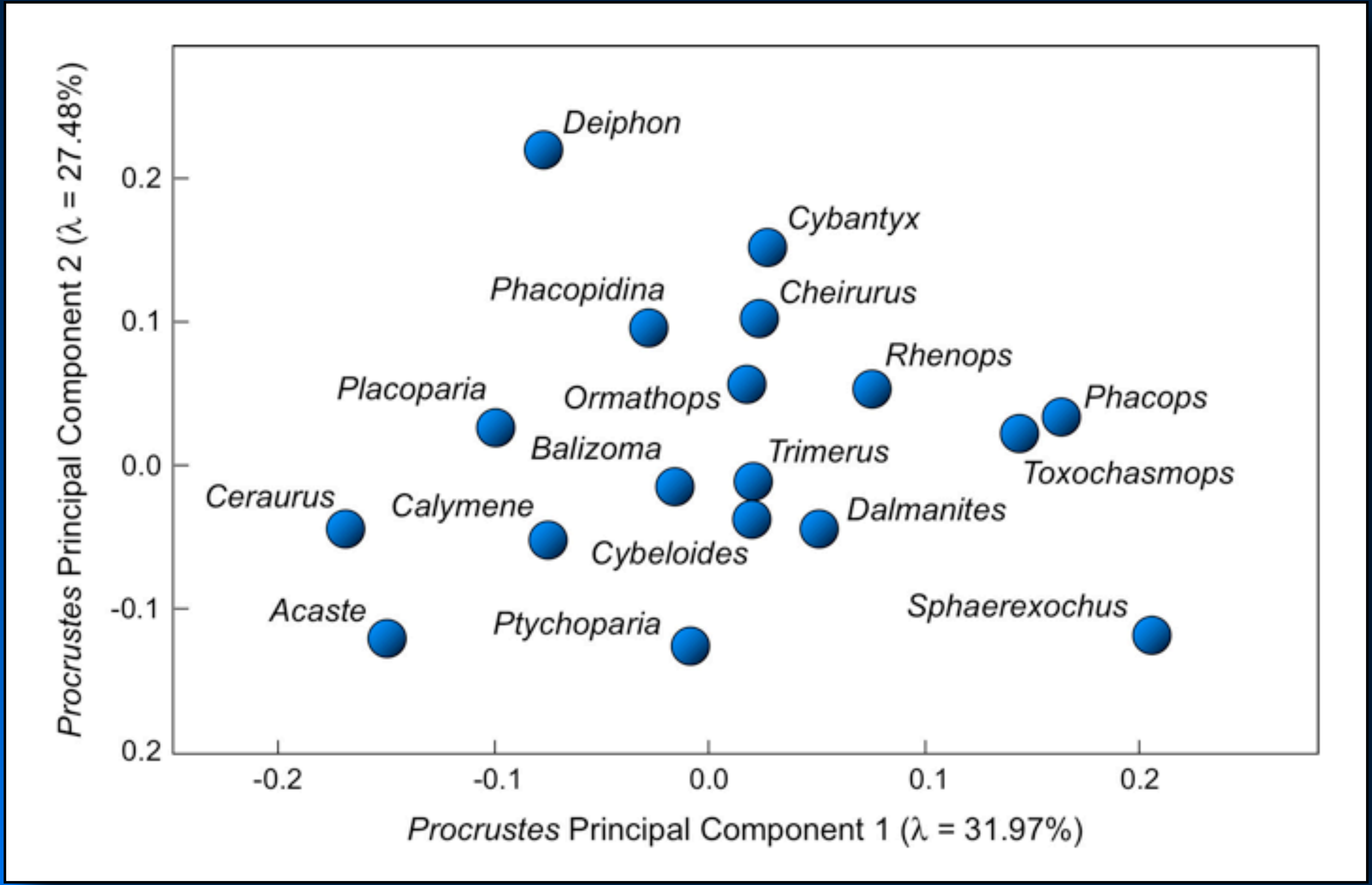
Geometric Morphometrics: An Example Analysis

Eigenvectors of Procrustes Shape Coordinate (Covariance) Matrix

Variable	PC–1	PC–2	PC–3	PC–4	PC–5	PC–6	PC–7
x1	−0.187	0.188	0.185	−0.142	0.085	0.491	−0.313
y1	−0.548	−0.134	−0.006	0.015	0.020	−0.032	0.178
x2	−0.153	0.004	0.110	−0.071	0.050	−0.486	0.278
y2	0.082	−0.008	−0.097	−0.499	−0.221	0.002	0.123
x3	0.191	−0.396	0.259	0.137	−0.153	0.076	0.206
y3	0.129	0.222	−0.290	0.354	−0.054	0.127	0.173
x4	−0.014	−0.201	−0.287	0.050	0.484	−0.288	−0.293
y4	0.124	0.158	0.363	0.166	0.275	0.018	0.024
x5	0.038	0.030	0.018	−0.095	0.069	0.091	0.207
y5	0.247	−0.312	−0.153	0.041	−0.065	0.359	−0.178
x6	−0.047	−0.036	−0.015	0.043	−0.063	−0.183	−0.319
y6	0.299	−0.031	0.277	−0.214	0.077	−0.292	−0.345
x7	−0.126	0.511	−0.192	−0.166	0.171	0.154	0.167
y7	0.029	0.131	−0.367	0.323	−0.207	−0.267	0.009
x8	−0.031	0.169	0.305	0.147	−0.616	−0.102	−0.051
y8	0.144	0.229	0.328	0.221	0.289	0.024	0.119
x9	0.161	0.099	−0.279	0.071	−0.160	0.067	−0.282
y9	0.057	0.010	−0.116	−0.521	−0.075	−0.059	0.079
x10	0.168	−0.368	−0.105	0.026	0.134	0.180	0.399
y10	−0.562	−0.265	0.060	0.114	−0.041	0.120	−0.183

Geometric Morphometrics: An Example Analysis

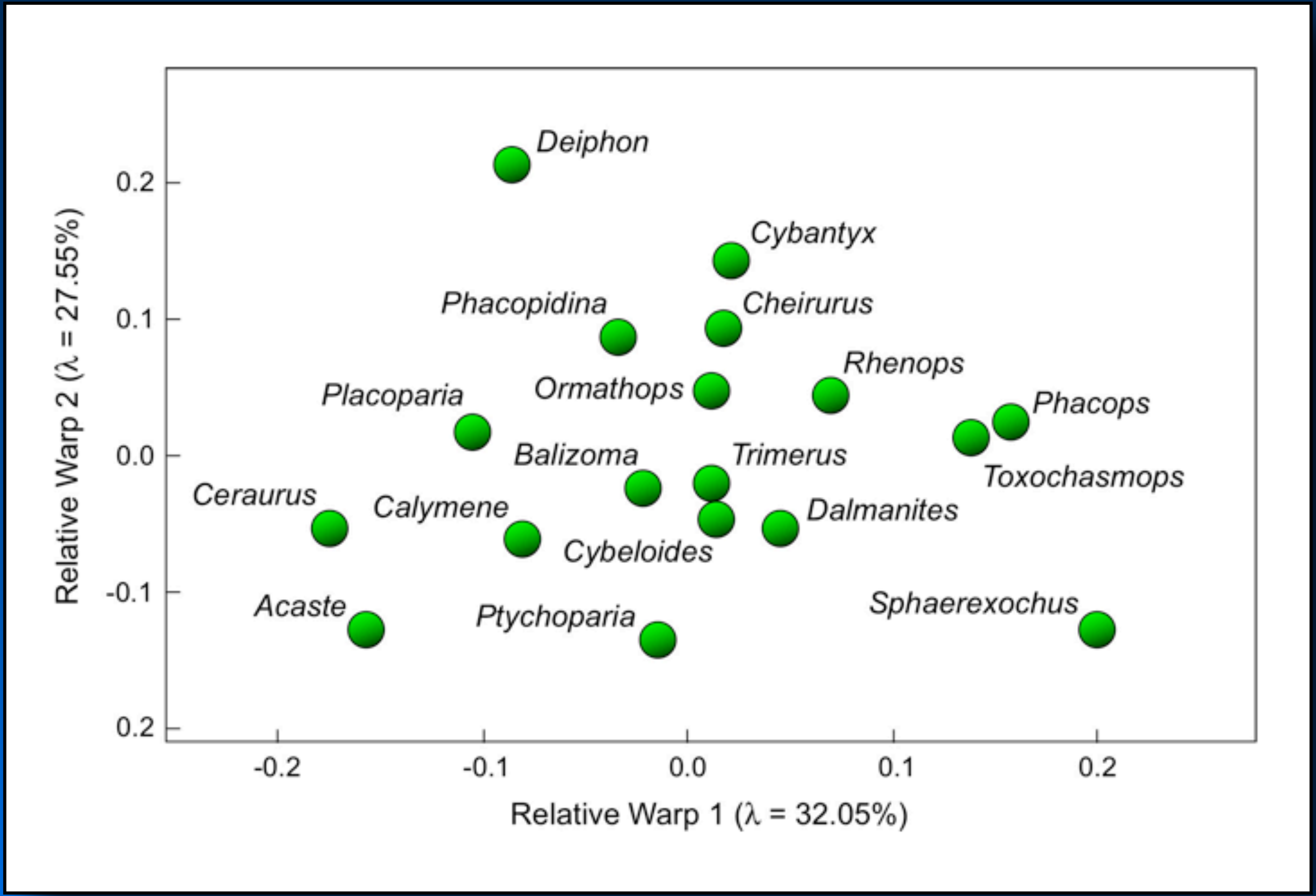
Eigenscores of Landmark Configurations Projected onto Procrustes Shape Coordinate Eigenvectors



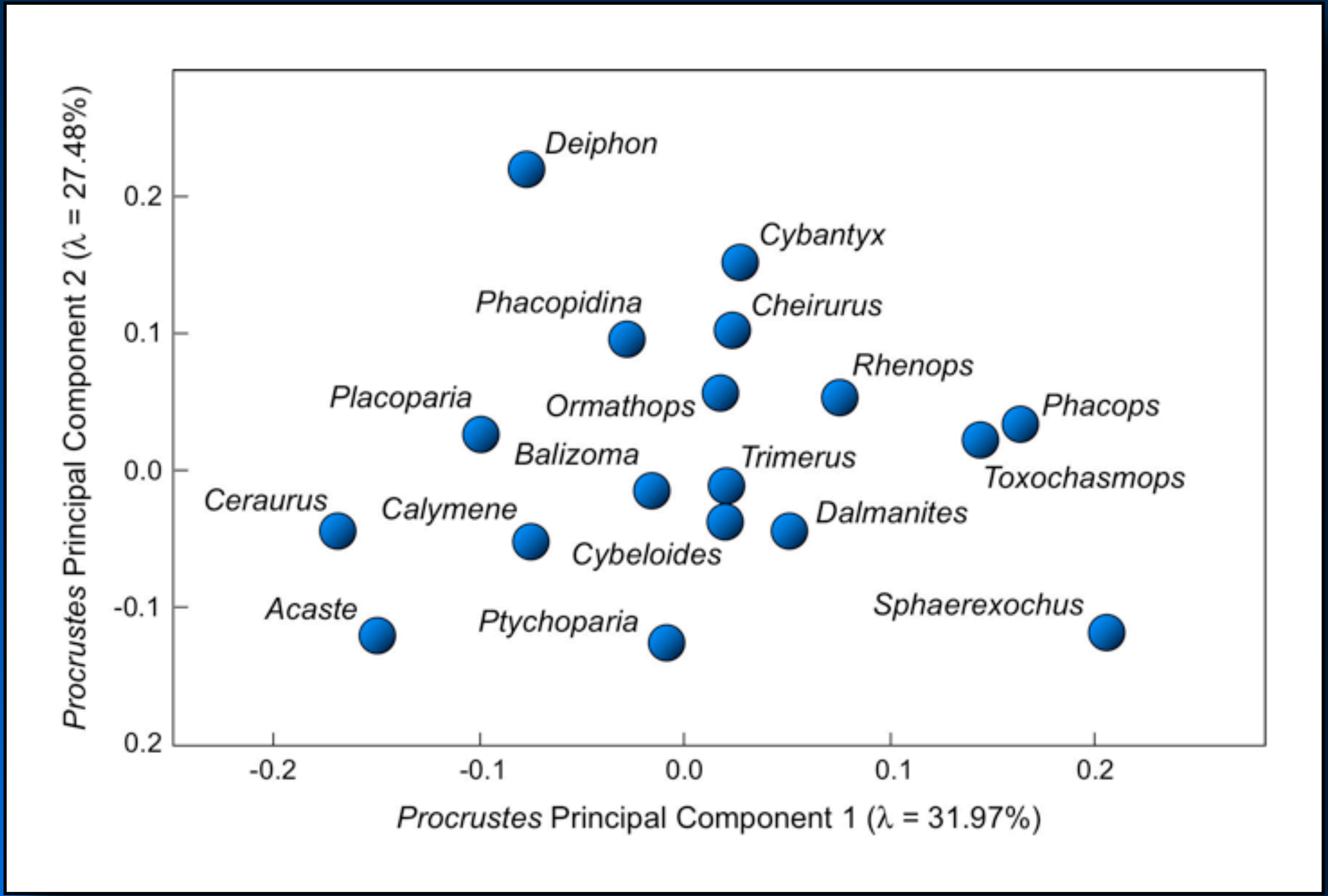
Geometric Morphometrics: An Example Analysis

Trilobite Dataset

Relative Warps Analysis of Partial Warps Scores



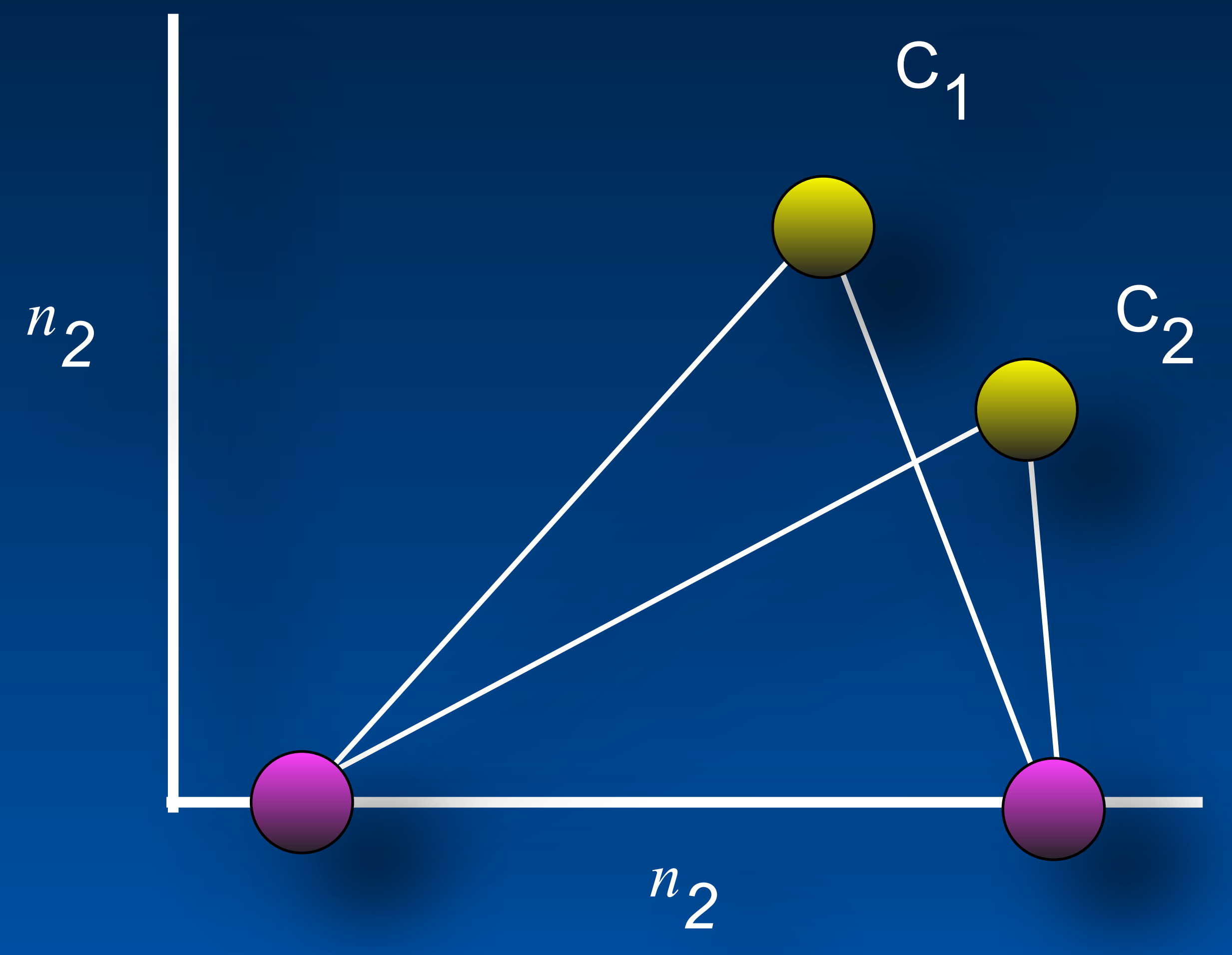
PCA Analysis of Procrustes Superposed Shape Coordinates

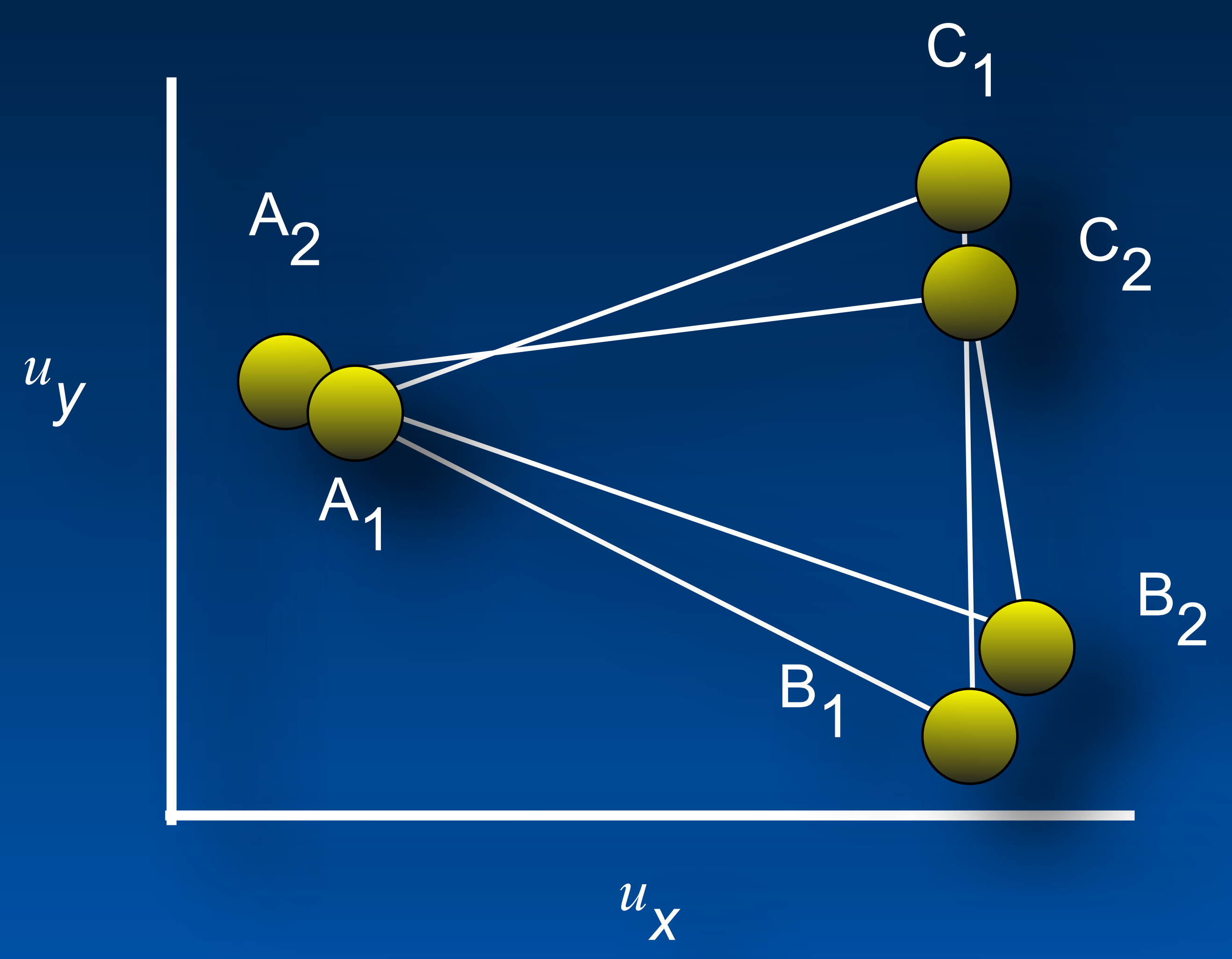


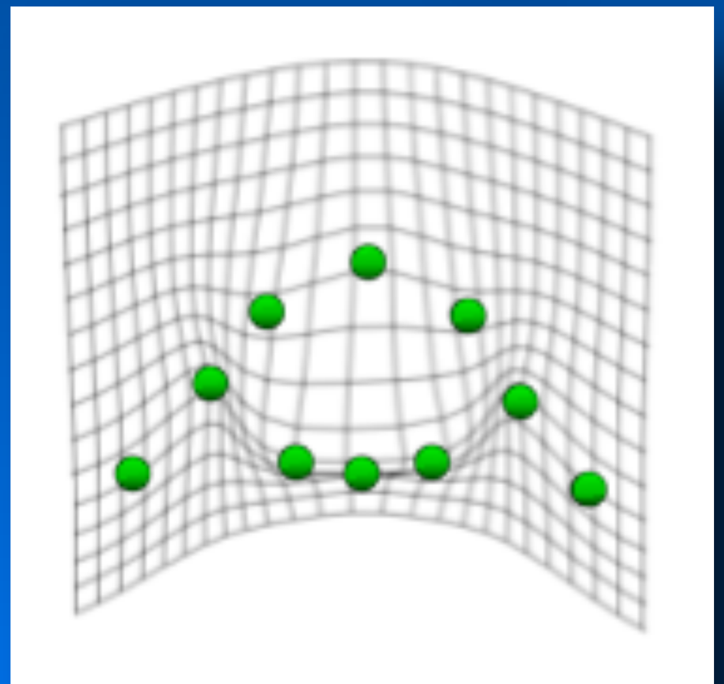
Form & Shape Analysis I

Multivariate Morphometrics, Thin-Plate Spines & Geometric Morphometrics

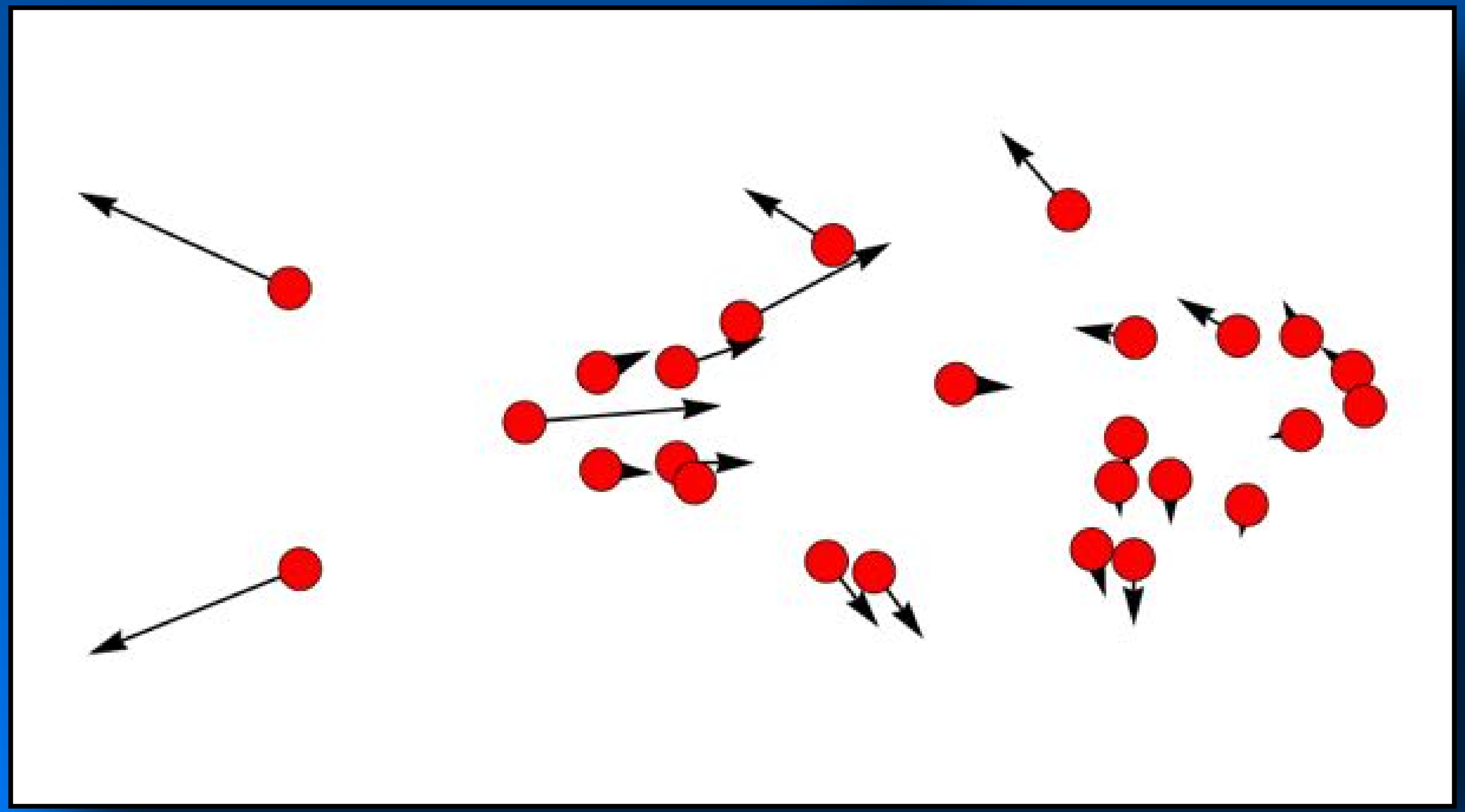
Prof. Norman MacLeod
School of Earth Sciences & Engineering, Nanjing University

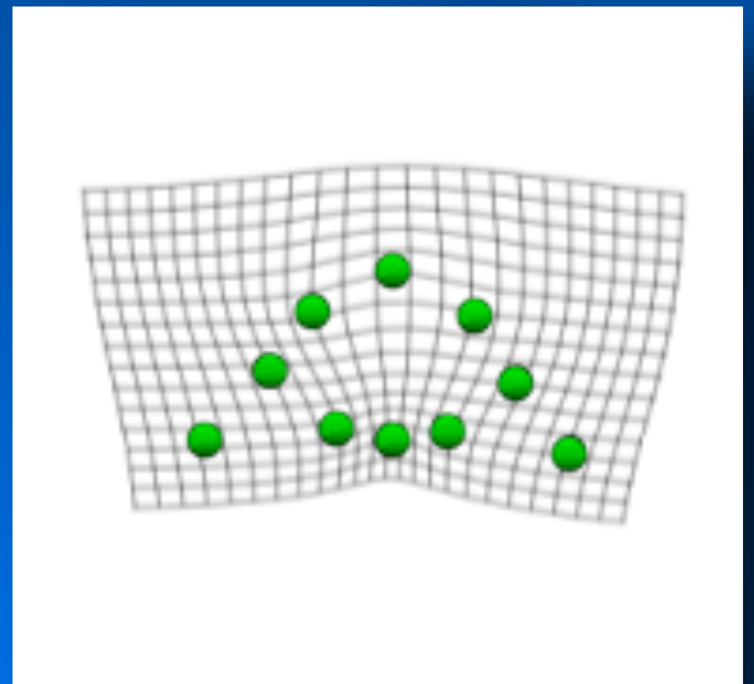






Acaste (0.0038)





Calymene (0.0002)