Dimensionality Reduction (Cont.)

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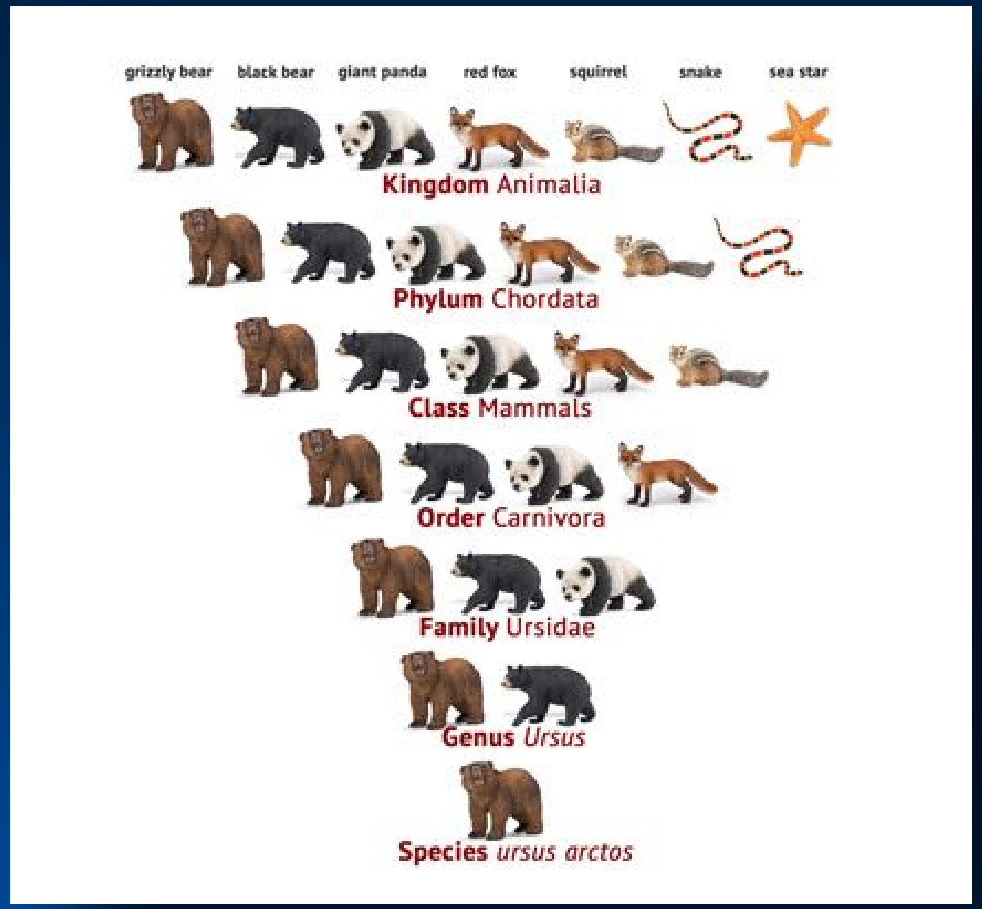
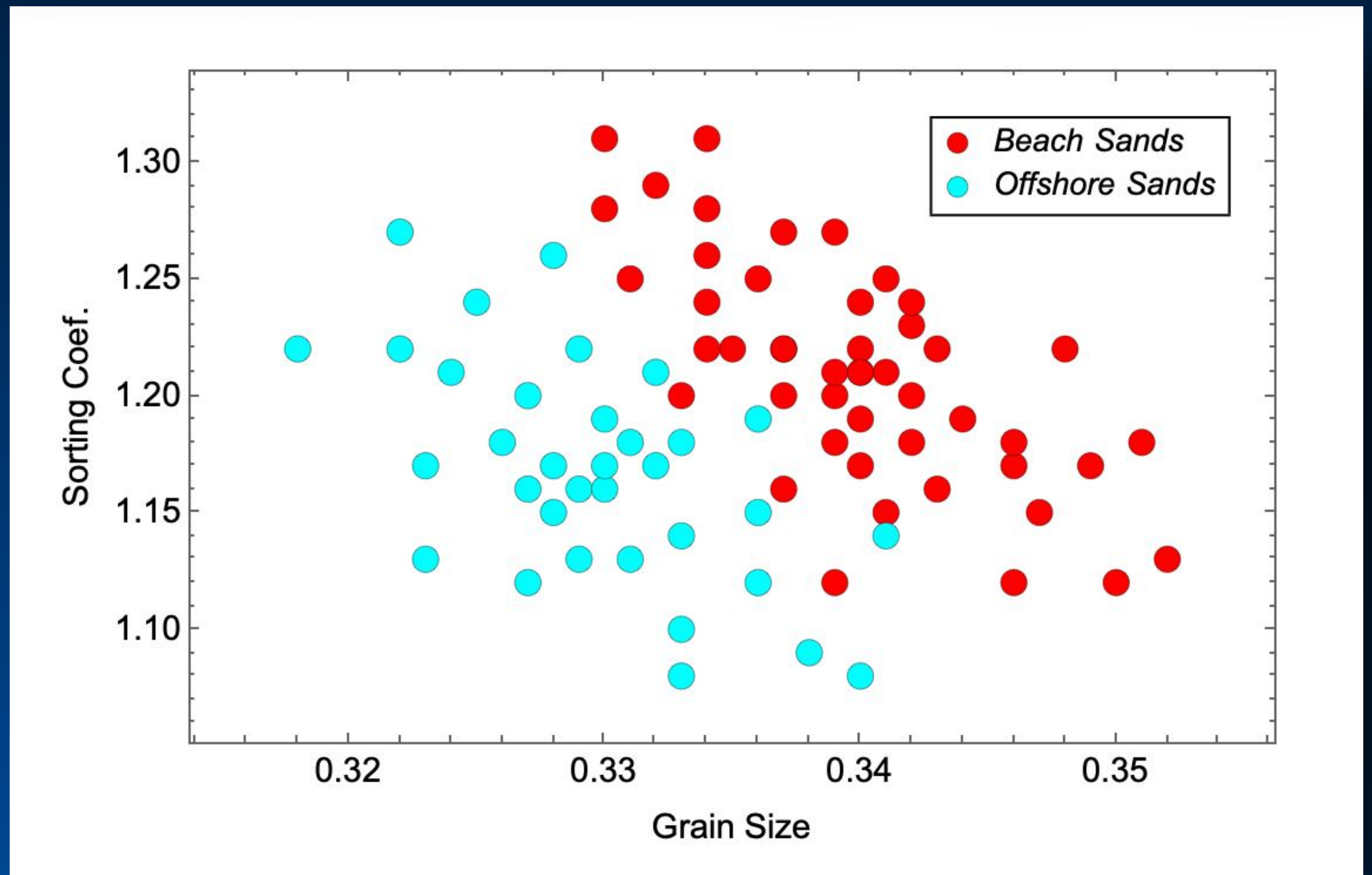
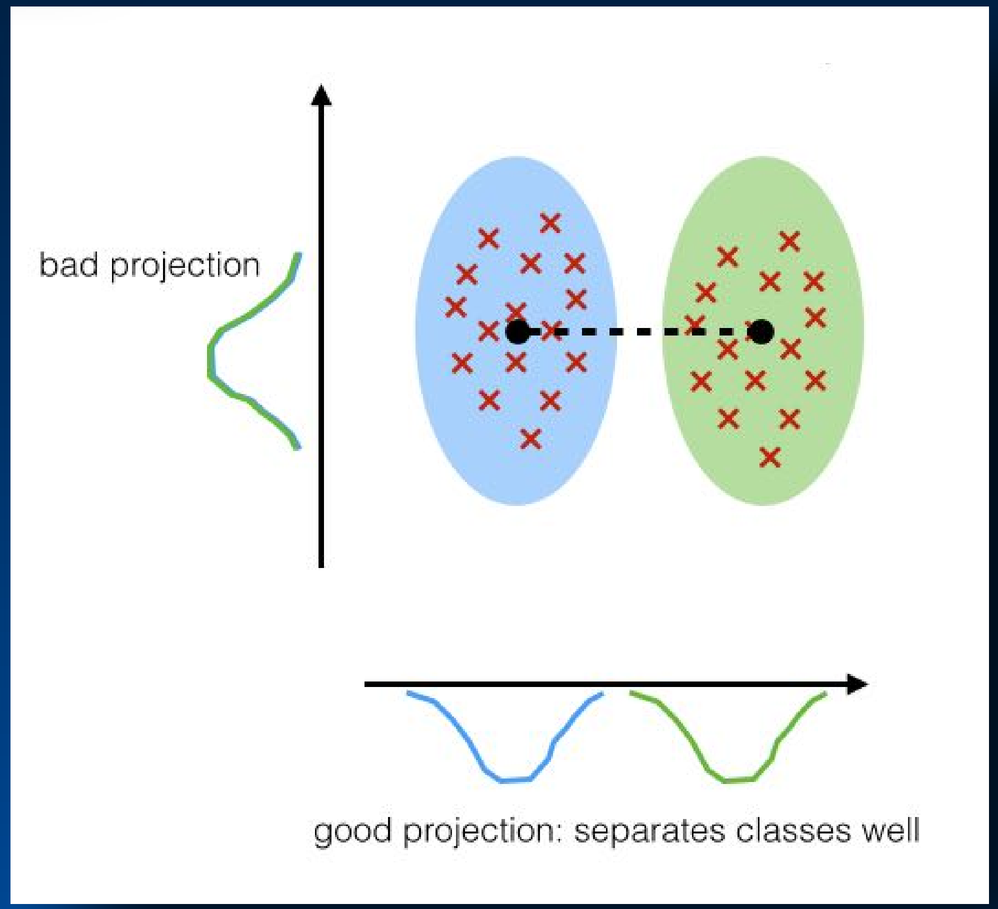
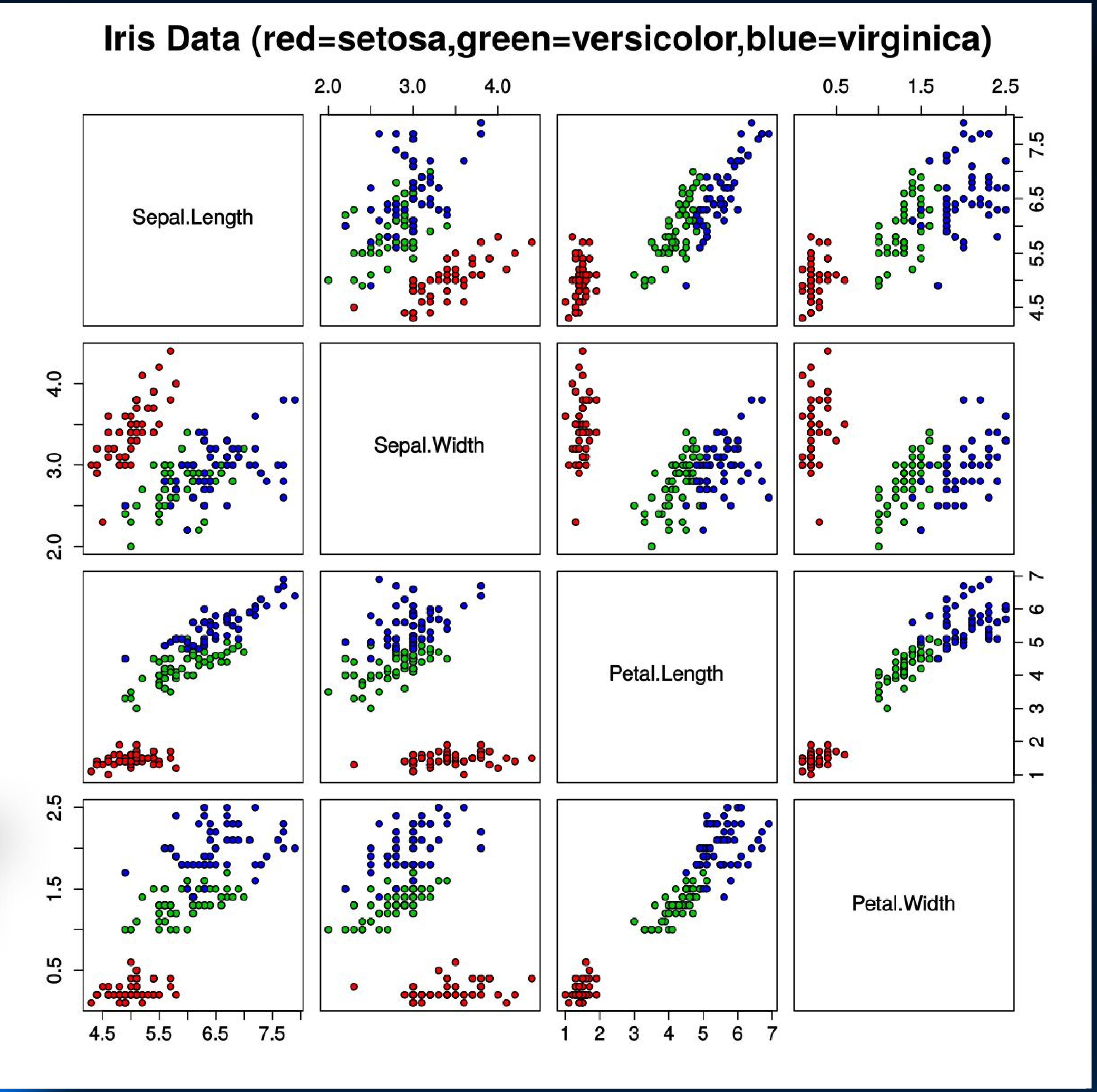






Discrimination & Classification

Discrimination - The numerical search for a combination of variables that separate groups to the maximum extent possible. Discrimination can be achieved passively as a result of many different types of numerical (ordination) analysis (e.g., PCA, PCoords, MDS) or it can be the basis of a deterministic estimate of single or sets of discriminators that find and/or optimize group differences.

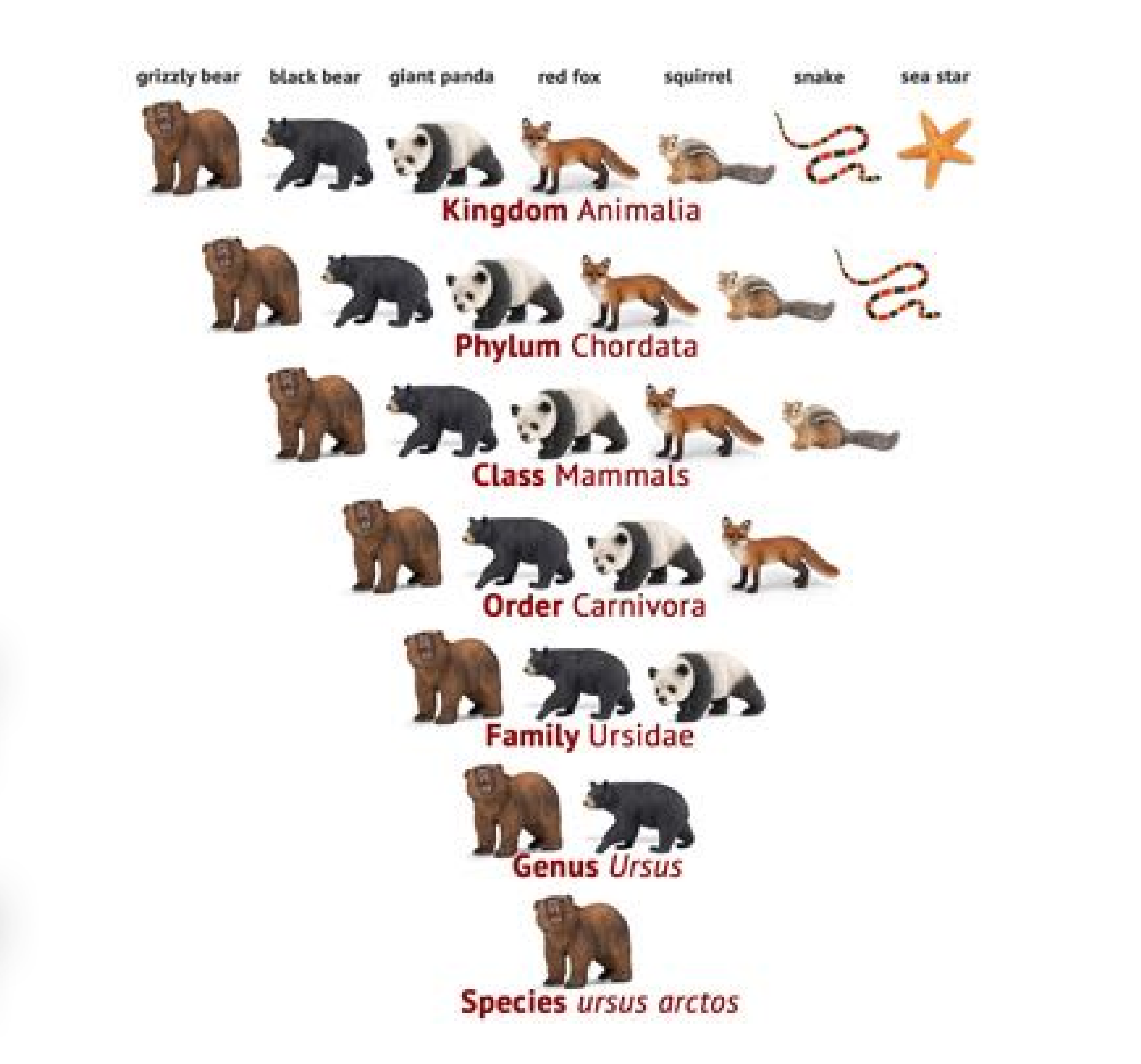


- Passive (Unsupervised) Discrimination - a posteriori group identifications that result from the passive or exploratory analyses of patterns in data, usually through visualization.
- Active (Supervised) Discrimination - group identifications achieved or confirmed via the active analysis of groups defined a priori.

Discrimination & Classification

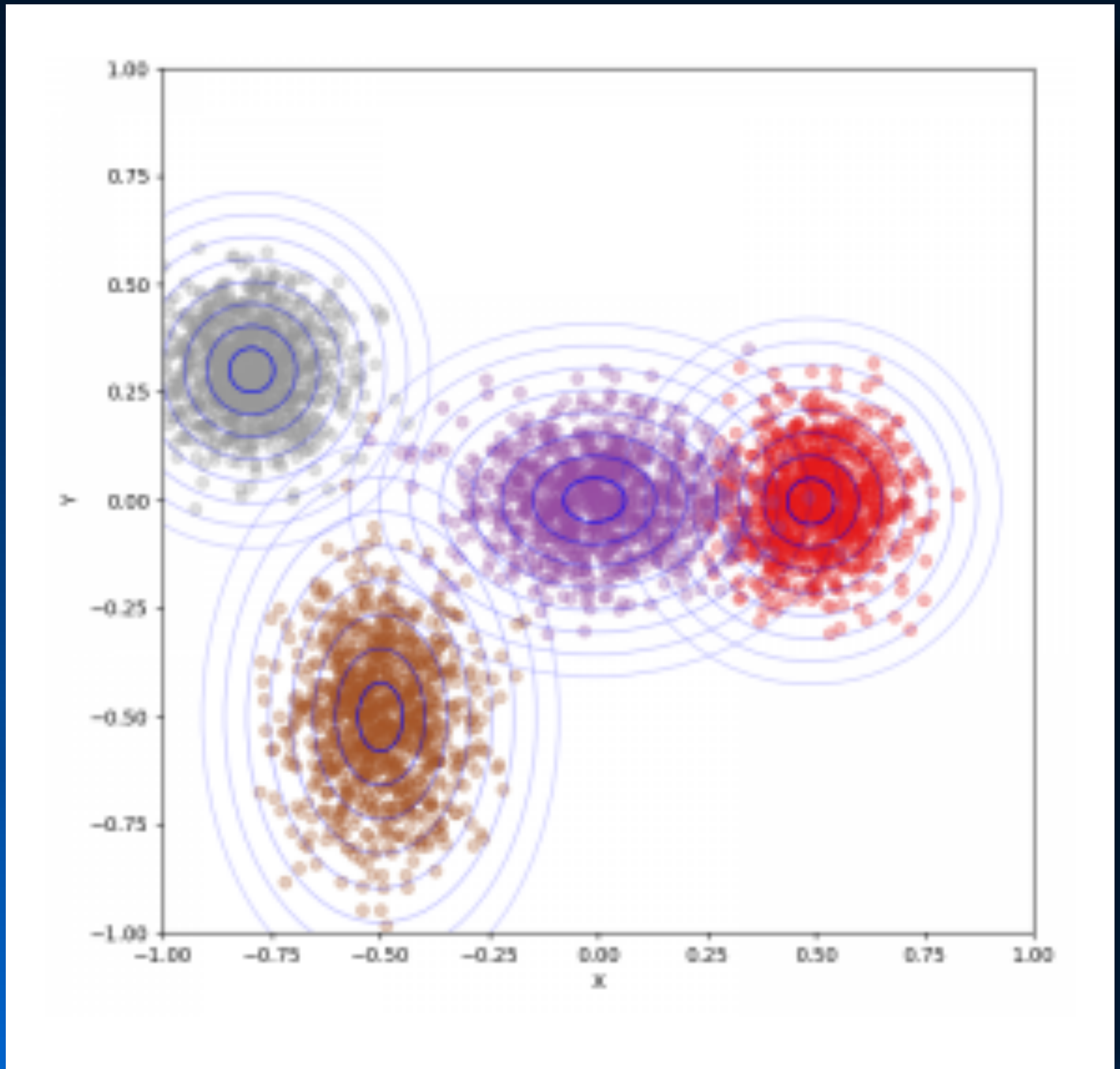
Classification - The process of placing an object or set of observations into one or more a priori-defined groups based on their similarity or dissimilarity relations with other group members. Ideally a correct classification is referenced to an unambiguous property that is unique to all members of the class or category.

- Ordinary Classification - a single or set of multiple categories that bear no consistent relation to one another.
- Hierarchical Classification - any system of classes or categories structured to include a strictly linear system of progressively more (or less) inclusive subcategories or subclasses.



Linear Discriminant Analysis

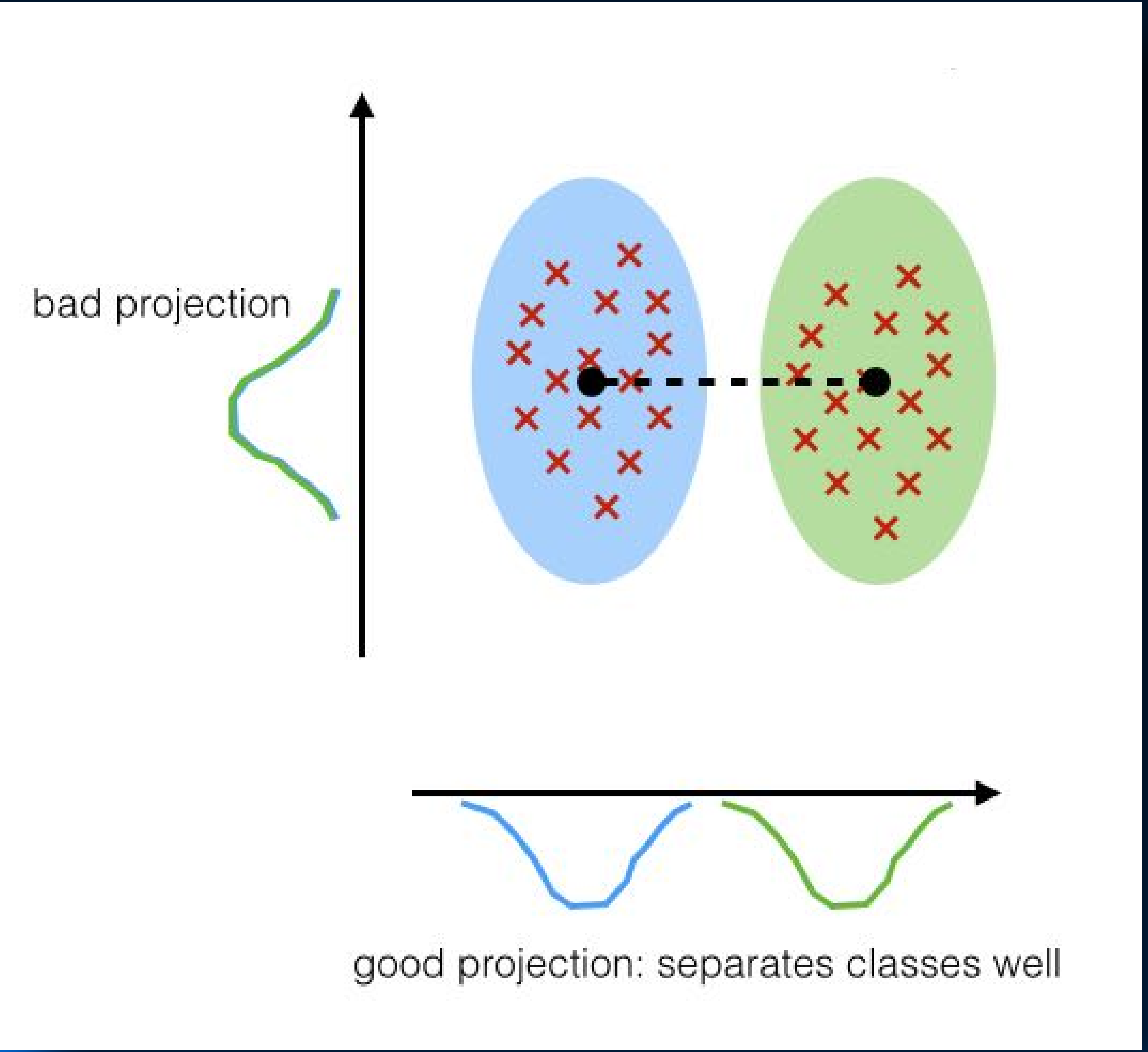
A family of data-analysis procedures that uses a wide variety of mathematical techniques (e.g., linear algebra, geometry kernel procedures, machine learning) to calculate linear vectors or filters that maximally separate two or more a priori-defined categories or classes characterized by two or more variables within a Euclidean, multivariable space.



- Binary or Dichotomous Discrimination - the original form of LDA, developed by Sir R. A. Fisher, to separate two groups optimally.
- Multi-Group Discrimination - eigenanalysis, regression or machine learning extensions of the binary discrimination concept to situations in which the separation of more than two groups is desired.

Binary Discriminant Analysis

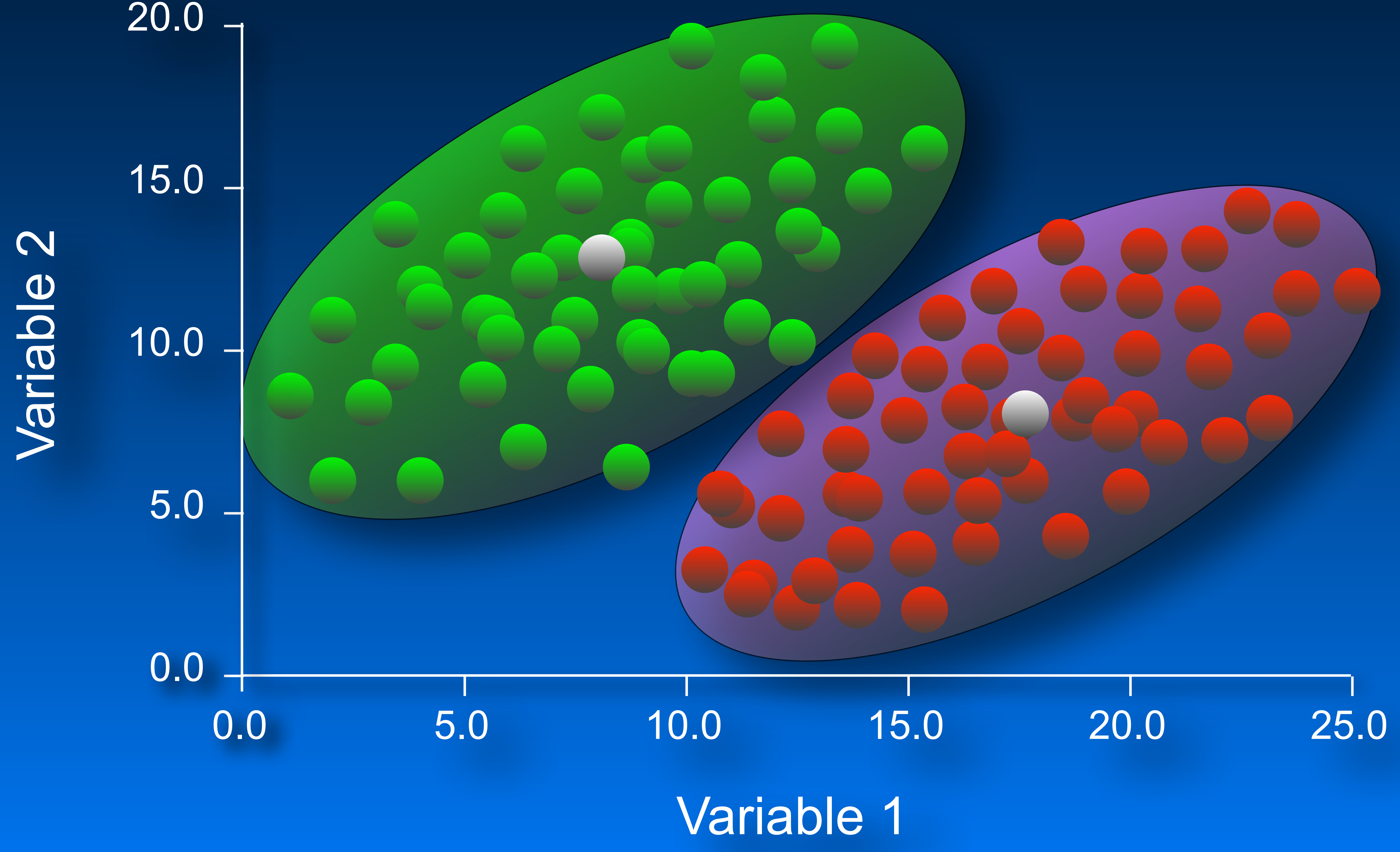
Binary discriminant analysis represents (essentially) a historically important approach to the problem of multivariate group separation insofar as it was the first successful mathematical solution to this problem and established the context for subsequent development various least-squares methods for solving this, and more complex multi-group discrimination situations.



Developed originally by Sir Ronald A. Fisher, binary linear discriminant analysis (LDA) was based on a simple matrix algebraic manipulation that combined the pooled sample covariance matrix and the group-difference vector to calculate a vector whose slope lies at right angles to pooled sample vector of maximum variance.

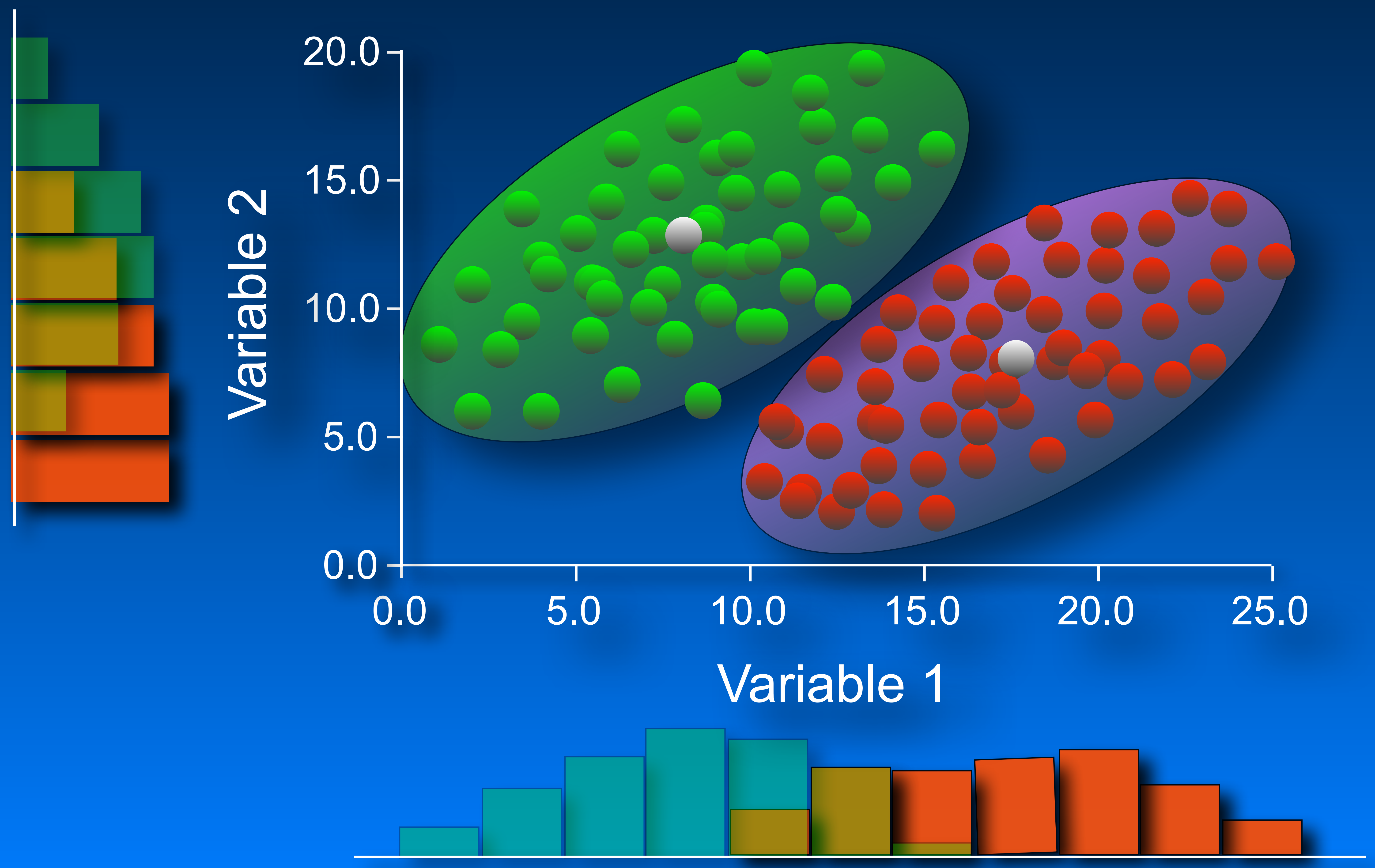
Binary Discriminant Analysis

Simple Two-Group Discrimination



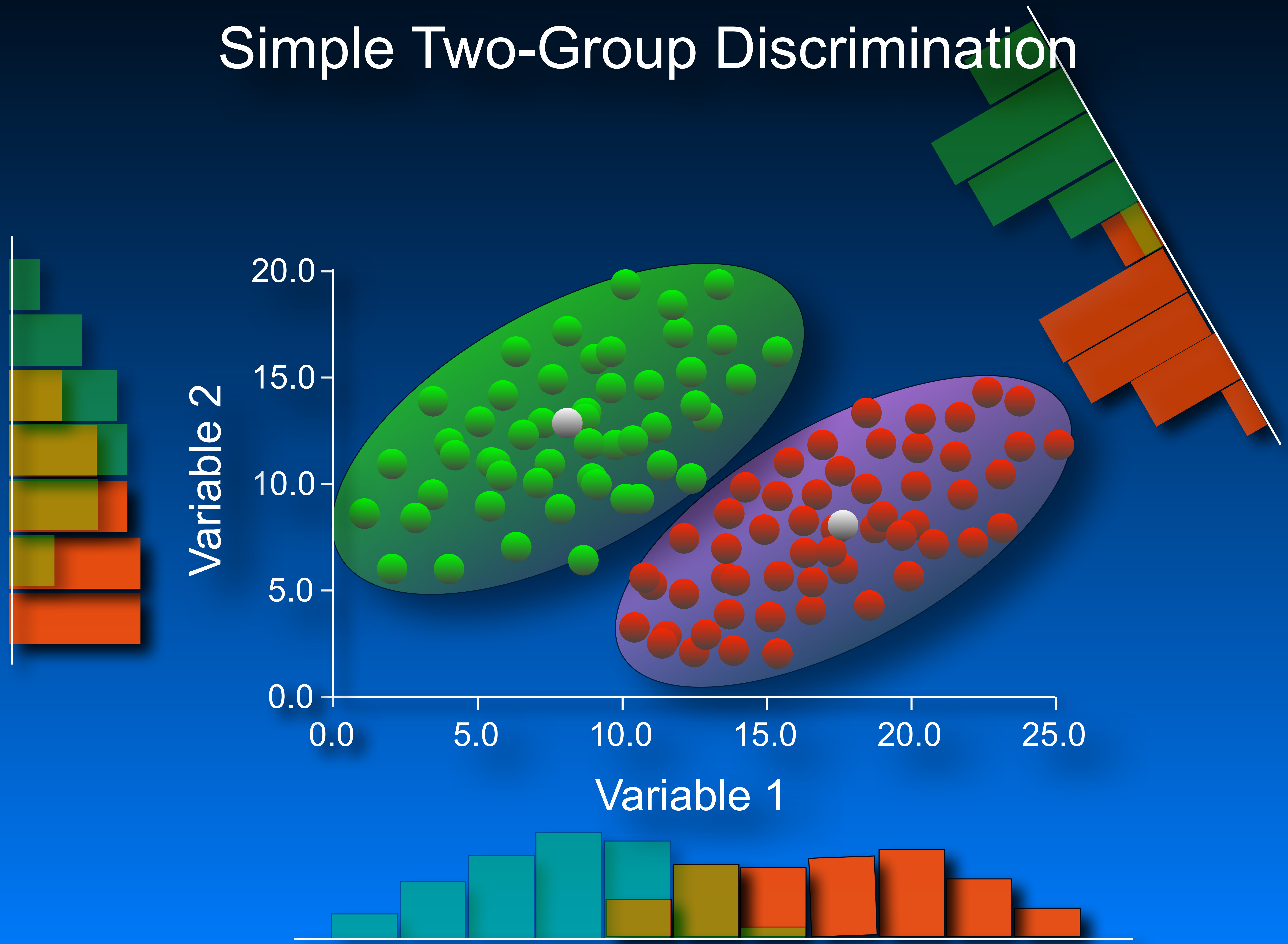
Binary Discriminant Analysis

Simple Two-Group Discrimination



Binary Discriminant Analysis

Simple Two-Group Discrimination



Binary Discriminant Analysis

Simple Two-Group Discrimination

$$S.L = D$$

Where: S = pooled covariance matrix;
L = matrix of discriminant functions;
D = group mean-difference vector.

$$L = S^{-1}.D$$

Mean-Difference Vector

$$d_j = \bar{a}_j - \bar{b}_j$$

Where: d = group mean-difference vector;
$\bar{a}$ = group a mean vector;
$\bar{b}$ = group b mean vector.

Binary Discriminant Analysis

Pooled Variance-Covariance Matrix

$$S = \frac{S_a + S_b}{n_a + n_b - 2}$$

$$S_{a_{jk}} = \sum_{i=1}^{n_a} a_{ij} \cdot a_{ik} - \frac{\sum_{i=1}^{n_a} a_{ij} \cdot \sum_{i=1}^{n_a} a_{ik}}{n_a}$$

$$S_{b_{jk}} = \sum_{i=1}^{n_b} b_{ij} \cdot b_{ik} - \frac{\sum_{i=1}^{n_b} b_{ij} \cdot \sum_{i=1}^{n_b} b_{ik}}{n_b}$$

Where: S_a = Group a covariance matrix;
S_b = Group b covariance matrix;
S = Pooled covariance matrix.

Binary Discriminant Analysis

Consider the Following Data

Offshore Sands

n	Grain Size (mm)	Sorting Coef.
1	0.333	1.08
2	0.340	1.08
3	0.338	1.09
4	0.333	1.10
5	0.323	1.13
6	0.327	1.12
7	0.329	1.13
8	0.331	1.13
9	0.336	1.12
10	0.333	1.14
11	0.341	1.14
12	0.328	1.15
13	0.336	1.15
14	0.327	1.16
15	0.329	1.16
16	0.330	1.16
17	0.323	1.17

n	Grain Size (mm)	Sorting Coef.
18	0.328	1.17
19	0.332	1.17
20	0.331	1.18
21	0.326	1.18
22	0.333	1.18
23	0.330	1.19
24	0.336	1.19
25	0.327	1.20
26	0.324	1.21
27	0.332	1.21
28	0.322	1.22
29	0.329	1.22
30	0.325	1.24
31	0.328	1.26
32	0.322	1.27
33	0.318	1.22
34	0.330	1.17

Binary Discriminant Analysis

Consider the Following Data

Beach Sands

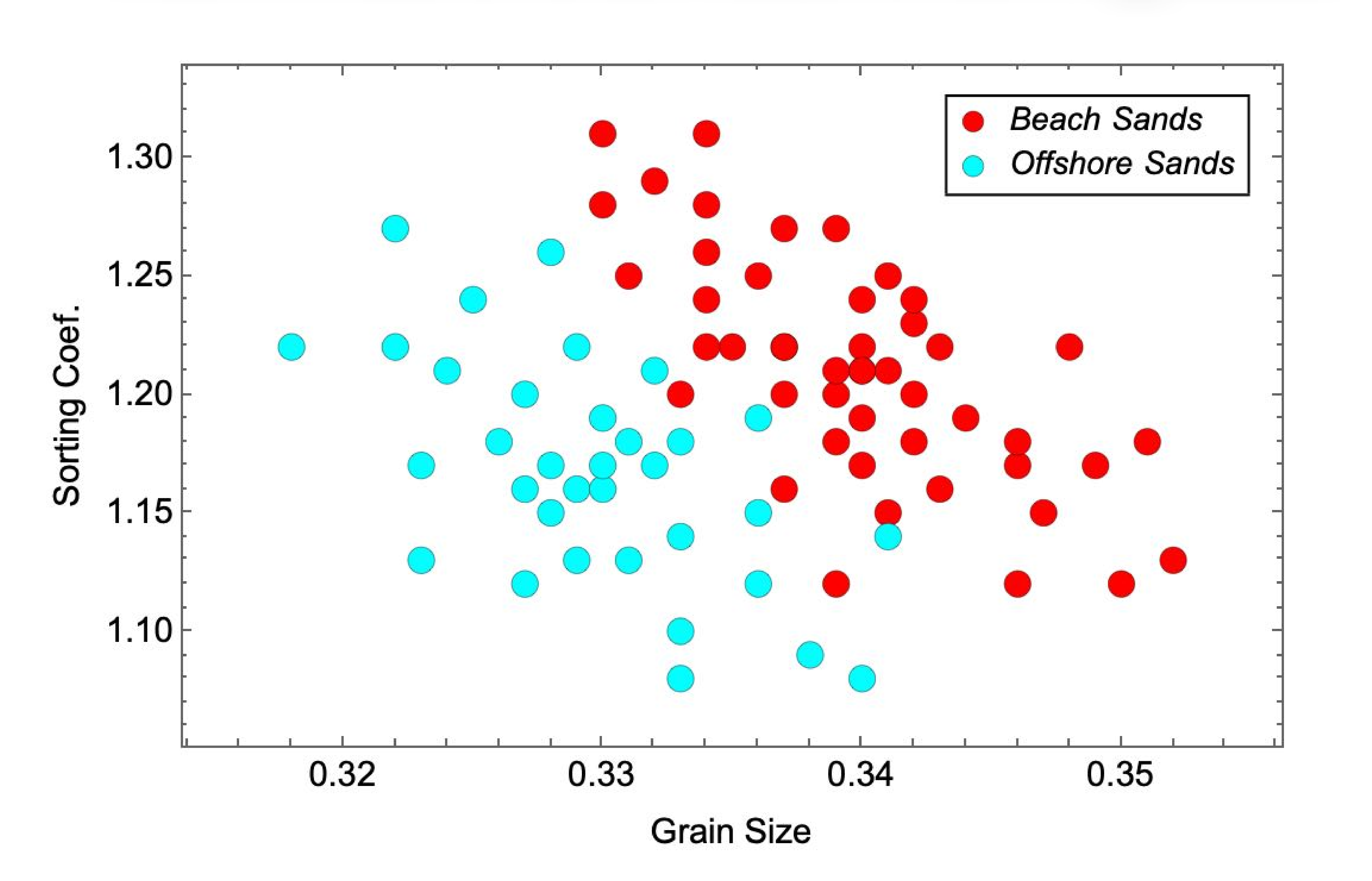
n	Grain Size (mm)	Sorting Coef.
1	0.339	1.12
2	0.346	1.12
3	0.350	1.12
4	0.352	1.13
5	0.341	1.15
6	0.347	1.15
7	0.337	1.16
8	0.343	1.16
9	0.340	1.17
10	0.346	1.17
11	0.349	1.17
12	0.339	1.18
13	0.342	1.18
14	0.346	1.18
15	0.351	1.18
16	0.340	1.19
17	0.344	1.19

n	Grain Size (mm)	Sorting Coef.
18	0.333	1.20
19	0.337	1.20
20	0.339	1.20
21	0.342	1.20
22	0.339	1.21
23	0.340	1.21
24	0.341	1.21
25	0.335	1.22
26	0.337	1.22
27	0.340	1.22
28	0.343	1.22
29	0.334	1.22
30	0.348	1.22
31	0.337	1.22
32	0.342	1.23
33	0.334	1.24
34	0.340	1.24

n	Grain Size (mm)	Sorting Coef.
35	0.342	1.24
36	0.331	1.25
37	0.336	1.25
38	0.341	1.25
39	0.334	1.26
40	0.337	1.27
41	0.339	1.27
42	0.330	1.28
43	0.334	1.28
44	0.332	1.29
45	0.330	1.31
46	0.334	1.31
47	0.340	1.21

Binary Discriminant Analysis

Marine Sand Grain Size & Sorting Data



Binary Discriminant Analysis

Marine Sand Grain Size & Sorting Data

Mean Vector: Beach Sands

n	Grain Size (mm)	Sorting Coef.
1	0.3399	1.2100

Mean Vector: Offshore Sands

n	Grain Size (mm)	Sorting Coef.
1	0.3297	1.1674

Mean Difference Vector

n	Grain Size (mm)	Sorting Coef.
1	-0.0101	-0.0426

Covariance Matrix: Beach Sands

	Grain Size (mm)	Sorting Coef.
Grain Size (mm)	0.0138	-0.0084
Sorting Coef.	-0.0084	0.1070

Covariance Matrix: Offshore Sands

	Grain Size (mm)	Sorting Coef.
Grain Size (mm)	0.0009	-0.0002
Sorting Coef.	-0.0002	0.0023

Binary Discriminant Analysis

Marine Sand Grain Size & Sorting Data

Pooled Covariance Matrix (S)

	Grain Size (mm)	Sorting Coef.
Grain Size (mm)	0.000925	-0.000169
Sorting Coef.	-0.000169	0.002312

	Grain Size (mm)	Sorting Coef.
Grain Size (mm)	59098.30	4311.64
Sorting Coef.	4311.64	747.06

Inverse of Pooled Covariance Matrix (S^{-1})

$$S\,.\,L = D$$

$$L = S^{-1}\,.\,D$$

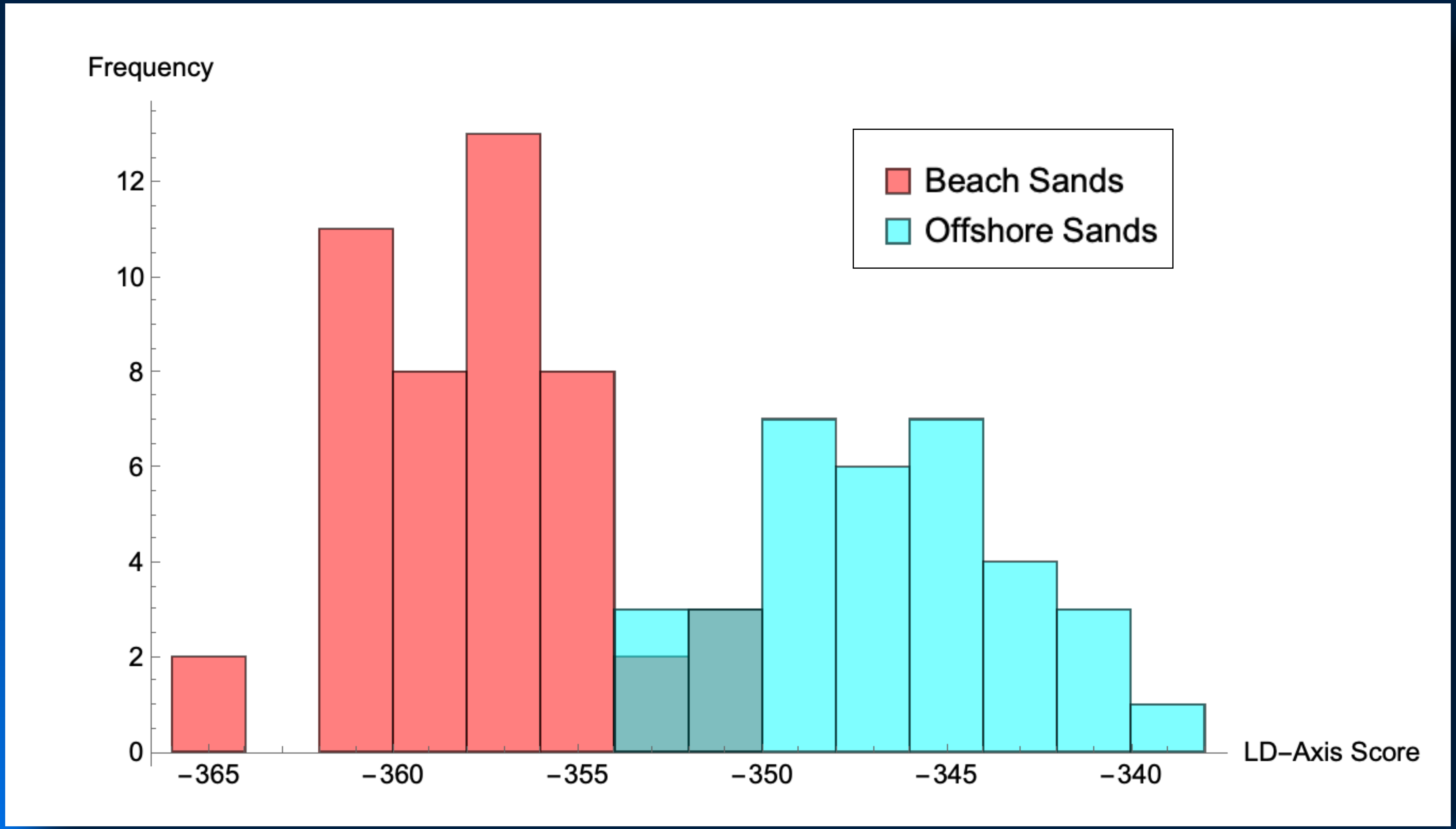
$$L = \begin{pmatrix} 59098.30 & 4311.64 \\ 4311.64 & 747.08 \end{pmatrix} . \begin{pmatrix} -0.0101 & -0.0426 \end{pmatrix}$$

$$L = \begin{pmatrix} -780.569 & -75.3723 \end{pmatrix}$$

Binary Discriminant Analysis

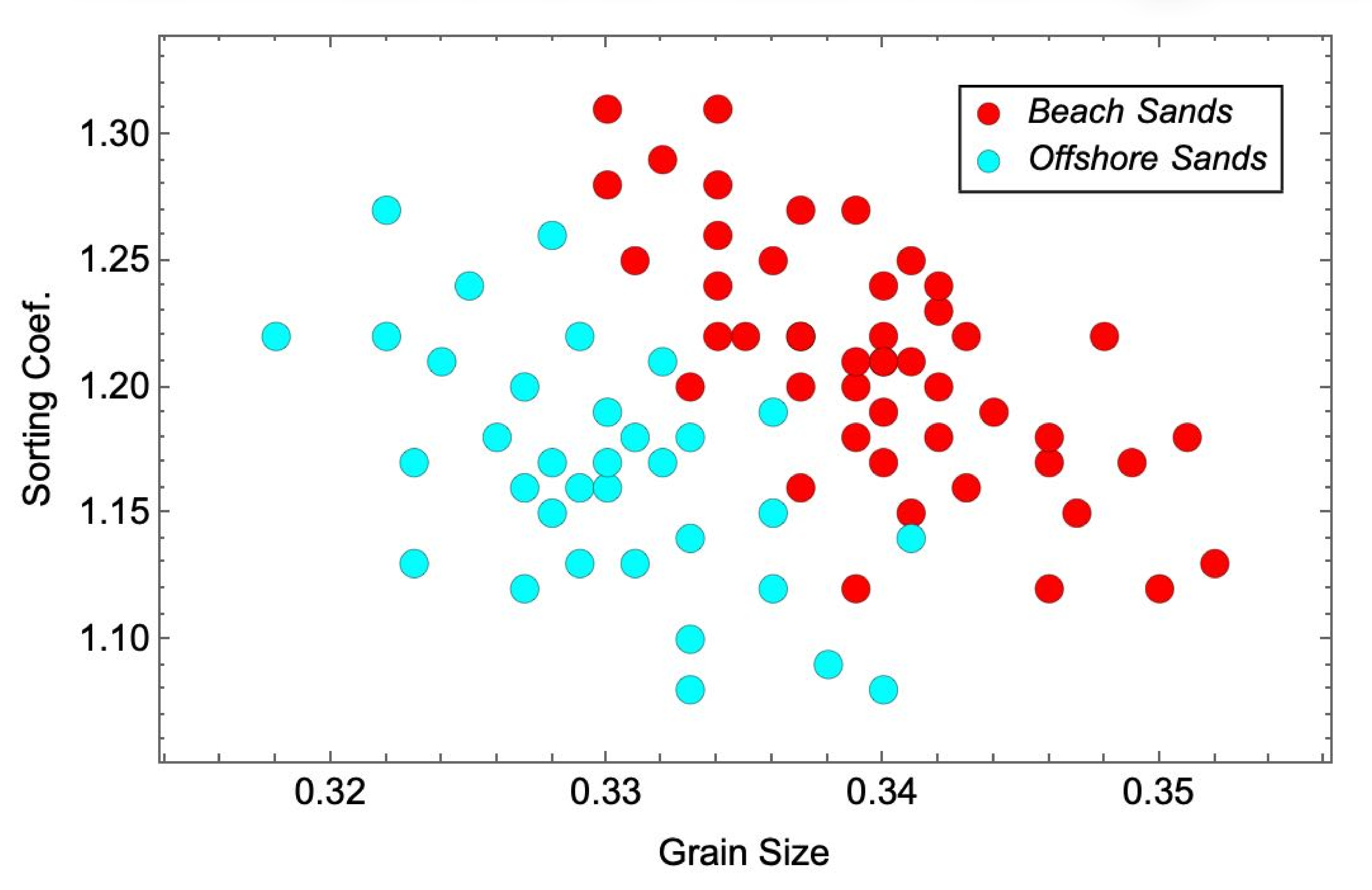
Marine Sand Grain Size & Sorting Data

$$Scores_a = X_a \cdot L$$
$$Scores_b = X_b \cdot L$$



Binary Discriminant Analysis

Marine Sand Grain Size & Sorting Data



Binary Discriminant Analysis

Mahalanobis Distance

$$D^2 = D' . S^{-1} . D$$

Where: D^2 = Mahalanobis distance;

D = Between groups mean vector;

S = pooled covariance matrix.

A generalization of the Euclidean distance, calculated in terms of standard-deviation units in an m-dimensional multivariate space, that takes covariances between variables into explicit consideration.

Binary Discriminant Analysis

Mahalanobis Distance

$$D^2 = D' \,.\, S^{-1} \,.\, D$$

$$D^2 = \begin{pmatrix} -0.0101 \\ = 0.0426 \end{pmatrix} \cdot \begin{pmatrix} 59898.30 & 4311.64 \\ 4311.64 & 747.06 \end{pmatrix} \cdot \begin{pmatrix} -0.0101 & -0.04 \end{pmatrix}$$

$$D^2 = 11.1724$$

Binary Discriminant Analysis

Hotelling's T^2 Test

$$T^2 = \frac{n_a \cdot n_b}{n_a + n_b} \cdot D^2$$

Where: n_a = group a sample size;
n_b = group b sample size
D^2 = Mahalanobis distance;
m = number of variables.

A multivariate extension of Students t-test which tests the null hypothesis (H_0) of no difference between multivariate mean vectors between two (or more) groups of observations. Distribution of the T^2 statistic is proportional to the F distribution.

Hotelling's T^2 Test

A multivariate extension of Students t-test which tests the null hypothesis (H_0) of no difference between multivariate mean vectors between two (or more) groups of observations. Distribution of the T^2 statistic is proportional to the F distribution.

$$T^2 = \frac{n_a \cdot n_b}{n_a + n_b} \cdot D^2$$

$$F = \left(\frac{n_a + n_b - m - 1}{(na + n_b - 2) \cdot m}\right) \cdot \frac{n_a \cdot n_b}{n_a + n_b} \cdot D^2$$

$$dof = m, (n_a + n_b - m - 1)$$

Hotelling's T^2 Test

A Test of Group-Separation Significance

Assumptions

- Specimens (observations) have been selected randomly.
- Probability of each specimens belonging to any group is equal.
- Variables exhibit normal distributions.
- Group variance-covariance matrixes are equal.
- No misclassifications are present in the samples.
- Distribution of deviations from the multivariate sample means are normal.

Binary Discriminant Analysis

Hotelling's T^2 Test

$$T^2 = \frac{47 \cdot 34}{47 + 34} \cdot 11.1724 = 220.413$$

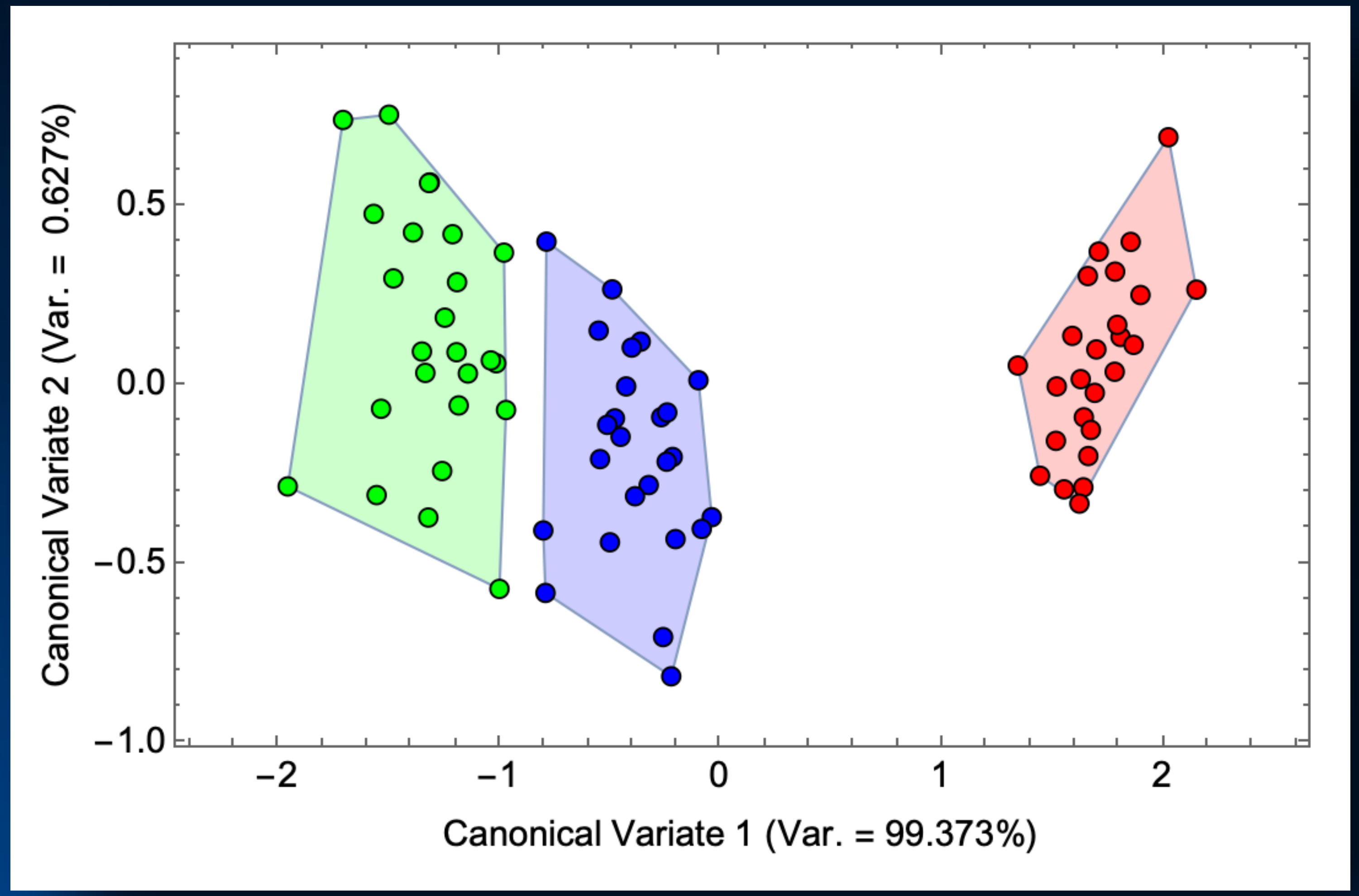
$$F = (\frac{47 + 34 - 2 - 1}{(47 + 34 - 2) \cdot 2}) \cdot \frac{47 \cdot 34}{47 + 34} \cdot 11.724 = 108.811$$

$$dof = 2, (47 + 34 - 2 - 1) = 2,78$$

$$F_{\alpha=0.05,dof=2,78} = 3.11$$

These sand samples have a very low probability of having the same mean-vector values.

Post-Hoc Category Assignment



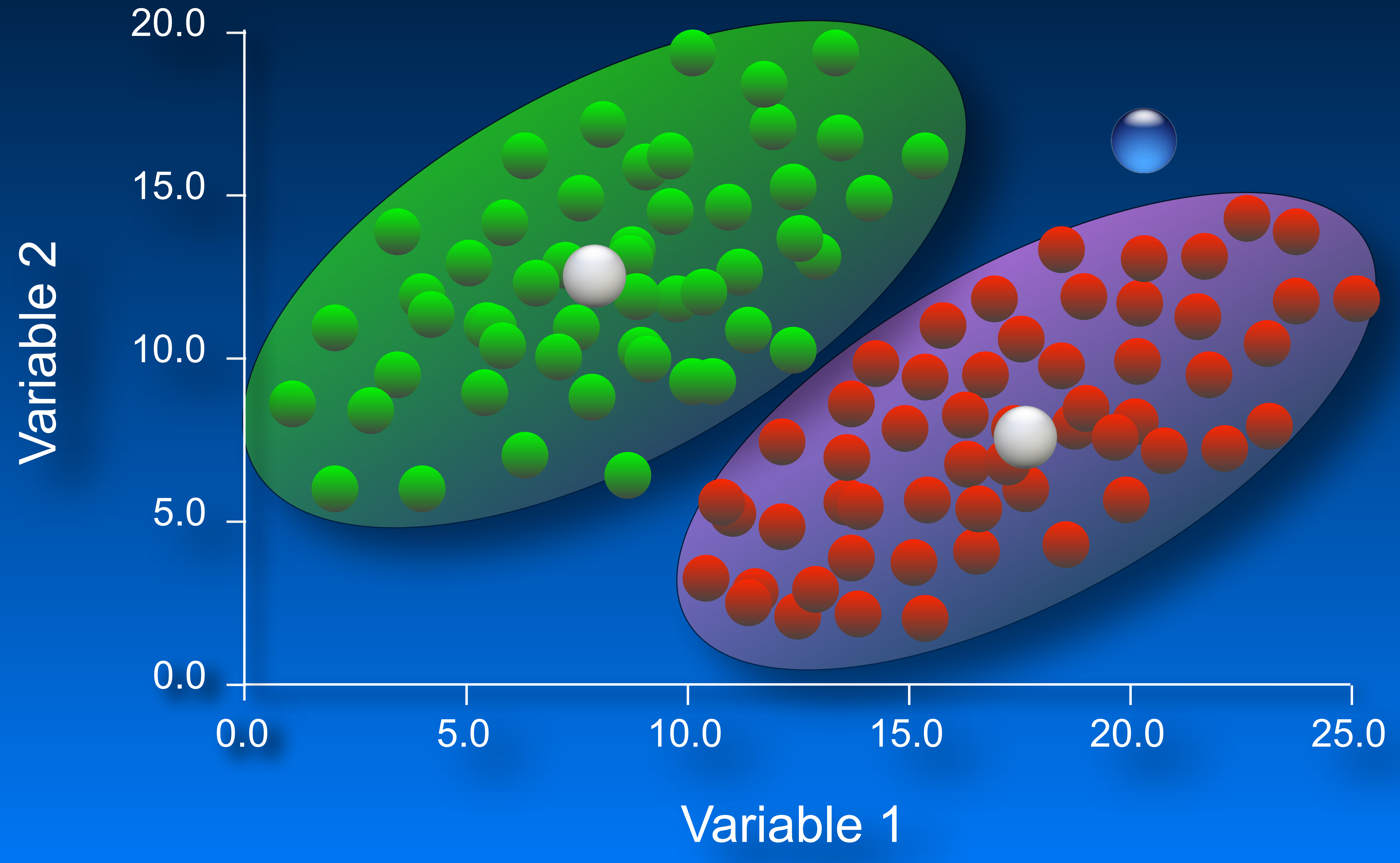
Once a discriminant analysis has found that groups of observations can be separated from one another, the linear discrimination regresssional lines that were used to quantify between-group differences can be used to project comparable sets of measurements collected from unknown individuals into the discriminant ordination space where they can be compared to the groups that exist there statistically. There are a variety of criteria that can be used to assign unknown individuals to groups in the context of an extended discriminant/classification analysis.

Criteria

- Minimum distance to group centroid
- Occupying a position within a groups 95% confidence ellipse
- Assigned to the group containing the n closest nearest neighbors

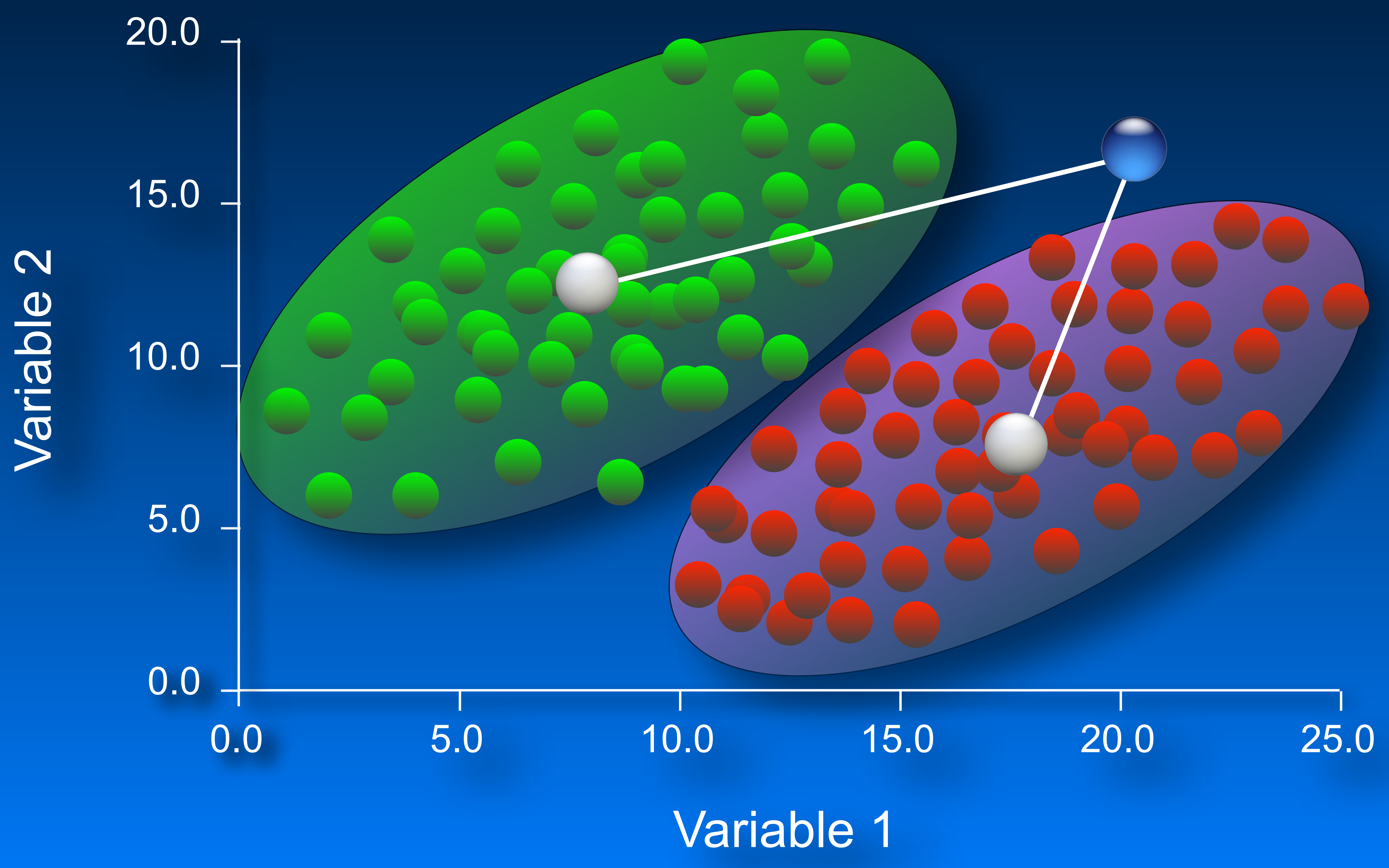
Binary Discriminant Analysis

Minimum Distance to Group Centroid



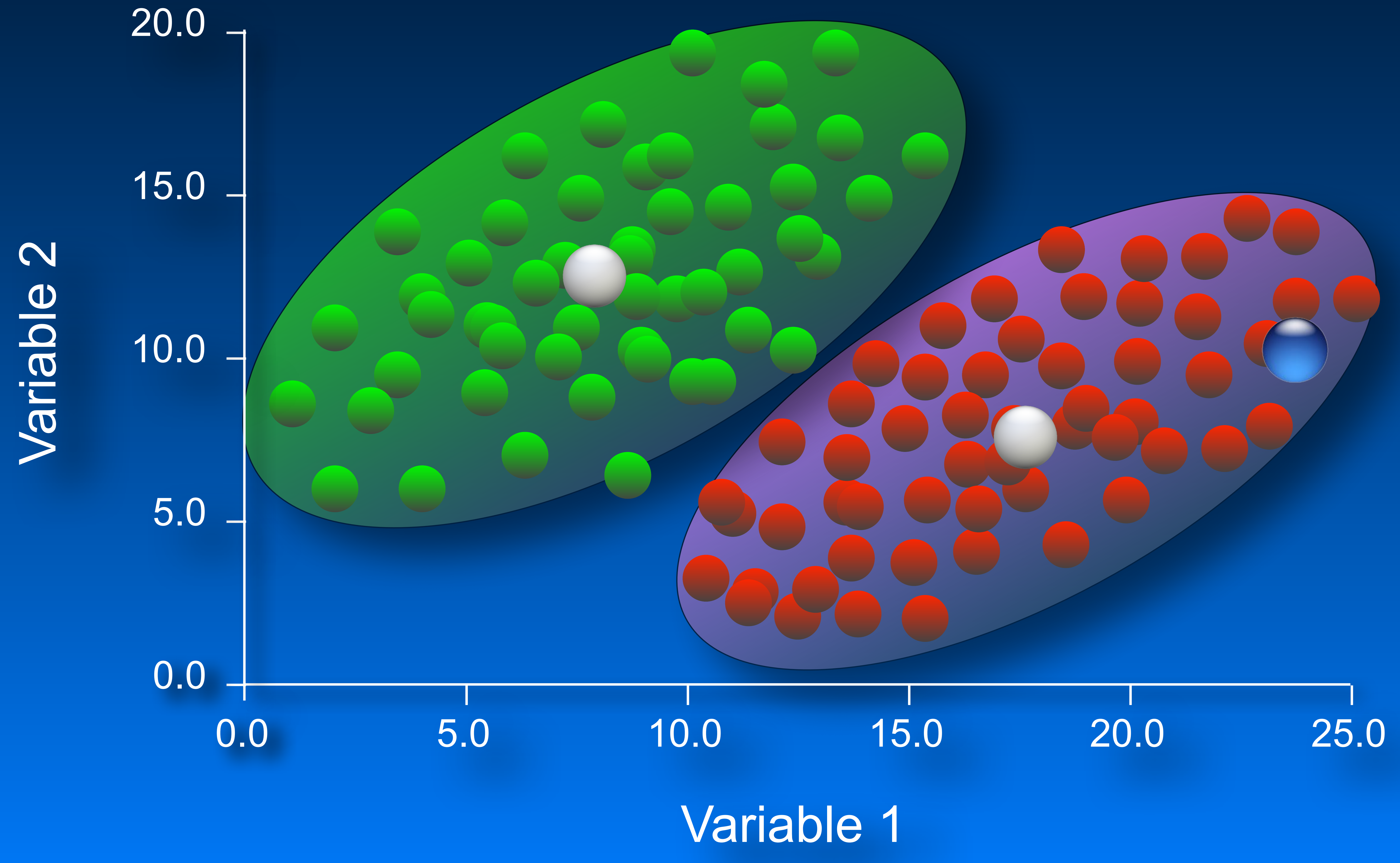
Binary Discriminant Analysis

Minimum Distance to Group Centroid



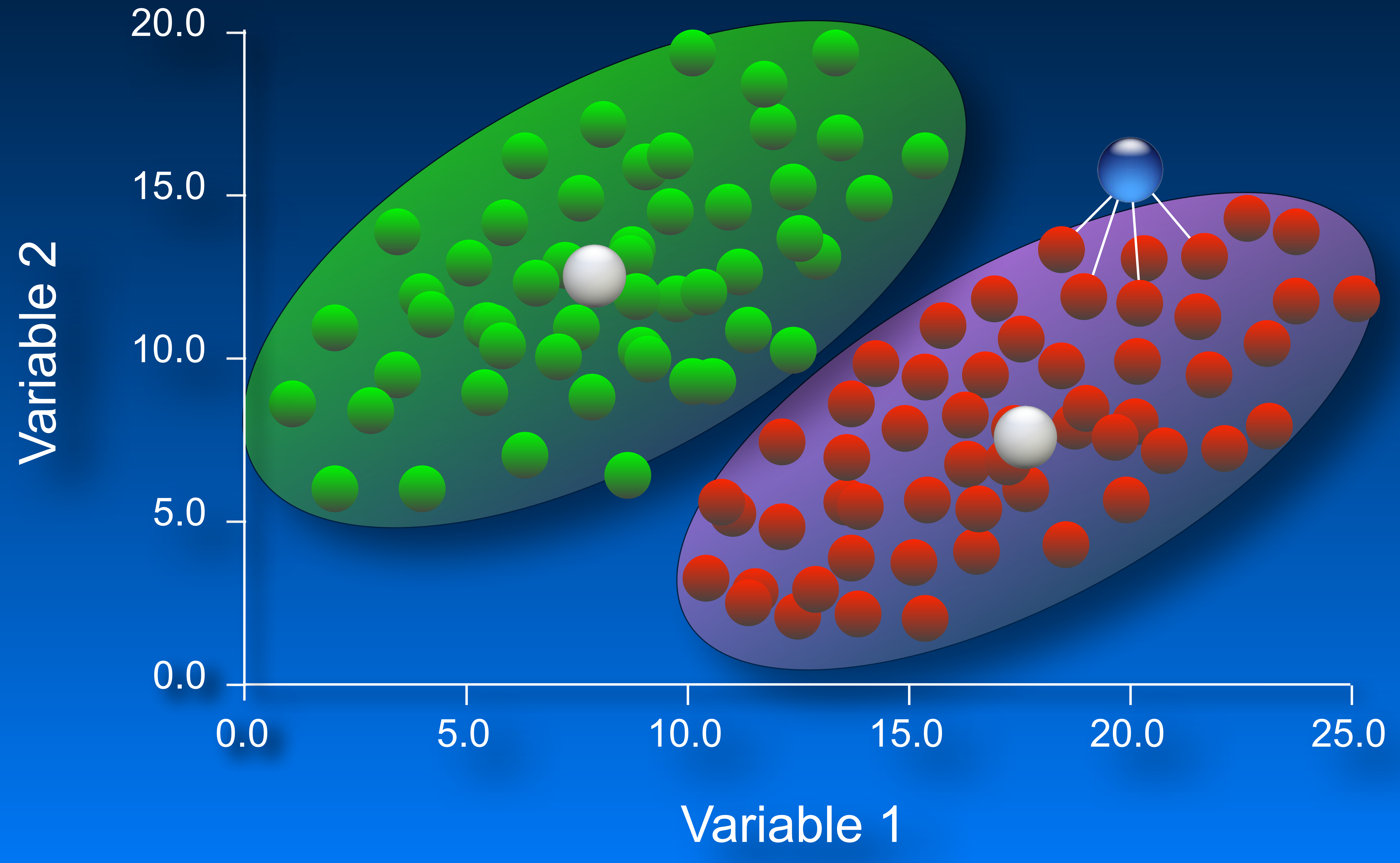
Binary Discriminant Analysis

Lies within 95% Confidence Ellipse



Binary Discriminant Analysis

Nearest Neighbor Analysis



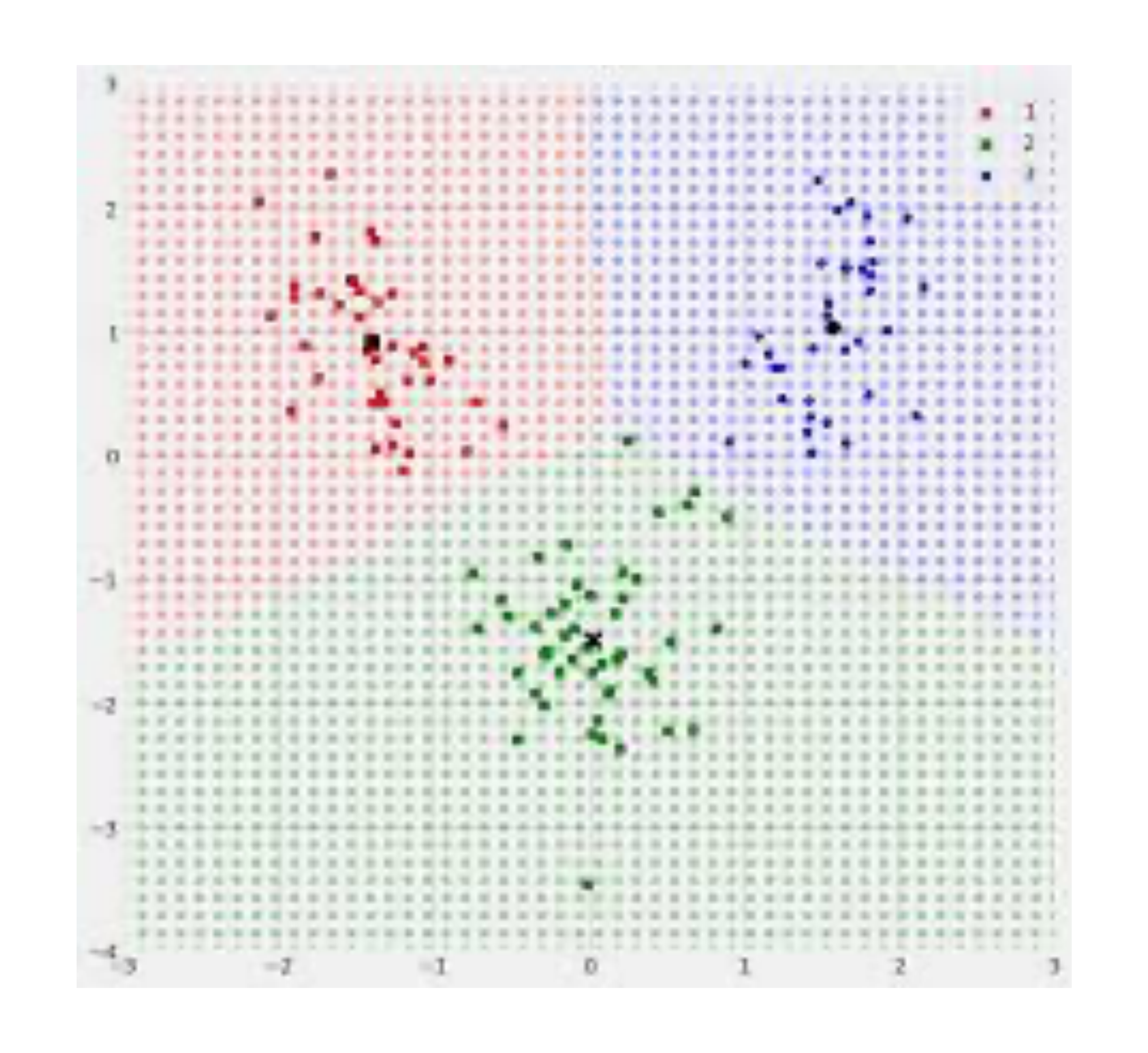
Confusion Matrix

A contingency table often employed in discriminant analysis and classification-focused machine learning analysis that produces a quick and easy performance summary for a set of group-assignment results.

	Beech Sand	Offshore Sand	No. Correct	Total	Percent
Beech Sand	45	2	45	47	95.74
Offshore Sand	2	32	32	34	94.12
No. Correct	45	32	77	81	95.06
Total	47	34	81		
Percent	95.74	94.12	95.06		

Multi-Group Discriminant Analysis

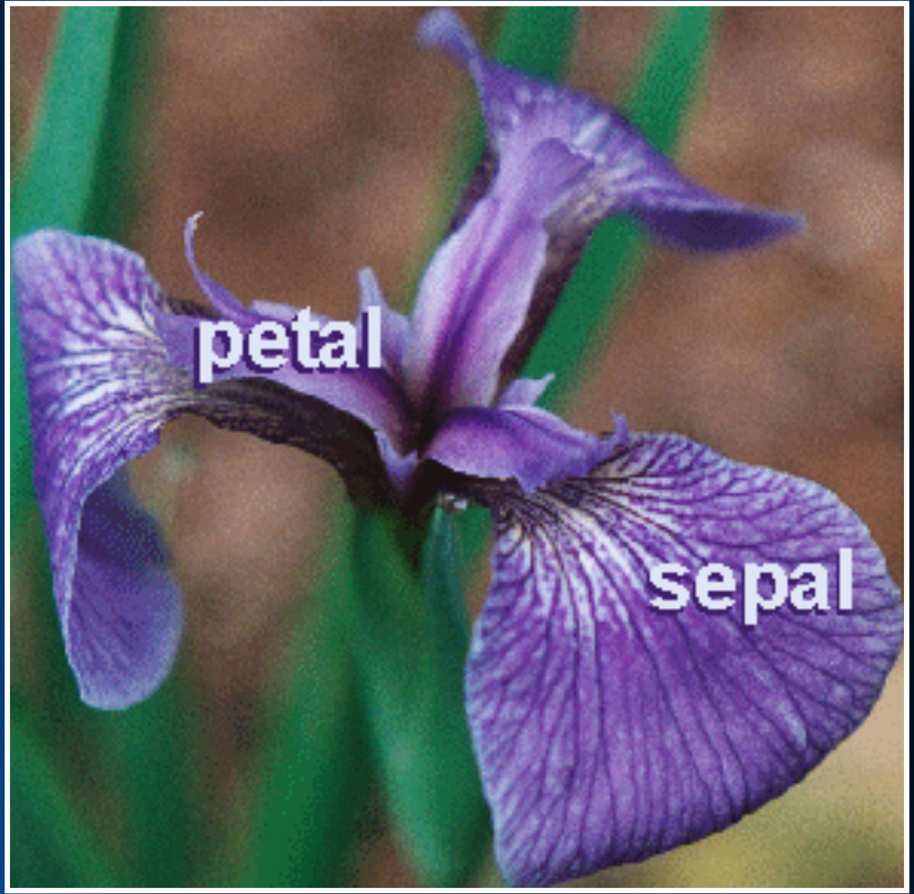
At present there are two alternative approaches to this issue: a linear eigenvector-based approach and a machine-learning approach. Here we'll focus on the former (reservingthe latter for an upcoming lecture). The linear algebra/eigen-analysis approach is based on a series of data transformations that maximize between-group (centroid) separation relative to within-group dispersion (= variance). In many ways this approach resembles PCA, but the distinction between the two is very important to keep in mind. PCA transforms a single group dataset into a variance optimized space. Multi-group DA transforms a multiple group dataset into a space that maximally separates group centroids relative to between-groups variance.



Canonical Variates Analysis

An Example: The Fisher *Iris* Data

Edgar S. Anderson
(1897-1969)



Ronald A. Fisher
(1890-1962)

Iris setosa

Iris versicolor

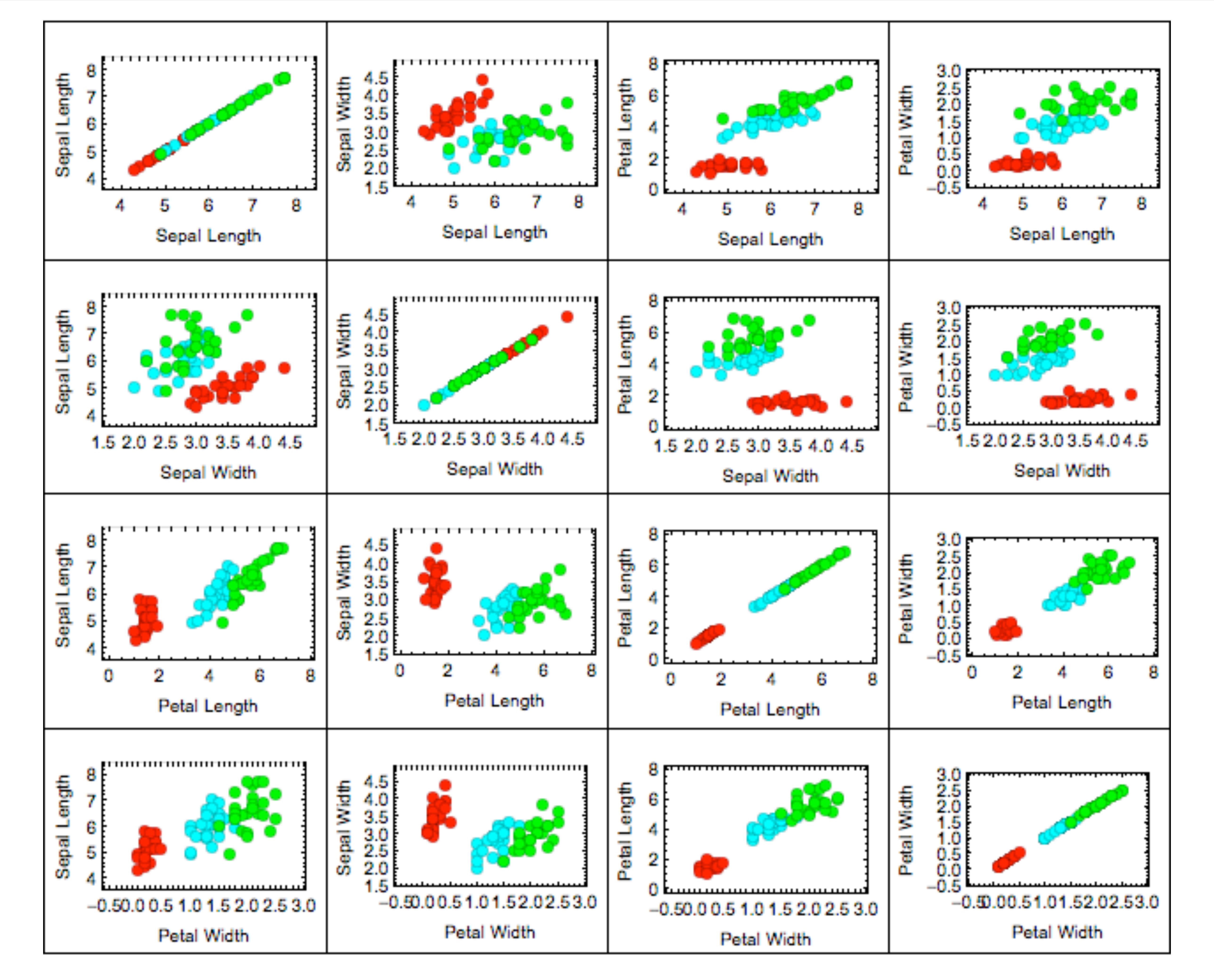
Iris virginica

Fisher *Iris* Data

	Iris setosa				*Iris versicolor*				*Iris virginica*			
	Petal		Sepal		Petal		Sepal		Petal		Sepal	
n	Length	Width	Length	Width	Length	Width	Length	Width	Length	Width	Length	Width
1	1.4	0.2	5.1	3.5	4.7	1.4	7.0	3.2	6.0	2.5	6.3	3.3
2	1.4	0.2	4.9	3.0	4.5	1.5	6.4	3.2	5.1	1.9	5.8	2.7
3	1.3	0.2	4.7	3.2	4.9	1.5	6.9	3.1	5.9	2.1	7.1	3.0
4	1.5	0.2	4.6	3.1	4.0	1.3	5.5	2.3	5.6	1.8	6.3	2.9
5	1.4	0.2	5.0	3.6	4.6	1.5	6.5	2.8	5.8	2.2	6.5	3.0
6	1.7	0.4	5.4	3.9	4.5	1.3	5.7	2.8	6.6	2.1	7.6	3.0
7	1.4	0.3	4.6	3.4	4.7	1.6	6.3	3.3	4.5	1.7	4.9	2.5
8	1.5	0.2	5.0	3.4	3.3	1.0	4.9	2.4	6.3	1.8	7.3	2.9
9	1.4	0.2	4.4	2.9	4.6	1.3	6.6	2.9	5.8	1.8	6.7	2.5
10	1.5	0.1	4.9	3.1	3.9	1.4	5.2	2.7	6.1	2.5	7.2	3.6
11	1.5	0.2	5.4	3.7	3.5	1.0	5.0	2.0	5.1	2.0	6.5	3.2
12	1.6	0.2	4.8	3.4	4.2	1.5	5.9	3.0	5.3	1.9	6.4	2.7
13	1.4	0.1	4.8	3.0	4.0	1.0	6.0	2.2	5.5	2.1	6.8	3.0
14	1.1	0.1	4.3	3.0	4.7	1.4	6.1	2.9	5.0	2.0	5.7	2.5

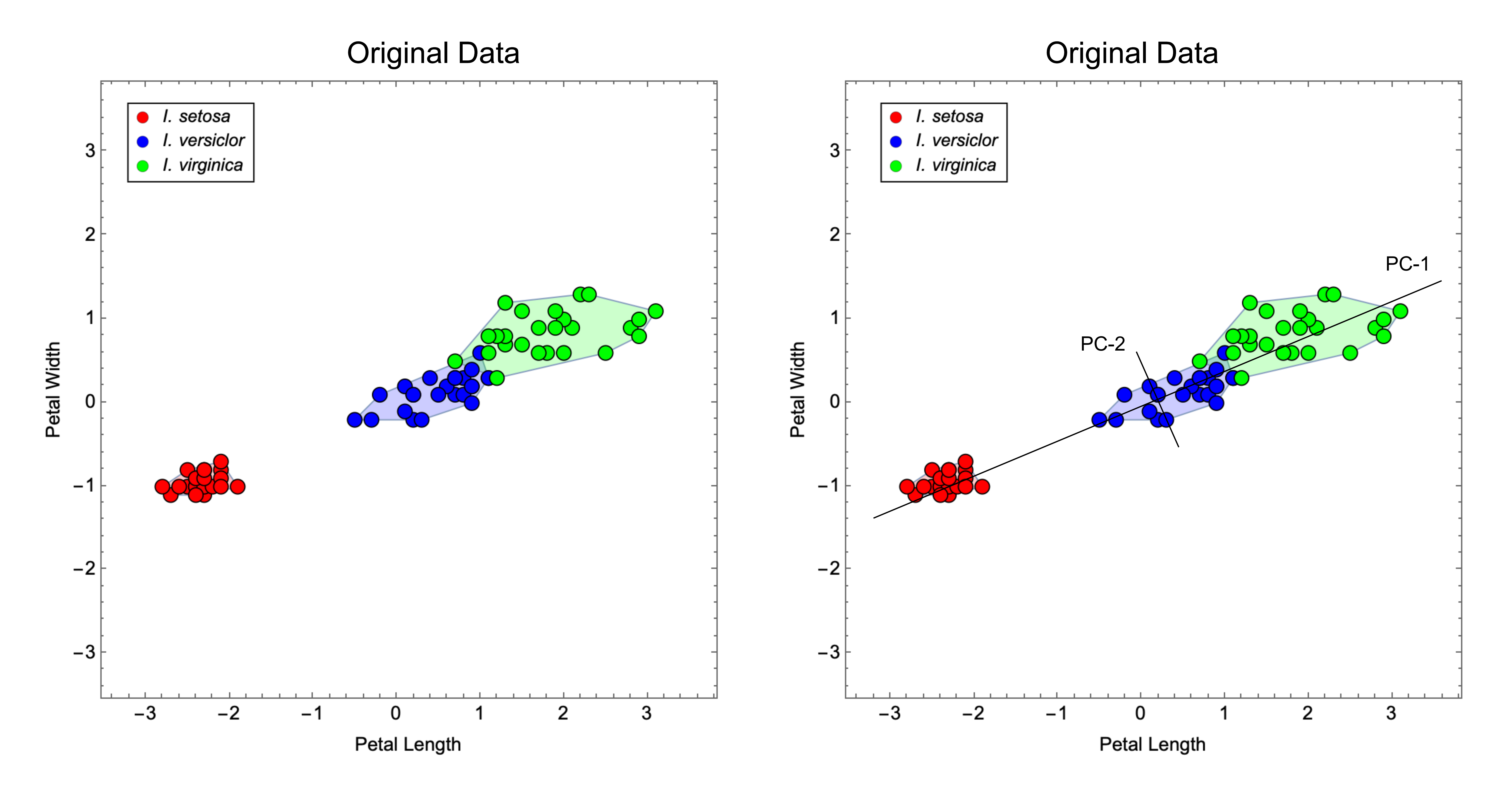
Multi-Group Discriminant Analysis

Fisher's *Iris* Data



Canonical Variates Analysis

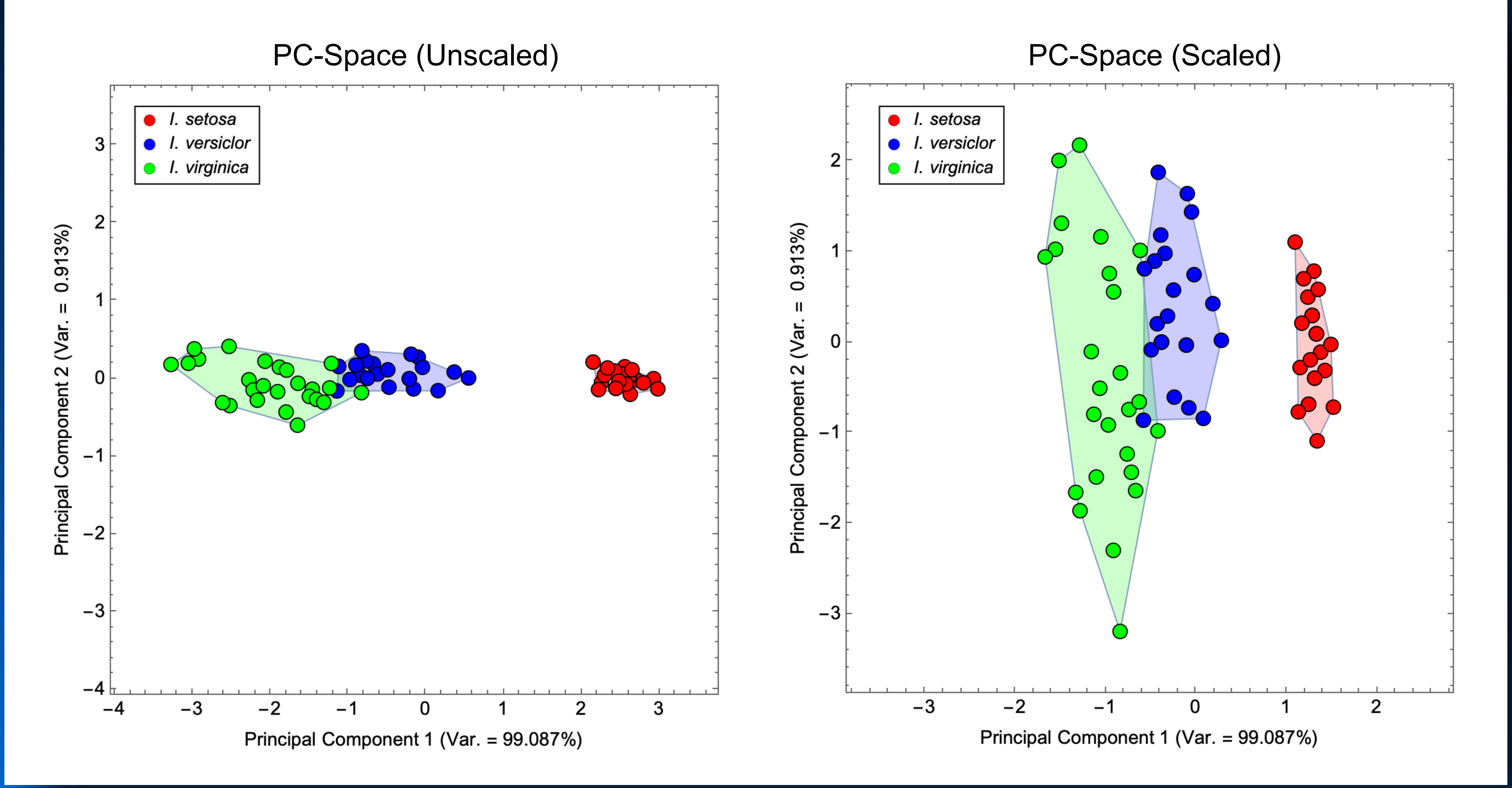
First, Plot Your Data!



Note, this operation already exhibits evidence for differentiation that, in this case, seems to be structured quite obviously along taxonomic lines. At the very least this provides encouragement that a discriminant analysis might be productive.

Canonical Variates Analysis

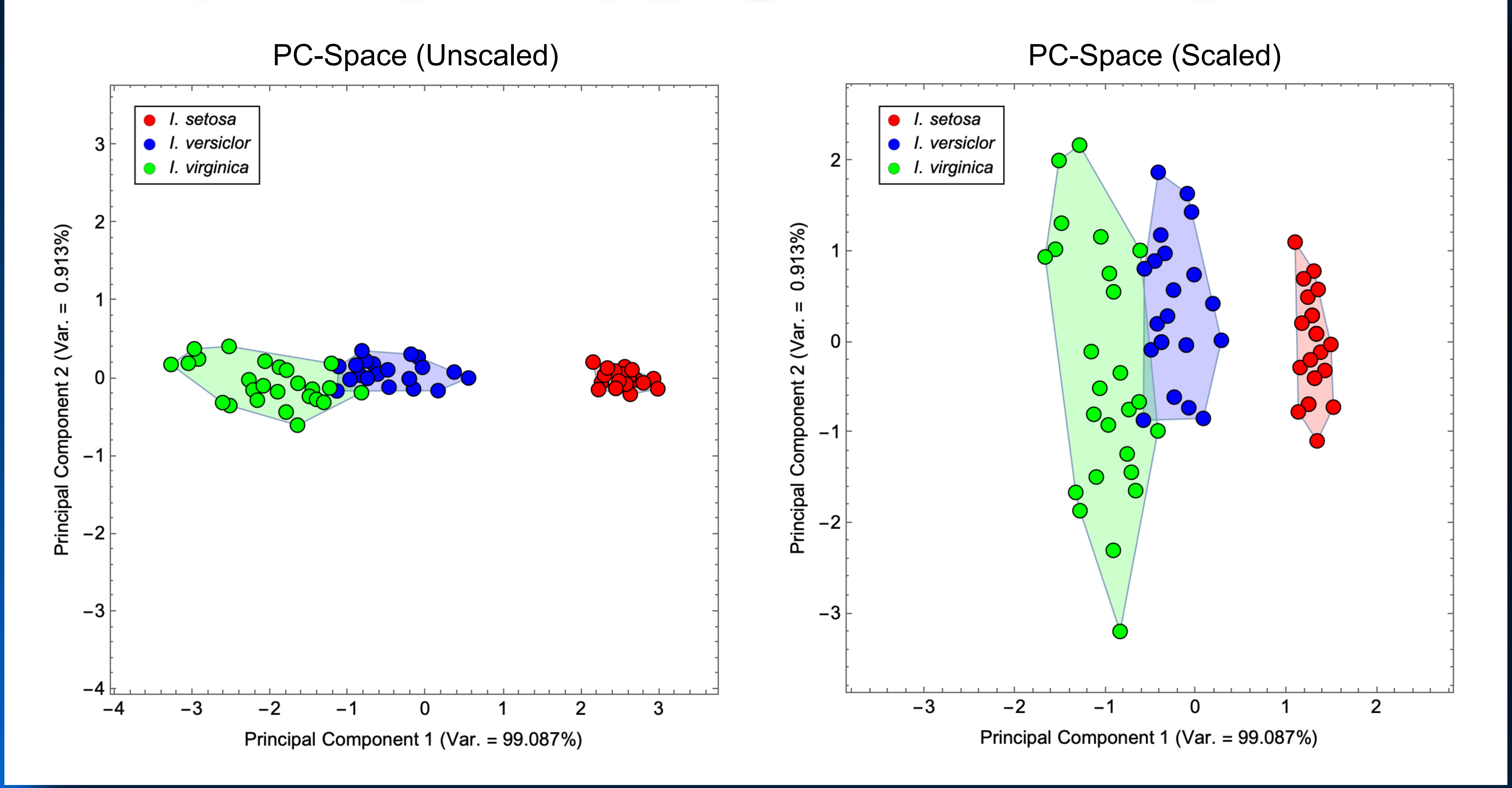
Perform a Standard PCA on the Pooled Data



This operation determines a rotation that aligns the PC axes with orthogonal directions of maximum variation and quantifies variation along those axes.

Canonical Variates Analysis

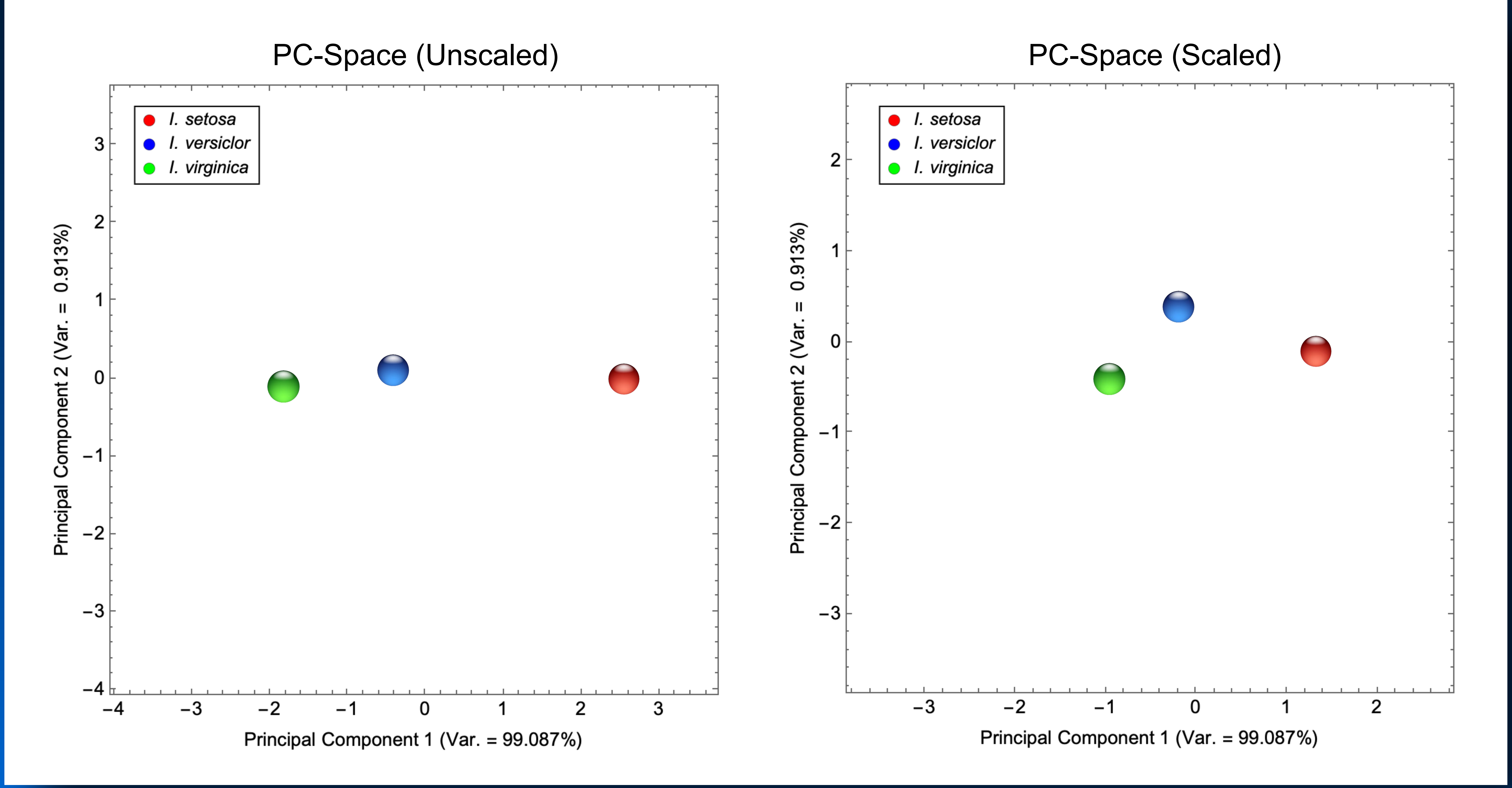
Scale space by multiplying each score by $1/\sqrt{evals}$



This operation distorts the variable space by standardizing the variance along the pooled (within) groups PC axes.

Canonical Variates Analysis

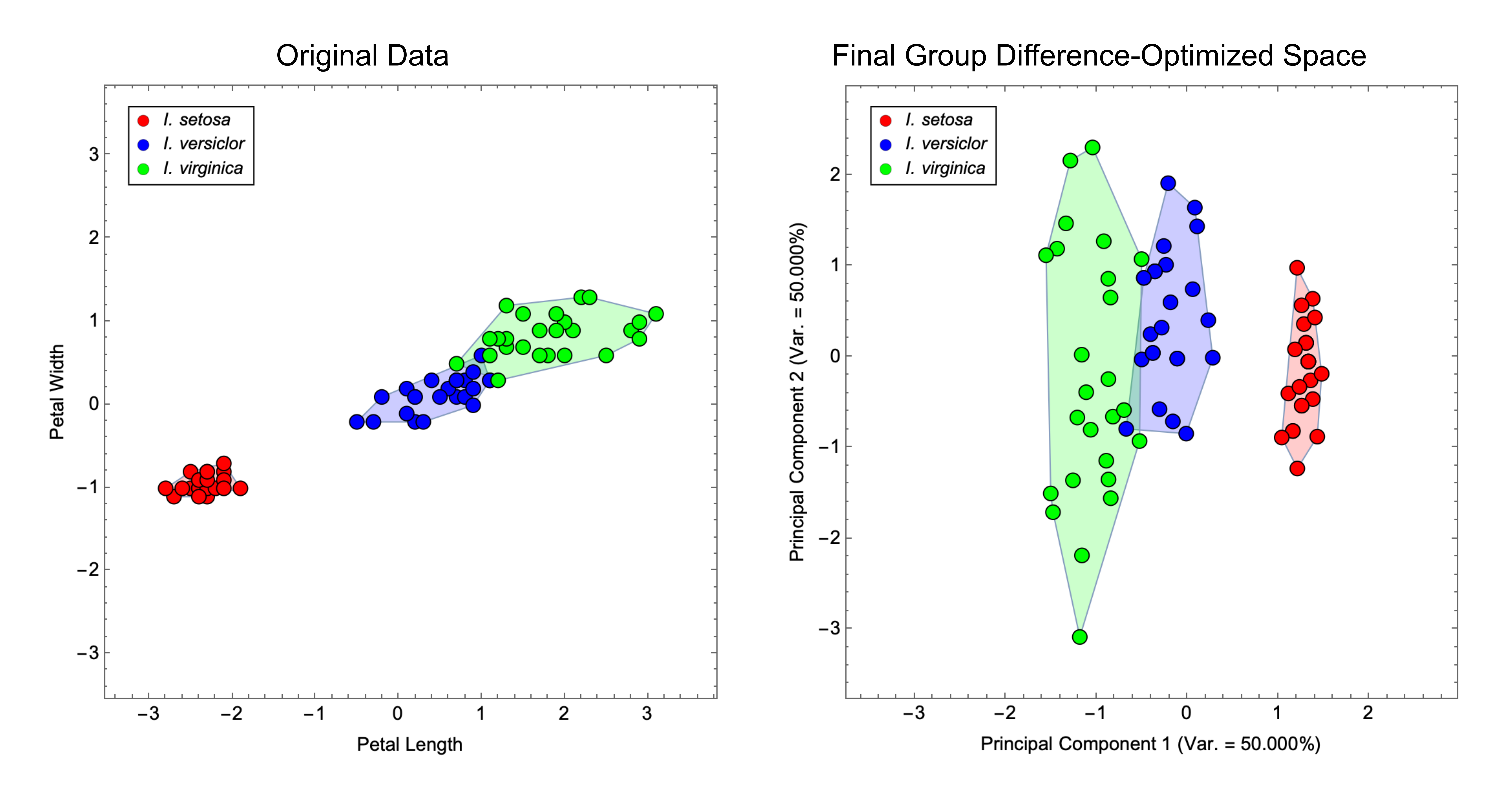
Calculate the group centroids



This operation localizes the positions of the groups relative to each other.

Canonical Variates Analysis

Perform Second PCA on Group Centroids



This operation represents a rotation that determines a set of axes maximally aligned with the between-groups separation. The result of this second rotation is reported as the CVA ordination.

Canonical Variates Analysis

Basic Equations

Grand Mean

$$\hat{X}_j = \frac{\sum_{k=1}^{g} \sum_{i=0}^{n} x_{ijk}}{N}$$

Where: g = no. of groups;
n = no. of objects or subjects in group k;
N = no. of objects or subjects in all groups.

Total Covariance (SSQ)

$$S_{pq} = \sum_{k=1}^{g} \sum_{i=0}^{n} (x_{ijk} - \hat{X}_p) \cdot (x_{ijk} - \hat{X}_q)$$

Canonical Variates Analysis

Basic Equations

$$S_{Total} = S_{Between} + S_{Within}$$

$$T = B + W$$

Where: T = total covariance matrix;

B = between-groups covariance matrix;

W = within-groups covariance matrix.

Canonical Variates Analysis

Basic Equations

Within-Groups Covariance (SSQ) Matrix

$$COV_{pq} = \sum_{k=1}^{g} \sum_{i=1}^{n} (x_{ijk} - \bar{X}_p) \cdot (x_{ijk} - \bar{X}_q)$$

Between-Groups Covariance (SSQ) Matrix

$$COV_{pq} = \sum_{k=1}^{g} n_k \cdot (\bar{X}_{pk} - \hat{X}_p) \cdot (\bar{X}_{qk} - \hat{X}_q)$$

Canonical Variates Analysis

Fisher's *Iris* Data

Total Covariance Matrix

	Petal Length (mm)	Petal Width (mm)	Sepal Length (mm)	Sepal Width (mm)
Petal Length (mm)	54.3312	-3.8012	98.9784	38.6452
Petal Width (mm)	-3.8012	16.2379	-29.6284	-10.7052
Sepal Length (mm)	98.9784	-29.6284	243.769	99.6664
Sepal Width (mm)	38.6452	-10.7052	99.6664	43.8192

Within-Groups Covariance Matrix

	Petal Length (mm)	Petal Width (mm)	Sepal Length (mm)	Sepal Width (mm)
Petal Length (mm)	23.6424	8.3780	15.4284	3.1996
Petal Width (mm)	8.3780	9.3760	4.6492	3.0996
Sepal Length (mm)	15.4284	4.6492	15.6864	3.3108
Sepal Width (mm)	3.1996	3.0996	3.3108	2.8456

Canonical Variates Analysis

Fisher's *Iris* Data

Total Covariance Matrix

	Petal Length (mm)	Petal Width (mm)	Sepal Length (mm)	Sepal Width (mm)
Petal Length (mm)	54.3312	-3.8012	98.9784	38.6452
Petal Width (mm)	-3.8012	16.2379	-29.6284	-10.7052
Sepal Length (mm)	98.9784	-29.6284	243.769	99.6664
Sepal Width (mm)	38.6452	-10.7052	99.6664	43.8192

Between-Groups Covariance Matrix

	Petal Length (mm)	Petal Width (mm)	Sepal Length (mm)	Sepal Width (mm)
Petal Length (mm)	30.6888	-12.1792	83.5500	35.4456
Petal Width (mm)	-12.1792	6.8619	-34.2776	-13.8048
Sepal Length (mm)	83.5500	-34.2776	228.082	96.3556
Sepal Width (mm)	35.4456	-13.8048	96.3556	40.9736

Canonical Variates Analysis

Fisher's *Iris* Data

Inverse of Pooled Within-Groups Covariance Matrix (W^{-1})

	Petal Length (mm)	Petal Width (mm)	Sepal Length (mm)	Sepal Width (mm)
Petal Length (mm)	0.1683	-0.1100	-0.1568	0.1130
Petal Width (mm)	-0.1100	0.2412	0.0875	-0.2409
Sepal Length (mm)	-0.1568	0.0875	0.2318	-0.1888
Sepal Width (mm)	0.1130	-0.2409	-0.1888	0.7065

Canonical Variates Basis Matrix ($W^{-1} . B$)

	Petal Length (mm)	Petal Width (mm)	Sepal Length (mm)	Sepal Width (mm)
Petal Length (mm)	-2.5889	-7.5359	6.8000	15.6641
Petal Width (mm)	-7.5359	3.3187	-2.8302	-6.3084
Sepal Length (mm)	6.8000	-2.8302	18.5856	42.6962
Sepal Width (mm)	15.6641	-6.3084	42.6962	18.0802

Canonical Variates Analysis

Fisher's *Iris* Data

Canonical Variates Basis Matrix ($W^{-1} \,.\, B$)

	Petal Length (mm)	Petal Width (mm)	Sepal Length (mm)	Sepal Width (mm)
Petal Length (mm)	-2.5889	-7.5359	6.8000	15.6641
Petal Width (mm)	-7.5359	3.3187	-2.8302	-6.3084
Sepal Length (mm)	6.8000	-2.8302	18.5856	42.6962
Sepal Width (mm)	15.6641	-6.3084	42.6962	18.0802

All previously illustrated rotations and scalings are accomplished by undertaking an eigenanalysis of the $W^{-1}B$ matrix

Canonical Variates Analysis

Fisher *Iris* Data

Eigenvalue (λ) Table

Canonical Variates	Eigenvalues	Percent Variance	Cum. Percent Variance
1	37.161	99.373	99.373
2	0.234	0.627	100.000

Because only three groups are involved only two canonical vacates are calculated. Owing to rounding error very small higher level eigenvalues may be reported. These should be ignored along with any negative eigenvalues.

Canonical Variates Analysis

Fisher *Iris* Data

Eigenvector (U) Table

Variables	Canonical Variate 1	Canonical Variate 2
Petal Length (mm)	0.137	-0.225
Petal Width (mm)	0.403	0.746
Sepal Length (mm)	-0.361	-0.068
Sepal Width (mm)	-0.830	0.623

These are the cosines that relate the orientation of the eigenvector axes to the variables from which they were calculated. Recall, however, that this basis matrix was the $W^{-1}.B$ matrix, not the matrix of original data values. Thus, these eigenvectors are orthogonal to the variables of the transformed data space, not the original variable space.

Canonical Variates Analysis

Calculation of Canonical Variate Scores

$$CV_{scores} = X \,.\, U'$$

Where: X = original data matrix;
U = eigenvectors of the $W^{-1} \,.\, B$ matrix.

This calculation needs to employ the original data in the form it had when it was used to calculate the T, W and B matrices.

Canonical Variates Analysis

Fisher *Iris* Data: Canonical Variate Scores

I. setosa

n	CV-1	CV-2
1	1.780	0.033
2	1.551	-0.295
3	1.640	-0.094
4	1.514	-0.160
5	1.806	0.130
6	1.707	0.368
7	1.588	0.133
8	1.690	-0.026
9	1.442	-0.257
10	1.638	-0.289
11	1.865	0.108
12	1.626	0.012
13	1.620	-0.335
14	1.660	-0.202
15	2.149	0.262
16	2.022	0.688
17	1.852	0.395
18	1.697	0.095
19	1.791	0.164
20	1.781	0.312
21	1.672	-0.129
22	1.658	0.300
23	1.896	0.247
24	1.342	0.050
25	1.518	-0.008

I. virginica

n	CV-1	CV-2
1	-0.270	-0.094
2	-0.362	0.117
3	-0.479	-0.097
4	-0.501	-0.443
5	-0.546	-0.211
6	-0.453	-0.149
7	-0.491	0.263
8	-0.041	-0.373
9	-0.326	-0.283
10	-0.428	-0.008
11	-0.261	-0.707
12	-0.403	0.101
13	-0.225	-0.817
14	-0.514	-0.115
15	-0.101	0.009
16	-0.243	-0.081
17	-0.552	0.148
18	-0.087	-0.405
19	-0.793	-0.584
20	-0.205	-0.434
21	-0.788	0.396
22	-0.218	-0.205
23	-0.803	-0.410
24	-0.388	-0.314
25	-0.245	-0.218

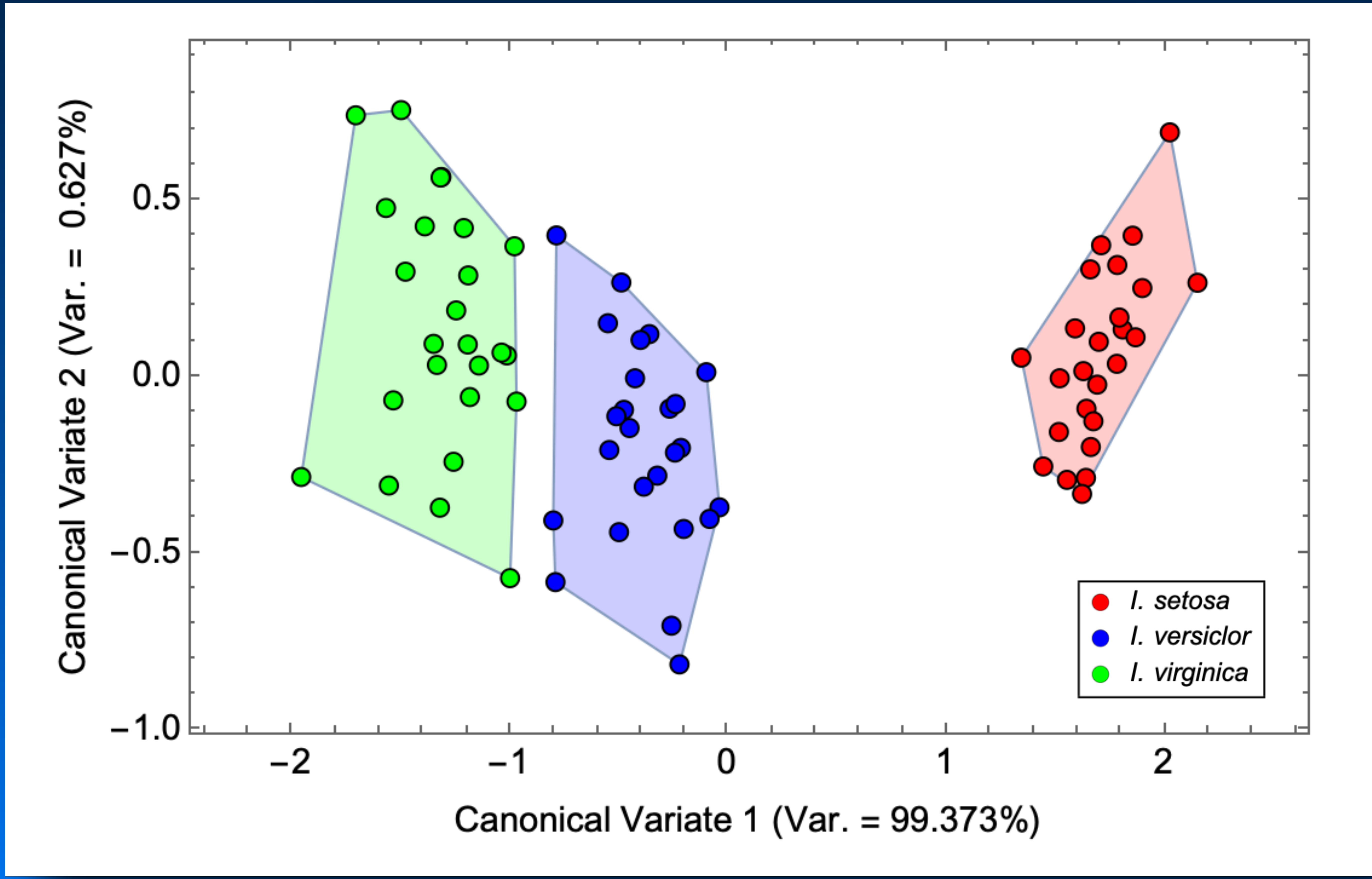
I. versicolor

n	CV-1	CV-2
1	-1.708	0.736
2	-1.195	0.088
3	-1.351	0.089
4	-1.143	0.028
5	-1.480	0.294
6	-1.536	-0.070
7	-1.015	0.057
8	-1.260	-0.244
9	-1.322	-0.374
10	-1.500	0.750
11	-0.980	0.365
12	-1.185	-0.061
13	-1.248	0.184
14	-1.336	0.030
15	-1.569	0.474
16	-1.315	0.561
17	-1.040	0.065
18	-1.319	0.560
19	-1.958	-0.287
20	-1.001	-0.573
21	-1.392	0.422
22	-1.192	0.283
23	-1.556	-0.311
24	-0.971	-0.074
25	-1.213	0.417

Canonical Variates Analysis

Fisher *Iris* Data

Canonical Variates Score Plot



Dimensionality Reduction (Cont.)

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